The basis choice and the reconstruction of combined refractive-kinetic inhomogeneities in the problems of ocean acoustic tomography

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The characteristics of the acoustical signal in ocean are determined by both the refractive inhomogeneities (the perturbation of sound velocity) and the presence of ocean currents. The methods of acoustical tomography can be applied to the simultaneous reconstruction of both refractive and kinetic inhomogeneities. In this paper the problem of the combined refractive-kinetic inhomogeneites reconstruction by the tomography methods is considered. For the realization of proposed scheme so called band basis consisted of a number of intersected stripes is applied. The advantage of the band basis in tomographic applications is conditioned by the simplicity of solving the direct problem and the possibility of describing all types of inhomogeneities in a unified manner. The results of the reconstruction of model combined refractive-kinetic inhomogeneities in band basis are presented. The possibility of complete tomography reconstruction of two-dimensional flows based on the scattering data only is illustrated.

1 Introduction

The characteristics of an acoustic signal transmitted through the oceanic medium are determined both by the parameters of refraction inhomogeneities (e.g., deviations of the velocity of sound $\Delta c(\mathbf{r})$), and by the presence of liquid flows with velocity $\mathbf{v}(\mathbf{r})$ in the medium [1]. This allows the application of tomographic methods for the reconstruction of combined scalar–vector inhomogeneities. For the first time, this approach to the problem of monitoring the oceanic medium was considered in [2] in the framework of the ray representation of acoustic field in the simple time-of-flight geometry. Later, the diffraction tomography methods, which are based on the use of the amplitude and phase of the scattered acoustic field as input data, were developed [3-5].

The solution of sound propagation problem in three-dimensional ocean can be reduced to a two-dimensional consideration. For example in the "vertical modes-horizontal rays" representation of acoustical field and valid adiabatic approximation the two-dimensional independent problems for single modes propagation along horizontal rays can be considered. Below the tomography problem is developed in horizontal plane.

2 Reconstruction of sound speed and current velocity: peculiar properties

The existing schemes of tomographic reconstruction of two-dimensional combined inhomogeneities are based on the separation of their effects and a subsequent separate reconstruction of $\Delta c(\mathbf{r})$ and $\mathbf{v}(\mathbf{r})$ [3-5], where $\mathbf{r} = \{x, y\}$ is the radius vector in the (x, y) plane. The reconstruction of the inhomogeneity of sound speed $\Delta c(\mathbf{r})$ is performed by the standard scalar tomography methods. However, the reconstruction of the flow velocity vector encounters some difficulties. In the general case, when the components $\mathrm{v}_x(\mathbf{r})$ and $\mathrm{v}_y(\mathbf{r})$ of the flow velocity vector $\mathbf{v}(\mathbf{r})$ are independent functions, the problem of reconstructing the moving inhomogeneities in terms of ray representation is underdetermined [4-5], whereas the methods of wave (diffraction) tomography allow an unambiguous reconstruction of the total field $\mathbf{v}(\mathbf{r})$ [3-5]. The tomographic reconstruction of vector inhomogeneities is considerably simplified when the flow of an incompressible liquid is considered, which is quite possible in application to many types of motion in the ocean. The incompressibility condition $\operatorname{div} \mathbf{v}(\mathbf{r}) = 0$ represents a direct definition of a solenoidal vector field. In other words, in the incompressible liquid approximation the velocity of large-scale motions in the ocean, including vortices and global currents of the Gulf Stream type, is a solenoidal vector field in a bounded region that does not contain the sources of this field. Then, the total field $\mathbf{v}(\mathbf{r})$ is uniquely determined by the vector potential $\mathbf{\Psi}(\mathbf{r})$: $\mathbf{v}(\mathbf{r}) = \operatorname{curl} \mathbf{\Psi}(\mathbf{r})$. As a result, it becomes unnecessary to reconstruct the potential component of currents, which requires additional measurements of the normal component of the field $\mathbf{v}(\mathbf{r})$ at the boundary of the water area under study [5-6]. Thus, the problem of reconstructing the current of an incompressible liquid is in this case reduced to the determination of its vector potential $\mathbf{\Psi}(\mathbf{r})$, or, in the two-dimensional case, its z component $\mathbf{\Psi}_z(\mathbf{r}) = \Psi_z(x, y)\hat{\mathbf{z}}$, where $\hat{\mathbf{z}}$ is the unit vector that is normal to the (x, y) plane and forms a righthanded set of three vectors with unit vectors $\mathbf{x}$ and $\mathbf{y}$. In ray tomography, the Fourier transform of projection scattering data allows one to determine the spatial spectral components $\widetilde{\mathbf{\Psi}}_z(\mathbf{k})$ [6] of the vector potential $\mathbf{\Psi}_z(\mathbf{r})$, i.e., to estimate the two-dimensional spatial spectrum $\widetilde{\mathbf{\Omega}}_z(\mathbf{k})$ of vorticity $\mathbf{\Omega}_z(\mathbf{r})$ (curl $\mathbf{v}(\mathbf{r})$ = $\mathbf{\Omega}_z(\mathbf{r})$, $\widetilde{\mathbf{\Omega}}_z(\mathbf{k}) = k^2 \widetilde{\mathbf{\Psi}}_z(\mathbf{k})$, $k^2 = |\mathbf{k}|^2$. After this, the flow velocity field $\mathbf{v}(\mathbf{r})$ is estimated as $\mathbf{v}(\mathbf{r}) = \operatorname{curl} \mathbf{\Psi}_z(\mathbf{r})$, or, through the inverse Fourier transform of its spectral components $\widetilde{\mathbf{v}}(\mathbf{k})$: $\widetilde{\mathbf{v}}(\mathbf{k}) = i[\mathbf{k} \times \widetilde{\mathbf{\Omega}}_z(\mathbf{k})]/k^2 = i[\mathbf{k} \times \widetilde{\mathbf{\Psi}}_z(\mathbf{k})]$. The results obtained for the ray problem are generalized to the case of the wave (diffraction) tomography [3,5]: the Fourier transform of the scattering data in the domain of wave vectors $\mathbf{k}$ allows one to estimate the spatial spectra $\widetilde{\mathbf{\Psi}}_z(\mathbf{k})$ and $\widetilde{\mathbf{\Omega}}_z(\mathbf{k})$, which uniquely determine the solenoidal field of the liquid flow velocity. Thus, the incompressibility condition imposed on the liquid makes it possible to perform the reconstruction of the total field of its flow velocity with both ray and wave representations of acoustic field by using only the scattering data. This fact considerably simplifies the construction of the tomographic scheme and the related mathematics, because, in this case, first, no additional measurements of the normal velocity component are required at the perimeter of the water area (which earlier seemed to be necessary) and, second, it is possible to describe all the kinetic inhomogeneities (vortices and global currents), as well as the refraction parameters of the ocean, in the framework of a single representation. In the present paper, we propose an algorithm of a combined tomographic reconstruction of two-dimensional scalar and vector inhomogeneities in the incompressible liquid approximation on the basis of their simple and concise representation. For this purpose, we use a nonorthogonal redundant basis with elements in the form of a set of intersecting stripes [7-8]. In [9], it was shown that such a basis is rather convenient for solving tomographic problems, because it facilitates the solution of the direct problem in constructing the perturbation matrix, i.e., the problem of the inhomogeneity reconstruction.

3 The basis choice

The combined refractive-kinetic inhomogeneities can be reconstructed by the suggested way both in the ray and wave presentation of the acoustic field expanding the characteristics upon the some basis. In solving the tomographic problem, we consider a circular water area with transmitting–receiving devices positioned along its perimeter. We assume that each of the sources in turn emits a signal, which is received by all of the receivers.

3.1 Ray presentation

In ray presentation the presence of an inhomogeneity distorts the ray trajectories and causes additional time delays, which serves as the initial data for the tomographic reconstruction of the inhomogeneity. The perturbation of the signal travel time along the ray connecting the *i*-th source–receiver pair due to the presence of flow velocity inhomogeneity $\Delta\mathbf{v}(\mathbf{r}) = \mathbf{v}(\mathbf{r}) - \mathbf{v}_0(\mathbf{r})$ ($\mathbf{v}_0(\mathbf{r}) \equiv 0$) and sound speed perturbation $\Delta c(\mathbf{r}) = c(\mathbf{r}) - c_0(\mathbf{r})$ with respect to their background values $\mathbf{v}_0(\mathbf{r})$ and $c_0(\mathbf{r})$ in case of $|\mathbf{v}(\mathbf{r})|/c_0(\mathbf{r}) << 1$ and $|\Delta c(\mathbf{r})|/c_0(\mathbf{r}) << 1$ has the form

$$\Delta t_i = -\int_{L_i^0} \frac{\Delta c(\mathbf{r})\, dl^0}{c_0^2(\mathbf{r})} - \int_{L_i^0} \frac{\mathbf{v}(\mathbf{r})\boldsymbol{\tau}_i^0(\mathbf{r})\, dl^0}{c_0^2(\mathbf{r})} \qquad (1)$$

where $\boldsymbol{\tau}_i^0(\mathbf{r})$ is the unit vectors tangential to the trajectory, L_i^0 in the unperturbed media. Performing sequential measurements with all of the source–receiver pairs, we obtain a set of measured quantities Δt_i, which serve as the input data for determining the unknown $\mathbf{v}(\mathbf{r})$ and $\Delta c(\mathbf{r})$. At one of other stage of solving integral equations (1) written for different pairs *i*, their algebraization is performed. In this case, the characteristics of the medium are represented in the form of linear combinations of a finite number of scalar $\Theta_j(\mathbf{r})$, $j = \overline{1, J}$ (for describing $\Delta c(\mathbf{r})$) and vectorial $\boldsymbol{\Theta}_k(\mathbf{r})$, $k = \overline{1, K}$ (for describing the currents and vortices) basis functions, not necessarily orthogonal but sufficiently complete for reconstructing the inhomogeneities with required accuracy. Then, $\Delta c(\mathbf{r})$ and $\mathbf{v}(\mathbf{r})$ are represented as

$$\Delta c(\mathbf{r}) = \sum_{j=1}^{J} x_j' \Theta_j(\mathbf{r}), \quad \mathbf{v}(\mathbf{r}) = \sum_{k=1}^{K} x_k'' \boldsymbol{\Theta}_k(\mathbf{r}) \qquad (2)$$

and the system of equations (1) determined for all the *i*-th source–receiver pairs takes the form $\Delta t_i = \sum_{j=1}^{J} A_{ij}' x_j' + \sum_{k=1}^{K} A_{ik}'' x_k'' = \sum_{m=1}^{M} A_{im} x_m$, $M = J + K$. For convenience, below we use the representation in the matrix form in terms of the Dirac notation: $|\ \rangle$ for a column vector and $\langle\ |$ for a row vector. In this form, the system of equations under study has the form

$$A\ |X\rangle = |\Delta T\rangle \qquad (3)$$

where the perturbation matrix $A = [A'\ \ A'']$ consists of two blocks with the elements $A_{ij}' = -\int_{L_i^0} \Theta_j(\mathbf{r}) c_0^{-2}(\mathbf{r}) dl$, $A_{ik}'' = -\int_{L_i^0} (\boldsymbol{\Theta}_k(\mathbf{r})\boldsymbol{\tau}^0(\mathbf{r})) c_0^{-2}(\mathbf{r}) dl$. The column vector $|\Delta T\rangle$ consists of the values of signal delay times Δt_i, and the vector $|X\rangle$ consists of unknown dimensionless expansion coefficients of the reconstructed inhomogeneity in terms of the basis functions $\Theta_j(\mathbf{r})$ and $\boldsymbol{\Theta}_k(\mathbf{r})$. The solution of system (3) is obtained by the least squares method (LSM) and can be regularized in the simplest case by adding a unit matrix *E* with weighting factor γ^2 to A^+A. As a result, the solution takes the form

$$|\hat{X}\rangle = (A^+ A + \gamma^2 E)^{-1} A^+ |\Delta T\rangle, \qquad (4)$$

where the plus sign denotes Hermitian conjugation and the regularizing coefficient γ^2 controls the balance between the minimization of the norm of the solution and the norm of discrepancy. Thus, integral equations (1) are reduced to systems of linear algebraic equations in unknown coefficients of expansion of the functions $\Delta c(\mathbf{r})$ and $\mathbf{v}(\mathbf{r})$ in terms of the chosen bases. It is important that, unlike [6], in the proposed approach, we do not explicitly separate the effects of refraction and liquid flow described by the scalar and vector potentials with a subsequent separate reconstruction of the solenoidal and vortex-free components of the flow velocity. Instead, we solve the problem of a complete reconstruction in a single procedure for all of the inhomogeneity components responsible for the perturbations observed in the received data. In the process of solution, it is unnecessary to operate with spatial spectra in the explicit form. The perturbation matrix *A* in Eq. (3), which is constructed for a sufficiently large number of transmission and reception aspects, in the course of the solution takes into account all the reconstruction possibilities considered in [6].

When solving a specific tomographic problem, it is important to choose the basis functions that are adequate to the problem, so that they allow one to describe with sufficient accuracy the distributions of $\Delta c(\mathbf{r})$ and $\mathbf{v}(\mathbf{r})$ with minimal requirements imposed on both the algorithmic part of the reconstruction process and the practical realization of the process of data acquisition and processing at a given accuracy of data. Below, in reconstructing the inhomogeneities, we use a stripe basis [8], which has the form of a set of parallel stripes uniformly covering the region under tomography and rotated at a constant angular step within the interval from 0 to π. The basis functions of perturbations, $\Theta_j(\mathbf{r})$ and $\boldsymbol{\Theta}_k(\mathbf{r})$, are preset within the stripes, where their values are assumed to be constant. The comparison of the stripe basis with conventional ones (of the type of nonoverlapping squares densely covering the region under consideration) showed that they are equally effective from the viewpoint of the quality of inhomogeneity reconstruction [9]. However, the use of the stripe basis has some advantages from the mathematical point of view: the rigorous completeness and orthogonality of the basis are not necessary, the construction of the perturbation matrix is simplified, the description of all the types of inhomogeneities, including the vector ones, is unified, and the continuity condition for the incompressible liquid (which is necessary for a unique reconstruction of vector inhomogeneities) is automatically taken into account.

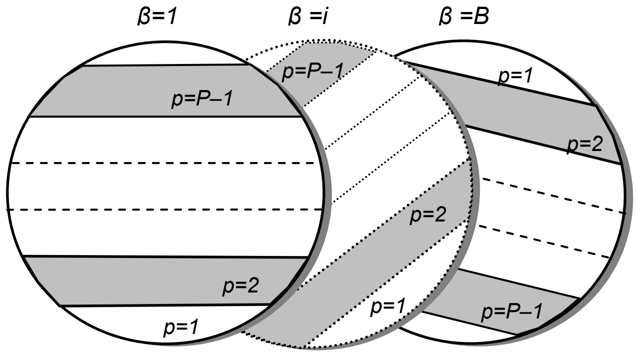

Fig.1 Strip basis.

As the input data for solving the tomographic problem in a given region, the background values of the parameters of the medium $c_0(\mathbf{r})$ and $\mathbf{v}_0(\mathbf{r})$, for example, the season-average values, may be used. In each stripe, the perturbation of sound speed is determined in the form of a base function $\Theta_j(\mathbf{r})$; it causes a perturbation of the received data (a variation of ray travel times or a resulting field perturbation), which is calculated for each of the source–receiver pair. Such calculations performed for all the basis stripes oriented at all the angles leads to the perturbation matrix A' determining the effect of scalar inhomogeneities $\Delta c(\mathbf{r})$ in Eq. (1). The perturbation of the flow velocity in the form of basis functions $\mathbf{\Theta}_k(\mathbf{r})$ determines the perturbation matrix A'' describing the effect of the vector inhomogeneities $\Delta\mathbf{v}(\mathbf{r})$ in Eq. (1). It is assumed that the flow direction $\mathbf{v}_k(\mathbf{r})$ coincides with the direction of the stripes, and the magnitude of its velocity is the same for all the basis functions. Solution (4) of system (3) yields the expansion coefficients $|X\rangle$ in terms of the bases introduced in Eqs. (2). The inhomogeneity is estimated using Eqs. (2) by weighted summation of perturbations in the stripes, i.e., by summation of the basis perturbations with weighting factors equal to the values of the reconstructed coefficients $|X\rangle$. To improve the quality of reconstruction, one can use a priori information about the desired inhomogeneities. Since real oceanic inhomogeneities are characterized by a smooth structure without sharp boundaries, the spatial spectrum of the reconstructed inhomogeneity can be corrected by filtering.

3.2 Wave presentation

The ray representation of acoustic field is approximate. The wave approach is more rigorous, exact, and practically realizable in the low-frequency range. The diffraction tomography is based on the wave equation. Within the first approximation by the Mach number ($|\mathbf{v}|/c_0$) and under the assumption that the flow is quasi-stationary, the wave equation for an inhomogeneous moving medium that is written for an acoustic field harmonic in time, $U(\mathbf{y},\mathbf{y}')$, is reduced to the Helmholtz equation:

$$\nabla^2 U(\mathbf{y},\mathbf{y}') + k_0^2(\mathbf{r})U(\mathbf{y},\mathbf{y}') = \varepsilon(\mathbf{r})U(\mathbf{y},\mathbf{y}') - \\ -\frac{2i\omega_0}{c^2(\mathbf{r})}\mathbf{v}(\mathbf{r})\nabla U(\mathbf{y},\mathbf{y}'). \qquad (5)$$

Here, $\mathbf{y}$ and $\mathbf{y}'$ are the points of reception and transmission, $k_0(\mathbf{r}) = \omega_0/c_0(\mathbf{r})$ is the wave number in the background medium, $\varepsilon(\mathbf{r}) = k_0^2(\mathbf{r}) - k^2(\mathbf{r})$ is the scatterer function, and the time dependence is ~exp(–*iωt*). The solution of Eq. (5) for the total field $U(\mathbf{y},\mathbf{y}')$, which consists of the incident field $U_0(\mathbf{y},\mathbf{y}')$ and the field $\Delta U(\mathbf{y},\mathbf{y}')$ scattered from the inhomogeneities of the medium that are localized in the region $\Re$, can be expressed through the Lippmann–Schwinger equation, which in the Born approximation has the form

$$U(\mathbf{y},\mathbf{y}') = U_0(\mathbf{y},\mathbf{y}') + \Delta U(\mathbf{y},\mathbf{y}') = U_0(\mathbf{y},\mathbf{y}') + \\ + \int_{\Re} G(\mathbf{y},\mathbf{r})\left[\varepsilon(\mathbf{r}) - \frac{2\omega_0}{c^2(\mathbf{r})}\left(\mathbf{v}(\mathbf{r})\mathbf{k}_0(\mathbf{r})\right)\right]U_0(\mathbf{r},\mathbf{y}')d\mathbf{r}. \qquad (6)$$

Here, $G(\mathbf{y},\mathbf{r})$ is the Green function for the background medium and $\mathbf{k}_0(\mathbf{r})$ is the wave vector determined by the direction of propagation of the incident wave. Then, we consider a system of equations of the type of Eq. (6) that is written for a given number of source–receiver pairs. Under the assumption that the deviation $\Delta c(\mathbf{r})/c_0(\mathbf{r})$ is small, the scatterer function $\varepsilon(\mathbf{r}) = \omega^2\left(\frac{1}{c_0^2(\mathbf{r})} - \frac{1}{c^2(\mathbf{r})}\right)$ is represented in the form $\varepsilon(\mathbf{r}) \approx \frac{2\omega^2}{c_0^3(\mathbf{r})}\Delta c(\mathbf{r})$. Expansion ((2) of inhomogeneities $\Delta c(\mathbf{r})$ and $\mathbf{v}(\mathbf{r})$ in terms of the bases $\Theta_j(\mathbf{r})$ and $\mathbf{\Theta}_k(\mathbf{r})$ allows us to obtain a system of linearized equations expressed in the matrix form as

$$\hat{A}\;|X\rangle = |\Delta U\rangle \qquad (7)$$

Here, the perturbation matrix $\hat{A} = \left[\hat{A}'\;\hat{A}''\right]$ consists of two blocks $\hat{A}'_{ij} = \int_{\Re}\frac{2\omega^2}{c_0^3(\mathbf{r})}G(\mathbf{y}_i,\mathbf{r})\Theta_j(\mathbf{r})U_0(\mathbf{r},\mathbf{y}'_i)d\mathbf{r}$ and $\hat{A}''_{ik} = -\int_{\Re}\frac{2\omega_0}{c_0^2(\mathbf{r})}G(\mathbf{y}_i,\mathbf{r})\left(\mathbf{\Theta}(\mathbf{r})\mathbf{k}_0(\mathbf{r})\right)U_0(\mathbf{r},\mathbf{y}'_i)d\mathbf{r}$ with the elements of the vector $|\Delta U\rangle$ consists of the values of acoustic field perturbations. The regularized LSM solution of system (7) is determined by Eq. (4) with the substitution $A \to \hat{A}$, $|\Delta T\rangle \to |\Delta U\rangle$. The use of the perturbation matrix constructed for the given number of insonification aspects inexplicitly takes into account the difference in the character of the effects of scalar and vector inhomogeneity components, thus providing the possibility of reconstructing the incompressible liquid flow velocity and the values of the phase velocity of sound [3, 5]

4 The reconstruction of combined refractive-kinetic inhomogeneities in ray acoustic tomography

The eikonal equation for an inhomogeneous moving medium allows the determination of ray trajectories L_i and the signal delay times Δt_i in the region containing scalar-vector inhomogeneities $\Delta c(\mathbf{r})$, $\mathbf{v}(\mathbf{r})$. We assume that the perturbations are limited in strength and possess no singularities (e.g., causing multipath propagation). This assumption is necessary for the unambiguous construction of the perturbation matrix A and solution of tomographic problem (3). In modeling, we considered a water area of radius $R_a = 10^5$ m surrounded by Z = 18 transmitting–receiving devices uniformly distributed along its perimeter. The perturbation matrix $A = [A' \; A'']$ consists of two separately calculated blocks A' and A''. The unperturbed medium with a constant velocity of sound is immobile. The magnitude of the basis flow velocity was 1 m/s, and the sound velocity perturbation in each of the stripes was assumed to be 5 m/s. The matrix $A = [A' \; A'']$ was determined by the geometric method, and the blocks A' and A'' were constructed using identical numbers of basis stripes P = 8 and angles of their rotation B = 31. The right-hand side of system (3) was determined from the eikonal equation. The regularizing coefficient γ^2 in Eq. (4) was 10^{-3} of the maximum eigenvalue of the matrix A^+A.

Figure 2 shows the results of a simultaneous reconstruction of the components of a combined inhomogeneity in the form of a rectilinear flow, a vortex, and a refraction component. The rectilinear parallel flow with a velocity magnitude of 0.5 m/s and a limited width of $0.3R_a$ is oriented at an angle of $5\pi/4$ to the abscissa axis (the chosen width and rotation angle do not coincide with the corresponding parameters of any of the basis stripes). The Oseen vortex is characterized by the azimuth velocity profile $v(|\mathbf{r}|) = \Omega_0 R_0^2\,[1 - \exp\left(-|\mathbf{r}-\mathbf{r}'|^2 / R_0^2\right)] / |\mathbf{r}-\mathbf{r}'|$ and the parameters Ω_0 = 0.00006 rad/s, $R_0 = 0.3R_a$, and $\mathbf{r}' = \{x', y'\} = \{0.4R_a, -0.4R_a\}$. The refraction inhomogeneity of the phase velocity of sound has a Gaussian form $c(\mathbf{r}) = c_0 + \Lambda \exp\left[-|\mathbf{r}-\mathbf{r}'|^2 / \sigma^2\right]$, Λ = 4.9 m/s, $\sigma = 0.25R_a$, and $\mathbf{r}' = \{-0.5R_a, 0.5R_a\}$.

As one can see from Fig. 2, the structure and the positions of the inhomogeneities under study and the values of the velocity vector for the flow and the vortex are reconstructed with acceptable accuracy.

5 Conclusion

The reconstruction of the flow velocity (both streams and eddies) with refraction inhomogeneities can be done by use of the same scattering data. This fact considerably simplifies the tomographic problem and allows to develop the new scheme for combined refractive-kinetic inhomogeneities reconstruction. It is important that, unlike common methods, in the proposed method the refraction and flow inhomogeneities are reconstructed in the unified approach. The application of the stripe basis, together with the widely discussed possibility of using the ocean noise instead of sound sources, may give the opportunity of development for a new tomographic scheme with reasonable requirements imposed on the practical realization.

Acknowledgments

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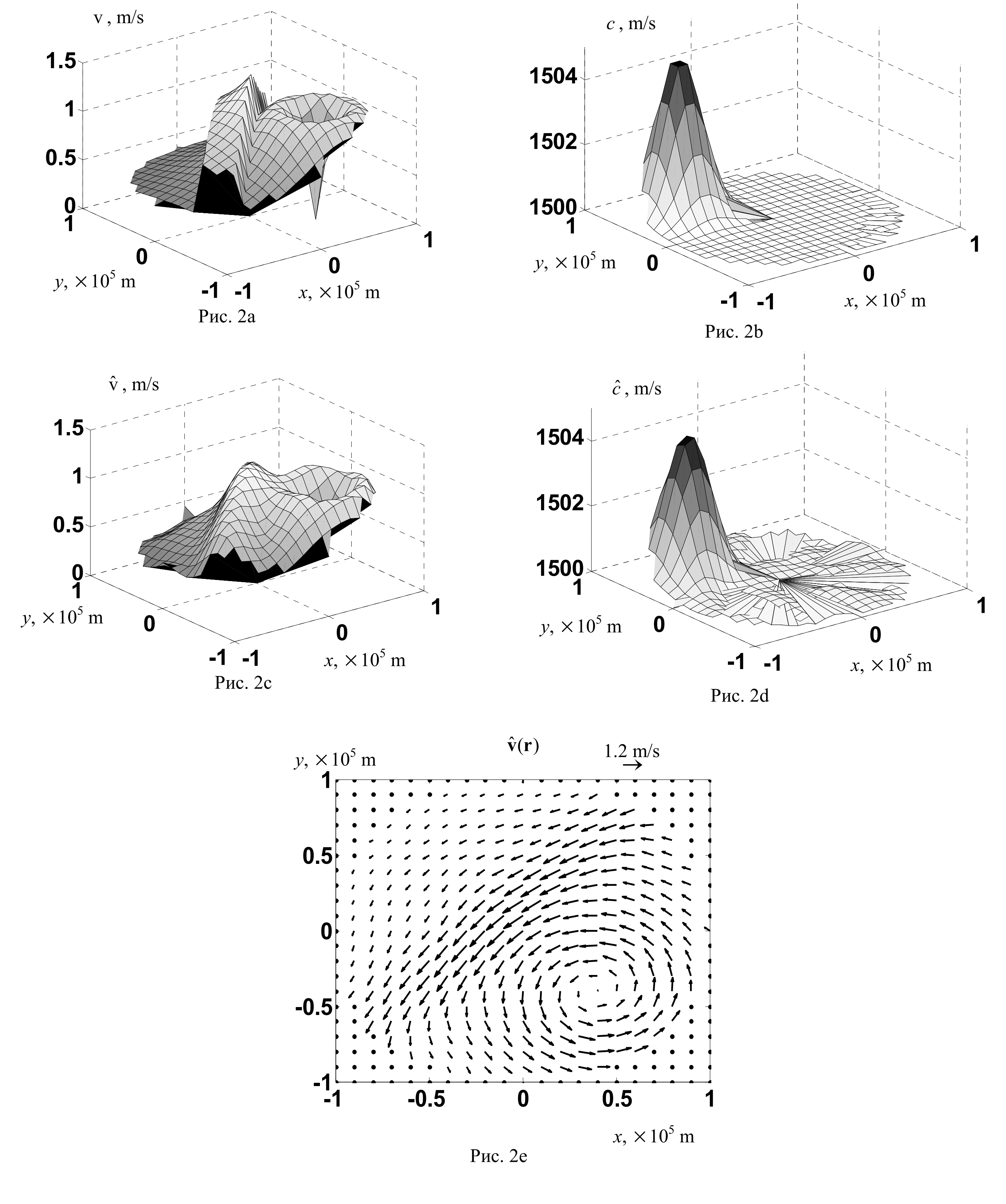

Fig.2 Distributions of the (a) flow and (b) refractive components of a combined inhomogeneity and the results of their reconstruction (c), (d). The reconstructed distribution of the flow velocity vector (e).