The passive mode tomography of the ocean using data from short vertical arrays bent by the ocean currents

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The possible realization of passive ocean tomography based on the widely discussed relation between the Green's function and ambient noise cross-coherence is discussed. The problem is considered in the mode representation of acoustic field in adiabatic approximation. It is shown that the use of the vertical arrays with vector receivers allows a decrease in the accumulation time to one or several hours, depending on the conditions of experiment. The mode structure of acoustic field is determined from the cross-correlation matrix of the noise field received by the hydrophones of short vertical arrays bent by the ocean currents and covering only the part of the sound channel. The proposed algorithm allows a compensation of antenna declination from the vertical profile and takes into account of the finite length of antenna aperture, that ordinary takes place in ocean experiments.

1 Introduction

Nowadays the practical implementation of ocean acoustic tomography is greatly restricted by the technical difficulties such as deployment of long vertical arrays, complexity of precise determination of hydrophones positions in space, problems of low frequency sound radiation and others. All these aspects considerably increase the cost of experiments and complicate its realization. As a result the ocean acoustic tomography is applied only in single scientific-research experiments without real opportunity of realization as day-to-day method for monitoring large ocean areas.

In this paper the attempt to find the solution some of mentioned above problems is made. The problems with the low frequency source could be solved by using the relation between the Green's function and coherence function of the noise field [1]. The key point of this problem is the time of noise field accumulation required for a reliable determination of the Green's function for the purposes of ocean tomography [2]. This question is poorly analyzed in literature and must be additionally investigated.

The solution of sound propagation problem in three-dimensional ocean in mode representation with valid adiabatic approximation can be restricted to the solution of two-dimensional independent problems for single modes propagation. In Section 2 the horizontal planar problem on the determination of the Green's function for a single mode from the noise cross-coherence function is considered. The approach proposed is somewhat different from previous considerations [3]; it is based on the integral expression of the Huygens principle in the Helmholtz formulation (for harmonic signals) with a further extension to the case of broadband processes. The special attention is paid to the determination of the Green's function type (outgoing or incoming), which makes it possible to offer in Section 3 the possibility of reduction of the noise signal accumulation time with the use of vector receivers. In Section 4 in vertical plane the problem of modal processing for noise field received by curved antennas, not covering the whole ocean waveguide, is considered. The developed method provides the information about ocean mode structure that is necessary for the solution of tomography problem. As distinct from the commonly applied methods of instrumental control of antenna declination [4], the method proposed allows algorithmic compensation of antenna curvature without application any additional devises. Below, a circular water area begirt around the periphery with vertical multielement arrays is considered (Fig. 1).

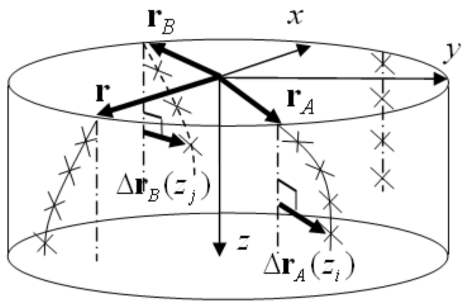

Fig.1 Schematic diagram of the passive mode tomography with short curved antennas.

2 Estimation of the Green's Function from the noise cross-coherence function

It is assumed that the noise field is created by uncorrelated sources uniformly distributed in space. The monochromatic component $U(\mathbf{r},\omega)$ of the signal received at the point $\mathbf{r}=\mathbf{r}_A$ inside the region S (Fig. 2) can be expressed in terms of the Helmholtz–Kirchhoff integral (noise sources in S are neglected in comparison with external ones):

$$U(\mathbf{r}_A)=\oint_L\left[U(\mathbf{r}_L)\frac{\partial}{\partial\mathbf{n}}-\frac{\partial}{\partial\mathbf{n}}U(\mathbf{r}_L)\right]G^{\pm}(\mathbf{r}_A,\mathbf{r}_L)dL,\quad \mathbf{r}_L\in L\ ,\ (1)$$

$\mathbf{n}$ – is the external unit normal to contour L at the current integration point. The close contour L is arbitrary and for simplicity it is a circle with the center at the poin $\mathbf{r}_A$ and the radius $\rho=|\mathbf{r}_A-\mathbf{r}_L|$ (Fig. 2). The functions $G^+(\mathbf{r}_A,\mathbf{r}_L,\omega)$ and $G^-(\mathbf{r}_A,\mathbf{r}_L,\omega)$ are the outgoing and incoming Green's functions, respectively. Below, the dependence on ω is omitted. Equation (1) with function $G^+(\mathbf{r}_A,\mathbf{r}_L)$ describes the field $U(\mathbf{r}_A)$ created by external sources and expressed in terms of the field $U_{\leftarrow}(\mathbf{r}_L)$ entering the region S. On the other hand, consideration of $G^-(\mathbf{r}_A,\mathbf{r}_L)$ in Eq. (1) allows representation of the field $U(\mathbf{r}_A)$ as a partial cause of signal appearance at the points of the circle $\mathbf{r}_L$. In this case Eq. (1) describes $U(\mathbf{r}_A)$ in terms of the field $U_{\rightarrow}(\mathbf{r}_L)$ outgoing from the region S (Fig. 2). Therefore, the choice of the Green's function type governs the propagation direction of the signal between the points under consideration. Using $G^+(\mathbf{r}_A,\mathbf{r}_L)$ in Eq. (1) it could be obtained:

$$<U(\mathbf{r}_A)U^*_{\leftarrow}(\mathbf{r}_B)> = \oint_L [<U_{\leftarrow}(\mathbf{r}_L)U^*_{\leftarrow}(\mathbf{r}_B)> \frac{\partial}{\partial \mathbf{n}} G^{+}(\mathbf{r}_A,\mathbf{r}_L) - G^{+}(\mathbf{r}_A,\mathbf{r}_L) < \frac{\partial U_{\leftarrow}(\mathbf{r}_L)}{\partial \mathbf{n}} U^*_{\leftarrow}(\mathbf{r}_B,)>]dL, \quad (2)$$

where brackets < > denote averaging over an ensemble of realizations of the noise field and the asterisk * denotes the complex conjugation. It is assumed that $k\rho >> 1$, where k is the wave number and the point A is the center of the circle L (Fig. 2). In this case $\frac{\partial}{\partial \mathbf{n}} U_{\leftarrow}(\mathbf{r}_L) \approx -ikU_{\leftarrow}(\mathbf{r}_L)$, $\frac{\partial}{\partial \mathbf{n}} G^{+}(\mathbf{r}_A,\mathbf{r}_L) \approx ikG^{+}(\mathbf{r}_A,\mathbf{r}_L)$ for all $\mathbf{r}_L$, and relation (2) takes the form:

$$<U(\mathbf{r}_A)U^*_{\leftarrow}(\mathbf{r}_B)> \approx 2ik\oint_L <U_{\leftarrow}(\mathbf{r}_L)U^*_{\leftarrow}(\mathbf{r}_B)> G^{+}(\mathbf{r}_A,\mathbf{r}_L)dL. \quad (3)$$

In general case the main contribution to the integral (3) is made by points $\mathbf{r}_L$ situated close by the point $\mathbf{r}_B$, because high-frequency oscillations of the real and imaginary parts of the integrand in Eq. (3) vanishe after integration along the contour L. In this case Eq. (3) can be rewritten as:

$$<U(\mathbf{r}_A)U^*_{\leftarrow}(\mathbf{r}_B)> \approx iD <U_{\leftarrow}(\mathbf{r}_B)U^*_{\leftarrow}(\mathbf{r}_B)> G^{+}(\mathbf{r}_A,\mathbf{r}_B),$$

where $D = 2kL_f$ is the dimensionless coefficient and L_f is the effective length of the contour L close to $\mathbf{r}_B$ along which the noise fields $U(\mathbf{r}_L)$ produce the coherent contribution to the Green's function. Considering the alternative form of Eq. (1) with the incoming Green's function $G^{-}(\mathbf{r}_A,\mathbf{r}_L)$ ($\frac{\partial}{\partial \mathbf{n}} G^{-}(\mathbf{r}_A,\mathbf{r}_L) \approx -ikG^{-}(\mathbf{r}_A,\mathbf{r}_L)$), and selecting the field propagating from $\mathbf{r}_A$ to $\mathbf{r}_B$ ($\frac{\partial}{\partial \mathbf{n}} U_{\rightarrow}(\mathbf{r}_L) \approx ikU_{\rightarrow}(\mathbf{r}_L)$), the relation can be obtained: $<U(\mathbf{r}_A)U^*_{\rightarrow}(\mathbf{r}_B)> \approx -iD <U_{\rightarrow}(\mathbf{r}_B)U^*_{\rightarrow}(\mathbf{r}_B)> G^{-}(\mathbf{r}_A,\mathbf{r}_B)$. Without special selection of the direction of noise signal arrivals, the coherence function of the fields will allow estimating the additive combination of the Green's functions for opposite directions of signal propagation between the hydrophones:

$$<U(\mathbf{r}_A)U^*(\mathbf{r}_B)> \approx iD\,[G^{+}(\mathbf{r}_A,\mathbf{r}_B) - G^{-}(\mathbf{r}_A,\mathbf{r}_B)] <U(\mathbf{r}_B)U^*(\mathbf{r}_B)>. \quad (4)$$

In the case of a broadband noise, one must consider the wave equation whose Green's function $G^{\pm}(\mathbf{r}_A,\mathbf{r}_B,t)$ is related to the Green's function of the Helmholtz equation $G^{\pm}(\mathbf{r}_A,\mathbf{r}_B,\omega)$ through the Fourier transform. Multiplying both sides of Eq. (4) by $\frac{1}{2\pi}e^{-i\omega\tau}$ and integrating the result with respect to ω within infinite limits, one can obtain

$$<U(\mathbf{r}_A,t)U^*(\mathbf{r}_B,t-\tau)> \approx iD\int_{-\infty}^{\infty}[G^{+}(\mathbf{r}_A,\mathbf{r}_B,\tau-\eta) - G^{-}(\mathbf{r}_A,\mathbf{r}_B,\tau-\eta)\,]C(\eta)d\eta, \quad (5)$$

where $C(\eta) = <U(\mathbf{r},t)U^*(\mathbf{r},t-\eta)>$ is the autocoherence function of noise. The factor $i = \exp(i\,\pi/2)$ on the right-hand side of Eq. (5) points to the phase shift $\pi/2$. This fact must be taken into account in tomographic problems.

In the case of the noise signal reception in a relatively narrow frequency band $[\omega_0 - \Delta\omega/2, \omega_0 + \Delta\omega/2]$, $\Delta\omega/\omega_0 << 1$, within which the noise characteristics vary slightly, the multiplication of both sides of Eq. (4) by $\frac{i\omega}{2\pi}e^{-i\omega\tau}$ and integration of the result with respect to ω leads to relation:

$$\frac{\partial <U(\mathbf{r}_A,t)U^*(\mathbf{r}_B,t-\tau)>_{\Delta\omega}}{\partial\tau} \approx -\omega_0 D\int_{-\infty}^{\infty}[G^{+}(\mathbf{r}_A,\mathbf{r}_B,\tau-\eta) - G^{-}(\mathbf{r}_A,\mathbf{r}_B,\tau-\eta)\,]C_{\Delta\omega}(\eta)d\eta. \quad (6)$$

From Eq. (6) it follows that the time derivative of the cross-coherence function of the narrow-band noise signal, received at two points allows estimating the time structure of narrowband fragments of both outgoing (for signal delays $\tau > 0$), and incoming (for $\tau < 0$) Green's functions at these points.

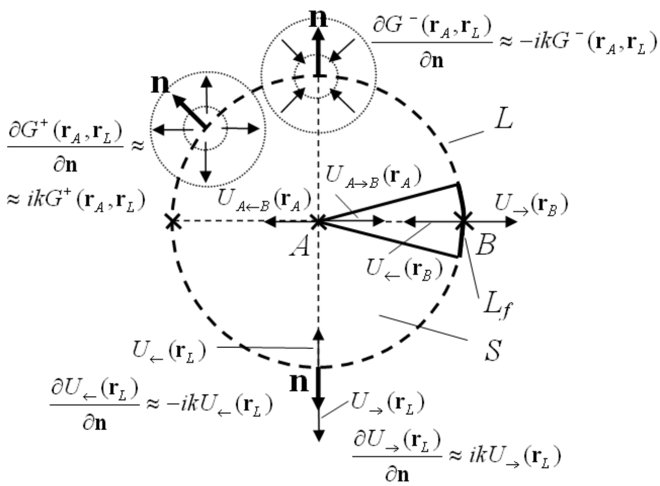

Fig.2 Schematic diagram of the tomography in the horizontal plane. Light arrows show the propagation directions of the fields entering the region S ($U_{\leftarrow}(\mathbf{r}_L)$) and leaving it ($U_{\rightarrow}(\mathbf{r}_L)$).

3 Estimation of required accumulation time

If the statistical characteristics of the noise field are such that the ergodicity condition is valid, the averaging over the ensemble of realizations in Eqs. (5), (6) can be replaced with averaging over time. Here, an important question arises about the time that is required to determine the Green's function from the coherence function of the noise field with the required output signal-to-noise ratio $S_{\text{out}}/N_{\text{out}}$. The squared input signal-to-noise ratio $(S_{\text{in}}/N_{\text{in}})^2$ in the correlator is proportional to the ratio of the effective length of the region L_f, where the sources of secondary noise field are coherent, to the length of the whole contour L. The value L_f can be estimated as $L_f \approx \oint_L <U(\mathbf{r}_L)U^*(\mathbf{r}_B)> dl \Big/ <U(\mathbf{r}_B)U^*(\mathbf{r}_B)>$. For a cylindrically isotropic noise field ($<U(\mathbf{r}_L)U^*(\mathbf{r}_B)>/<U(\mathbf{r}_B)U^*(\mathbf{r}_B)> = J_0(k|\,\mathbf{r}_B - \mathbf{r}_L|)$, $J_0(k|\,\mathbf{r}_B - \mathbf{r}_L|)$ is the zero-order Bessel function), the estimated value is $L_f \approx 2/k$, where $k = 2\pi/\lambda$. So it can be obtained:

$$S_{\text{out}}/N_{\text{out}} \approx 2\Delta f T\left(S_{\text{in}}/N_{\text{in}}\right)^2 = 2\Delta f T\left(\lambda/2\pi^2\rho\right).$$

For parameters $\rho = 10^5$ м, $\Delta f = 50$ Hz, $\lambda = 15$ м, $S_{\text{out}}/N_{\text{out}} = 10$, the collection time can be estimated as T ≈ 13000 s. For reduction of the achieved value an additional consideration is proposed.

As was shown in Sect. 2, the reception of the noise signal with nondirectional hydrophones and the subsequent calculation of the noise cross-coherence function results in a simultaneous estimation of both outgoing $G^{+}(\mathbf{r}_A, \mathbf{r}_B)$ and incoming $G^{-}(\mathbf{r}_A, \mathbf{r}_B)$ Green's functions. Thus, in the case of nondirectional hydrophones application for reconstructing either $G^{+}(\mathbf{r}_A, \mathbf{r}_B)$ or $G^{-}(\mathbf{r}_A, \mathbf{r}_B)$ only a portion of the received signal is significant. It reduces the signal-to-noise ratio $S_{\text{out}}/N_{\text{out}}$ and, as a consequence, overestimates the required accumulation time. A possible way to improve the $S_{\text{out}}/N_{\text{out}}$ ratio consists in using vector receivers, that offers a possibility of selecting the fields coming in and outgoing from the region of interest. It yields an additional (approximately by a factor of two) increase in the $S_{\text{in}}/N_{\text{in}}$ ratio of each receiver and leads to a fourfold improvement of the signal-to-noise ratio. As a result, the use of vector receivers offers a possibility of decreasing the accumulation time by a factor of four till T ≈ 1 hour.

4 Selection of modes by short curved vertical arrays

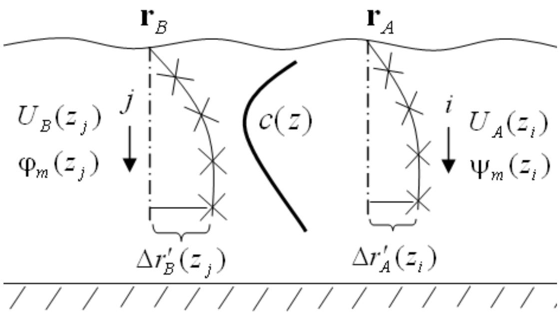

Fig.3 Vertical section of the region of tomography. Two arrays positioned at the points $\mathbf{r}_A$ and $\mathbf{r}_B$ have $i = \overline{1, I}$, and $j = \overline{1, J}$ receiving hydrophones.

Bellow the problem is considered in vertical plane passing through the two vertical arrays deployed in $\mathbf{r}_A$ and $\mathbf{r}_B$, respectively (Fig. 3). Displacements of hydrophones $\Delta\mathbf{r}_A(z)$, $\Delta\mathbf{r}_B(z)$ from strict vertical position lead to additional phase shifts of the signals. In approximation $|\Delta\mathbf{r}_A(z)|/\rho \ll 1$, $|\Delta\mathbf{r}_B(z)|/\rho \ll 1$, $\rho = |\mathbf{r}_A - \mathbf{r}_B|$, the field received on antennas can be represented as: $U(\mathbf{r}_A, z, t) = \sum_{k=1}^{M} \breve{a}_k(\mathbf{r}_A, t)\breve{\psi}_k(z)$, $U(\mathbf{r}_B, z, t) = \sum_{m=1}^{M} \breve{b}_m(\mathbf{r}_B, t)\breve{\varphi}_m(z)$, where $\breve{\varphi}_m(z) \equiv \varphi_m(z)\exp[i\kappa_m \Delta r'_B(z)]$, $\breve{\psi}_k(z) \equiv \psi_k(z)\exp[i\,æ_k\,\Delta r'_A(z)]$ are the profiles of "curved" modes and $\psi_k(z)$, $\varphi_m(z)$ are exact modes at the points $\mathbf{r}_A$ and $\mathbf{r}_B$. The coefficients $\breve{a}_k(\mathbf{r}_A, t)$ and $\breve{b}_m(\mathbf{r}_B, t)$ contain information about space and power-spectrum parameters of mode excitation; M is the number of propagating modes; $æ_k$ and κ_m are the horizontal wave numbers of modes in the places of antennas deployments. The projection of hydrophones displacement $\Delta\mathbf{r}_A(z)$, $\Delta\mathbf{r}_B(z)$ on the plane of consideration are $\Delta r'_A(z)$, $\Delta r'_B(z)$.

Representation of the noise field in a finite frequency band Δf as a sum of curved modes is valid in the approximation of the quasimonochromatic modes. In this case it is necessary that the mode profile will not change essentially on the boundary frequencies of Δf and the difference of propagation times for a mode with the fixed number but different frequencies must be less than $1/\Delta f$. More over, additional signal propagation times caused by the hydrophone displacement should not exceed $1/\Delta f$, i.e. the signal distortion caused by the antenna curvature can be expressed as phase shifts.

Elements $\Gamma_{ij}(\mathbf{r}_A, \mathbf{r}_B, \tau)$ of the cross-coherence matrix $\Gamma(\mathbf{r}_A, \mathbf{r}_B, \tau)$ of the noise field received by the i th hydrophone of the array at $\mathbf{r}_A$ and j th hydrophone of the array at $\mathbf{r}_B$ are defined as $\Gamma_{ij}(\mathbf{r}_A, \mathbf{r}_B, \tau) = \left\langle U(\mathbf{r}_A, z_i, t)U^*(\mathbf{r}_B, z_j, t-\tau)\right\rangle$, $i = \overline{1, I}$, $j = \overline{1, J}$. If the modes are incoherent (that is true for incoherent noise sources), then $\left\langle \breve{a}_k(\mathbf{r}_A, t)\breve{b}^*_m(\mathbf{r}_B, t-\tau)\right\rangle = \delta_{km}\mu_m(\mathbf{r}_A, \mathbf{r}_B, \tau)$, where function $\mu_m(\mathbf{r}_A, \mathbf{r}_B, \tau)$ is defined by the mode number m and horizontal range between antennas under consideration; symbol δ_{km} is the Kronecker delta. In this case

$$\Gamma_{ij}(\mathbf{r}_A, \mathbf{r}_B, \tau) = \sum_{m=1}^{M} \mu_m(\mathbf{r}_A, \mathbf{r}_B, \tau)\,\breve{\psi}_m(z_i)\,\breve{\varphi}^*_m(z_j). \qquad (7)$$

Eq. (7) is very similar to well known expression of acoustic field in a wave guide as a sum of modes:

$$U_{ij}(\mathbf{r}_A, \mathbf{r}_B, \tau) = \sum_{m=1}^{M} a_m(\mathbf{r}_A, \mathbf{r}_B, \tau)\,\breve{\psi}_m(z_i)\,\breve{\varphi}^*_m(z_j).$$

Relation between functions $a_m(\mathbf{r}_A, \mathbf{r}_B, \tau)$ and $\mu_m(\mathbf{r}_A, \mathbf{r}_B, \tau)$ can be obtained from Eq. (6): $\frac{\partial\,\mu_m(\mathbf{r}_A, \mathbf{r}_B, \tau)}{\partial\tau} \sim \omega_0 \int_{-\infty}^{\infty} a_m(\mathbf{r}_A, \mathbf{r}_B, \tau-\eta)\,C_{\Delta\omega}(\eta)d\eta$. If noise characteristics vary slightly in the frequency band $\Delta\omega$ then $|\mu_m(\mathbf{r}_A, \mathbf{r}_B, \tau)| \sim |a_m(\mathbf{r}_A, \mathbf{r}_B, \tau)|$, and

$$|\mu_m(\mathbf{r}_A, \mathbf{r}_B, \tau)|^2 \sim |a_m(\mathbf{r}_A, \mathbf{r}_B, \tau)|^2. \qquad (8)$$

From Eq. (8) follows that the functions $a_m(\mathbf{r}_A, \mathbf{r}_B, \tau)$ and $\mu_m(\mathbf{r}_A, \mathbf{r}_B, \tau)$ achieve their maximum values at the same times τ. As result, the problem of propagation time definition for different modes can be reduced to the problem of coefficients $\mu_m(\mathbf{r}_A, \mathbf{r}_B, \tau)$ reconstruction using noise measurement data. In matrix representation with the use of Dirac's notation ($|\ \rangle$ for the column vector and $\langle\ |$ for the row vector), the matrix $\Gamma(\mathbf{r}_A, \mathbf{r}_B, \tau)$, for the fixed time delay τ, assumes the form:

$\Gamma(\tau)=\sum_{m=1}^{M}\mu_m(\tau)\left|\breve{\psi}_m\right\rangle\left\langle\breve{\varphi}_m\right|$, where $\left|\breve{\psi}_m\right\rangle$ and $\left|\breve{\varphi}_m\right\rangle$ the column vectors composed of the values of "curved" modes at different depths. For brevity the dependence of the functions μ_m and Γ on the delay time τ and radius-vectors $\mathbf{r}_A$, $\mathbf{r}_B$ bellow is omitted. The consideration of the square matrix $\Gamma\Gamma^+$ (sign «+»stands for the Hermittian conjugation) allows to obtain the relations for determination of functions $\left|\mu_m(\mathbf{r}_A,\mathbf{r}_B,\tau)\right|^2$:

$$\Gamma\Gamma^+=\sum_{m=1}^{M}\mu_m\mu_m^*\left\langle\breve{\varphi}_m\left|\breve{\varphi}_m\right\rangle\right|\breve{\psi}_m\rangle\langle\breve{\psi}_m\right|+$$
$$+2\sum_{m=1}^{M-1}\sum_{k=m+1}^{M}\mathrm{Re}\left[\mu_m\mu_k^*\left\langle\breve{\varphi}_m\left|\breve{\varphi}_k\right\rangle\right|\breve{\psi}_m\rangle\langle\breve{\psi}_k\right|\right]= \qquad (9)$$
$$=\sum_{n=1}^{N}\lambda_n\left|\hat{\psi}_n\right\rangle\left\langle\hat{\psi}_n\right|,$$

where N is the number of nonzero eigenvalues λ_n of matrix $\Gamma\Gamma^+$, $N\geq M$. It must be pointed out that curved mode profiles $\left|\breve{\varphi}_m\right\rangle$, $\left|\breve{\psi}_m\right\rangle$ and values $\mu_m\mu_m^*=\left|\mu_m\right|^2$ in general case are not equivalent to the eigenvectors $\left|\hat{\varphi}_n\right\rangle$, $\left|\hat{\psi}_n\right\rangle$ and eigenvalues λ_n. If approximation $\breve{\psi}_m(z)\breve{\psi}_k^*(z)\approx\psi_m(z)\psi_k(z)$, $\left\langle\breve{\varphi}_m\middle|\breve{\varphi}_k\right\rangle\approx\left\langle\varphi_m\middle|\varphi_k\right\rangle$, $m\neq k$ is valid, then from Eq. (9) one can obtain the system of linear equations:

$$\Psi_{(2)}\left|\mu\mu^*\right\rangle=\left|B\right\rangle, \qquad (10)$$

where $\left|B\right\rangle\equiv\begin{pmatrix}\sum_{n=1}^{N}\lambda_n\hat{\psi}_n(z_1)\hat{\psi}_n^*(z_1)\\ \dots\\ \sum_{n=1}^{N}\lambda_n\hat{\psi}_n(z_I)\hat{\psi}_n^*(z_I)\end{pmatrix}$, $\left|\mu\mu^*\right\rangle\equiv\begin{pmatrix}\mu_1\mu_1^*\\ \mu_2\mu_2^*\\ \dots\\ \mathrm{Re}\left[\mu_1\mu_2^*\right]\\ \dots\end{pmatrix}$.

The elements of the matrix $\Psi_{(2)}\equiv\left[\left\langle\varphi_1\middle|\varphi_1\right\rangle\left|\psi_1\psi_1\right\rangle \;\dots\; 2\left\langle\varphi_1\middle|\varphi_2\right\rangle\left|\psi_1\psi_2\right\rangle \;\dots\right]$ are assumed to be defined from the measurements of the sound speed profile at the points of antenna deployment (for example, using CTD measurements). The right-hand part $\left|B\right\rangle$ is defined by eigenvectors $\hat{\psi}_n$ and eigenvalues λ_n of matrix $\Gamma(\tau)\Gamma^+(\tau)$ for the fixed τ; $\left|\mu\mu^*\right\rangle$ is a vector of unknowns. The solution of the system (10) can be obtained by the least squares method (LSM) and then regularized in the simplest case by summing $\Psi_{(2)}^+\Psi_{(2)}$ with diagonal matrix D with elements γ_m^2 on the diagonal: $\left|\widehat{\mu\mu^*}\right\rangle=\left(\Psi_{(2)}^+\Psi_{(2)}+D\right)^{-1}\Psi_{(2)}^+\left|B^{\mathrm{noise}}\right\rangle$. In the numerical modelling coefficients $\gamma_m^2=0$ when $m=\overline{1,M}$ and for $m=\overline{M+1,M(M+1)/2}$ values γ_m^2 are equal to 10^{-8} of maximum eigenvalue of matrix $\Psi_{(2)}^+\Psi_{(2)}$. Such selective regularization is appeared to be more optimal and gives less estimate variance.

Thus the problem of mode structure reconstruction using data from short curved antenna is reduced to the solution of the system of linear equation relatively unknown squared modules of mode propagation coefficients.

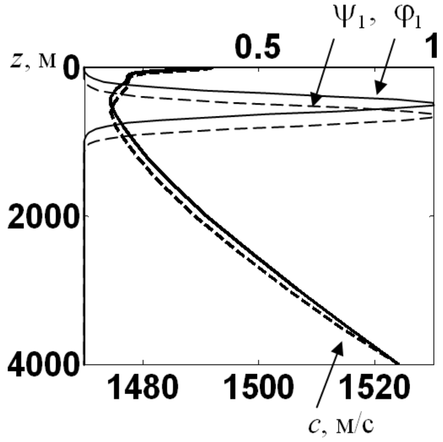

Fig.4 Hydrologies at the points of antennas deployment and corresponding them normalized profiles of the first modes.

In numerical modeling two vertical antennas separated by $\rho=2\times10^5$ м are considered. At the places of antenna deployment the sound speed profiles are different, that yields the different mode profiles $\psi_k(z)$, $\varphi_m(z)$ (Fig. 4). Two configurations of antennas are investigated. Firstly, both antennas are uncurved and cover the whole waveguide, have the same length of 4000 м and same numbers of hydrophones $I=J=100$. In the case of short curved antennas they have the same length of 500 м and the same numbers of hydrophones $I=J=50$. The top ends of the both antennas are deployed at the depth of 200 м. It is assumed that arrays are forced by ocean flows with constant with depth projection of velocity on the plane of consideration $\mathrm{v}_A=0.2$ м/с and $\mathrm{v}_B=0.3$ м/с, respectively; to the bottom ends of antennas the weights are attached with the same mass 30 кг. Under these conditions the curvatures of antennas profile were calculated (Fig. 5). The hydrophones

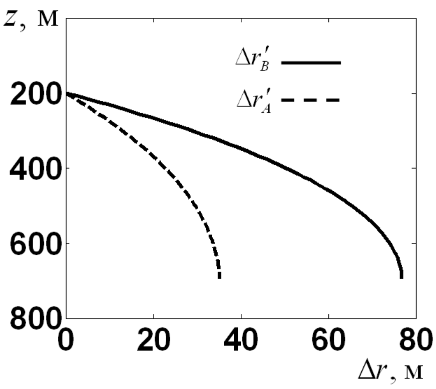

Fig.5 Declinations of profiles for considered antennas.

receive the noise signals propagating in the plane of consideration with uniform density power in frequency band $\Delta f=10$ Hz with central frequency $f_0=80$ Hz during $T=400$ s. The signal was consisted from $M=10$ incoherent modes. Figure 6a, 6b shows the time dependence of normalized squared module of the total field $\left|\Gamma^{\mathrm{norm}}(\tau)\right|^2\equiv\sum_{i,j}\left|\Gamma_{ij}(\tau)\right|^2\Big/\max_{\tau\in[\tau_1,\tau_2]}\sum_{i,j}\left|\Gamma_{ij}(\tau)\right|^2$ received by all hydrophones of antenna in $\mathbf{r}_A$, as if this field was radiated by all hydrophones of antenna in $\mathbf{r}_B$. Figure 7a, 7b shows the time dependence of normalized squared modules of mode propagation coefficients for all ten modes $\left|\hat{\mu}_m^{\mathrm{norm}}(\tau)\right|^2 \equiv \sum_{i,j}\left|\Gamma_{ij}(\tau)\varphi_m(z_j)\right|^2 \Big/ \max_{\tau\in[\tau_1,\tau_2]}\sum_{i,j}\left|\Gamma_{ij}(\tau)\varphi_m(z_j)\right|^2$, that are obtained using standard procedure of mode filtering by multiplication of the field received by vertical array on mode profiles. When there are no hydrophone displacements and antennas are long enough $\left|\hat{\mu}_m^{\mathrm{norm}}(\tau)\right|^2 \cong \left|\mu_m(\tau)\right|^2 \Big/ \max_{\tau\in[\tau_1,\tau_2]}\left|\mu_m(\tau)\right|^2$ (Fig. 7a) and standard mode processing provides excellent result. But it is clearly shown on Fig. 7b that in the case of short curved antennas common approach of mode filtering yields unsatisfactory results. Figure 8a, 8b shows the dependence of $\left|\mu_m^{\mathrm{norm}}(\tau)\right|^2 \equiv \left|\mu_m(\tau)\right|^2 \Big/ \max_{\tau\in[\tau_1,\tau_2]}\left|\mu_m(\tau)\right|^2$, where functions $\left|\mu_m(\tau)\right|^2 = \mu_m\mu_m^*$, $m=\overline{1,10}$, are estimated using new method (10). The maximum values of $\left|\mu_m^{\mathrm{norm}}(\mathbf{r}_A,\mathbf{r}_B,\tau)\right|^2$, $m=\overline{1,10}$, are clearly seen, and propagation times of different modes in case of both long vertical (Fig. 8a) and short curved (Fig. 8b) antennas can be identified unambiguously. It should be mentioned that on Figure 7, 8 only nine peaks separated in time are observed instead of ten expected. Point is that the sixth and seventh modes are not separated in propagation times for considered hydrologies and frequencies but they can be identified using developed method of mode processing.

5 Conclusion

The cross-coherence matrix of the noise field received by the hydrophones of two short curved vertical arrays yields the information about the mode structure of field in the ocean. The estimated time of signal accumulation required to determine the Green's function shows the possibility of implementing the schemes of the passive mode tomography of the ocean. The use of the vertical arrays with vector receivers allows a decrease in the accumulation time to one or several hours, depending on the complexity, required accuracy, and resolution of the tomographic problem to be solved.

For decreasing the accumulation time the wide frequency band should be considered. On the other hand the modes within this frequency band must be quasi monochromatic and compensation signal processing should give opportunity to neutralize the possible antenna declinations. Antipathy of these conditions leads to the necessity of additional investigations of methods to synthesize data obtained for some narrow frequency bands.

The very important simplification for considered problems is the adiabatic approximation. A violation of adiabaticity results in an increase in the accumulation time and a complication of the algorithms of tomographic reconstruction. In this case new problems arise that must be solved before discussing the efficiency of the proposed approach.

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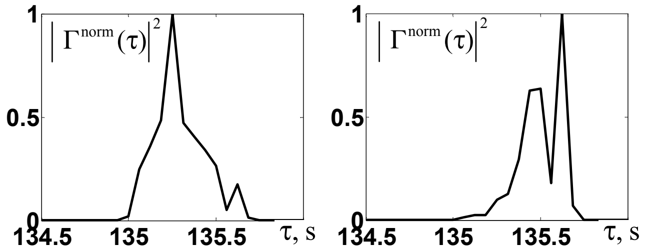
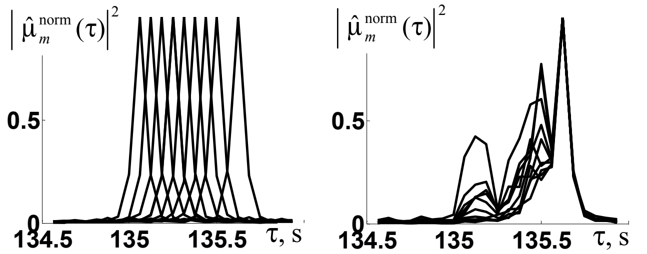


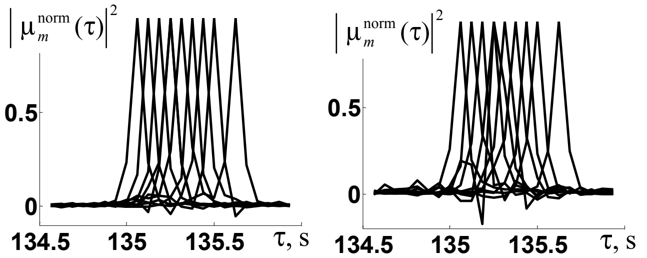
6a 6b

Fig.6 Time dependence of total field received by long uncurved (a) and short bent (b) antennas.



7a 7b

Fig.7 Results of mode filtering using mode orthogonality in the case of long uncurved (a) and short bent (b) antennas.



8a 8b

Fig.8 Results of mode filtering based on eigenvalues and eigenvectors of noise cross-coherence matrix consideration in the case of long uncurved (a) and short bent (b) antennas.

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