coustics'08
Paris
June 29-July 4, 2008
www.acoustics08-paris.org

Performance of high-resolution sensor array processing algorithms in the localization of acoustic sources

Joseph Lardies[a], Hua Ma[b], Marc Berthillier[b] and Emmanuel Foltete[a]

[a]FEMTO-ST Applied Mechanics, 24 chemin de l'épitaphe, 25000 Besançon, France
[b]University of Franche-Comté, Institute FEMTO - LMARC, 24 rue de l'Epitaphe, 25000 Besançon, France
joseph.lardies@univ-fcomte.fr

Abstract The localization of noise sources from a specified direction may often be accomplished with an array of sensors. One commonly used processor consists of delay and add networks: a conventional beamformer, however its spectrum suffers from the Rayleigh resolution and its performance is highly degraded, specially in lower frequency range. In the communication, the performance of some typical high-resolution sensor array processing algorithms: Minimum Variance and MUSIC algorithms are investigated for wideband source location. Their performances are compared with a new source localization algorithm which is based on a sparse representation of sensor measurements with an overcomplete basis composed of samples from the array manifold. The key of the method is the use of the SVD for data reduction and the formulation of a joint multiple-sample sparse representation problem in the signal subspace domain. Increased resolution and improved robustness to noise is obtained. Numerical examples are presented.

1 Introduction

Arrays of sensors are used in many fields to detect signals, to resolve closely spaced targets, to estimate the bearing, the position, the strength and other properties of radiating sources whose signals arrive from different directions. The emitted source of energy can be acoustic (sound-waves), electro-magnetic (radio-waves), vibrations (signals of geophysical nature), chemical (detection of vapours and gaseous pollutants from different substances), and so on, and the receiving sensors may be any transducers that convert the received energy to electrical signals. The type of sensors used to detect these signals differ accordingly: microphones for the acoustic signals, electromagnetic antennas for radio waves, accelerometers/seismometers for the detection of earthquakes, ultrasonic probes and X-ray detectors in medical imaging, containers with membranes or biosensors for gas and vapour detection and so forth. In all these highly diverse applications of array signal processing, the sensors are designed with one basic objective in mind: to provide an interface between the environment in which the array is embedded and the signal processing part of the system and the physical manner in which this interface is established depends on the application of interest. In these applications, the goal is to determinate the distribution of the emitted energy in the medium (air, water, rock, etc) that surrounds the array. For example, in industrial environments to localize complex noise fields, in underwater surveillance with sonar systems, in communications to separate speakers, in seismology for the monitoring and analysis of global earthquakes, in all these cases signals received by sensor arrays are processed to obtain estimates of their strength and direction of arrival.

In this paper we are concerned in industrial noises localization which have significant effects on labor's health and community living quality. It is highly desired to develop methods that are capable of locating noise sources in an accurate and systematic manner before any noise control measure can be applied. Conventional ways of noise source identifications include, for example, sound pressure measurement, sound intensity measurement and acoustic holography. These methods suffer from the drawbacks of being either inaccurate or being restrictive in only small areas or short distances when applied to complex noise fields in industrial environment.

In this communication, noise source localization techniques by using an array of sensors are presented. The performances of these methods in the localization of closely spaced broadband sources and in a considerable background noise are presented. The approach taken here is to assume that the signal field at the array is comprised of N independent plane wave arrivals from unknown directions and the problem reduces to estimating the K directions in a background noise environment. One main of the work described in the paper is to provide estimators which are simple to implement on line. These estimators are based on acoustic processing algorithms: the conventional beamforming, the Minimum Variance (MV), the Multiple Signal Classification (MUSIC) and the sparse signal reconstruction (or l_1-SVD) estimators are presented and their performances are compared. Some additional estimators (Root-Music and ESPRIT) require the restrictive assumption that the array of sensors is linear. This is assumption is not necessary with algorithms developed here. Finally, we introduce the following typical assumptions: signals are homogenous in the vicinity of the array, all receivers have the same sensitivity, the array does not distort the signal field, the medium in which the signals propagate is non-dispersive and the sources and the array lie in the same plane.

2 Signal representation and spatial correlation matrix

The receiving array considered in this communication has M omnidirectional sensors and is immersed in an acoustic noise field which consists of K independent discrete sources. Because of the geometric positions of the sensors, the total signal power incident on each sensor is the same, but the phase information is different on each receiver. The purpose of any estimator is to use the phase information in some way to infer which signals reached the receiving array and the goal of sensor array source localization is to find the locations of sources of wavefields that impinge on the acoustical array. The available information is the geometry of the array, the parameters of the medium where wavefields propagate and the time measurements or outputs of the sensors. For purposes of exposition, we first focus on the narrowband scenario. For a set of K sources, the signals observed at the outputs of the M sensors array are represented by the M-dimensional vector [1,2,3]

$$\mathbf{x}(t) = \sum_{i=1}^{K} \mathbf{a}(\theta_i)\, \mathbf{s_i(t)} + \mathbf{n}(t) \qquad (1)$$

where $s_i(t)$ is the complex amplitude of the i^{th} source. It is a zero-mean complex random variable : $E[s_i(t)] = 0$. The signal power p_i of the i^{th} source which we wish to localize is represented by its variance $p_i = Var[s_i(t)]=E[s_i(t)s_i(t)^*]$. Here E[ ] denotes an ensemble average and the superscript* represents the complex conjugate. The direction of arrival of the i^{th} signal is represented by the M-dimensional vector $\mathbf{a}(\theta_i) = [a_1(\theta_i)\ a_2(\theta_i) \ldots a_M(\theta_i)]^T$, often called the array manifold vector or the steering vector or the directional mode vector and $\mathbf{n}(t)$ is the additive noise. The noise is assumed to be spatially white (independent or uncorrelated from sensor to sensor) and the same power level of noise is present in each receiver. With these assumptions, the cross-spectral matrix for the noise alone is $\mathbf{R_N} = E[\mathbf{n}(t)\mathbf{n}(t)^H] = p_N\mathbf{I}$ where p_N is the noise power, $\mathbf{I}$ is the (MXM) identity matrix and the superscript H denotes the complex-conjugate transpose operation. Equation (1) may we rewritten in the matrix form

$$\mathbf{x}(t) = \mathbf{A}\,\mathbf{s}(t) + \mathbf{n}(t) \quad t \in \{t_1, t_2, \ldots t_N\} \qquad (2)$$

The (MXK) matrix $\mathbf{A}$ where each column is a source direction vector, is the so-called array manifold matrix. For any single plane wave arrival, the outputs from the M individual receivers will differ in phase by an amount determined by the geometry of the array and the arrival direction. In other words, the elements A_{qr} of the matrix $\mathbf{A}$ are functions of the signal arrival angles and the array elements locations. Thus, one has $A_{qr} = \exp(j\phi_{qr})$ where ϕ_{qr} is the phase of the signal at the q^{th} receiver from the r^{th} source, measured relative to some arbitrary reference point. That is, A_{qr} depends on the q^{th} array element, its position relative to the origin of the coordinate system, and its response to a signal incident from the direction of the r^{th} source. $\mathbf{s}(t)$ is the K-dimensional vector, the components of which are the complex amplitudes of the sources. It can readily be seen that the output signal from the q^{th} sensor may be written as

$$x_q(t) = \sum_{r=1}^{K} A_{qr}(\theta)\, s_r(t) + n_q(t) \qquad (3)$$

Since the K arrivals are by assumption independent, the correlation matrix between the different signal sources is

$$\mathbf{R_S} = E[\mathbf{s}(t)\mathbf{s}(t)^H] = \mathrm{diag}(p_1, p_2, \ldots, p_K) \qquad (4)$$

and at the operating frequency, the spatial correlation matrix (or covariance matrix) of the receiver outputs may be expressed, for signals uncorrelated of each other and of noise, as

$$\mathbf{R} = E[\mathbf{x}(t)\mathbf{x}(t)^H] = \mathbf{A}\mathbf{R_S}\mathbf{A}^H + \mathbf{R_N} \qquad (5)$$

In practice, the spatial correlation matrix is estimated by a finite number of time domain samples (snapshots) and the following estimated form is used

$$\hat{\mathbf{R}} = \frac{1}{N}\sum_{i=1}^{N} \mathbf{x}(t_i)\mathbf{x}(t_i)^H \qquad (6)$$

Where $\mathbf{x}(t_i)$ is the array signal vector sampled at time t_i and N is the number of such samples. The caret (^) denotes an estimated value.

We can now derive a variety of estimators used in sensor array source localization.

3 Estimators for source localization

3.1 The conventional beamformer

Signal direction estimation is one of the main tasks in array processing and the most commonly used method is the conventional beamformer, also called the ''time delay and sum'' or ''unweighted add-squarer'' beamformer. The major step of the conventional estimator consists of delaying and summing the outputs from each sensor to yield the total array output [1,2,3]

$$y(t) = \mathbf{a}(\theta)^H\mathbf{x}(t) \qquad (7)$$

It follows that the output power of the array is

$$p_{CB}(\theta) = E[\,|y(t)|^2\,] = \mathbf{a}(\theta)^H\ \mathbf{R}\,\mathbf{a}(\theta) \qquad (8)$$

The signal direction is indicated by the value of θ at which the output power is maximized. It follows that the beamforming estimates of the direction of arrivals are given as the K largest values of the scalar function $p_{CB}(\theta)$. Equation (8) is also known as the angular power spectrum of the conventional beamformer. The resolution of this beamformer, which is usually defined by its 3-dB beamwidth, is proportional to the reciprocal of the array aperture measured in wavelengths. So, for small array the resolution is poor. The beamformer also has high sidelobes that may cause a signal leakage problem: weak signals may be hidden by the presence of strong signals in the sidelobes. Despite the simplicity, the conventional beamforming algorithm suffers from resolution problems when the sources are close to each other. Furthermore, the beamwidth of this beamformer is very sensitive to frequency variations and then unsuitable for broadband source localization.

3.2 The minimum variance estimator

The minimum variance (MV) estimator was originally proposed by Capon who conducted a frequency wavenumber analysis on earthquake data analysis. The conventional beamformer can be considered as a kind of linear spatial filter with data-independent coefficients. In contrast, the minimum variance method (or Capon's method) can be considered as a kind of data-dependent spatial filter, in which the coefficients are chosen such that the filter has constant gain at a particular direction while its output power is minimized. The underlying principle of the method amounts to finding an optimal steering vector $\mathbf{w}_{opt}$ such that the array output power is minimized while maintaining the gain along the look direction to be constant, say unity [1,2]. That is,

$$\min_{\mathbf{w}}\ \mathbf{w}^H\,\mathbf{R}\,\mathbf{w} \qquad (9)$$

$$\text{subject} \quad \mathbf{w}^H \mathbf{a}(\theta) = 1 \qquad (10)$$

Minimizing the resulting beam energy reduces the contributions to this energy from sources and noise not propagating in the direction of look. The solution of this constrained optimization problem occurs often in the derivation of adaptive array processing algorithms. The solution technique is to use a Lagrange multiplier λ and a cost function

$$H(\mathbf{w}) = \mathbf{w}^H \mathbf{R}\, \mathbf{w} + 2\, \lambda\, (\mathbf{w}^H \mathbf{a}(\theta) - 1) \qquad (11)$$

The gradient with respect to **w** is

$$\mathbf{grad}\, H(\mathbf{w}) = 2\, \mathbf{R}\, \mathbf{w} + 2\, \lambda\, \mathbf{a}(\theta) \qquad (12)$$

and the minimum of the cost function is obtained when

$$\mathbf{w}_{opt} = -\, \lambda\, \mathbf{R}^{-1}\, \mathbf{a}(\theta) \qquad (13)$$

But $\mathbf{w}_{opt}$ must satisfy the constraint, one then has

$$\lambda = (-\mathbf{a}^H(\theta)\, \mathbf{R}^{-1}\, \mathbf{a}(\theta))^{-1} \qquad (14)$$

$$\mathbf{W}_{opt} = \mathbf{R}^{-1}\, \mathbf{a}(\theta)(\mathbf{a}^H(\theta)\, \mathbf{R}^{-1}\, \mathbf{a}(\theta))^{-1} \qquad (15)$$

and the corresponding array output power is

$$p_{MV}(\theta) = E[\, |y(t)|^2\,] = (\mathbf{a}^H(\theta)\, \mathbf{R}^{-1}\, \mathbf{a}(\theta))^{-1} \qquad (16)$$

The goal of the minimum variance estimator is that the contributions of the signals from directions other than θ to the array output are minimized while the signal at direction θ passes through without any distortion. Equation (16) is also known as the angular power spectrum of the minimum variance estimator and the signal directions are found by the locations of the spectrum peaks. The peaks level of the spectrum give a good estimate of the true targets power and the spectrum also has uniformly low sidelobes. Simulations have shown that this estimator gives satisfactory resolution if the number of snapshots is high. The algorithm does not require any knowledge of the number of sources present and can also be used with irregular arrays. It is expected that this estimator performs better than the classical beamforming and has super-resolution provided that the SNR is moderately high, the sources are not strongly correlated and the number of snapshots is sufficient.

The two estimators described previously do not make any assumption on the covariance structure of the data and they assume that the functional form of the array's transfer vector $\mathbf{a}(\theta)$ is known. The array performs a spatial sampling of the income wavefront, which is analogous to the temporal sampling done by the tapped-delay line implementation of a temporal finite impulse response (FIR) filter. The functional form of $\mathbf{a}(\theta)$ characterizes the array as a spatial sampling device and, assuming it is known, should not be considered to be parametric information. An array for which the functional form of $\mathbf{a}(\theta)$ is known is said to be calibrated.

The common advantage of the two estimators developed previously is that they do not assume anything about the statistical properties of the data and, therefore, they can be used in situations where we lack information about these properties. The parametric approach to direction of arrival estimation is the subject of the next section.

3.3 The multiple signal classification estimator

The parametric methods for direction estimation explicitly assume a certain structure of the array output covariance matrix **R** and the multiple signal classification algorithm (MUSIC) [4] is based on the eigensystem analysis of this spatial correlation matrix. It is assumed that the theoretical covariance matrix has the expression given by (5) and that the signal sources covariance matrix $\mathbf{R_S}=E[\mathbf{s}(t)\mathbf{s}(t)^H]$ has full rank K. Assume that M>K and that the directional mode vectors $\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \ldots, \mathbf{a}(\theta_K)$ are linearly independent. Then the array manifold matrix **A** has full column rank K. This fact, together with the assumption that $\mathbf{R}_S$ has full rank K, yields

$$\text{rank}\, (\mathbf{A}\mathbf{R_S}\mathbf{A}^H) = K \qquad (17)$$

It follows that $\mathbf{A}\mathbf{R_S}\mathbf{A}^H$ has K strictly positive eigenvalues, the remaining (M-K) eigenvalues all being equal to zero. Assume that the eigenvalues λ_i, i=1, 2,…,M of the spatial correlation matrix **R** are arranged in decreasing order : $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_M$. If $\tilde{\lambda}_j$ is an eigenvalue of the matrix $\mathbf{A}\mathbf{R_S}\mathbf{A}^H$ and $\mathbf{u}_j$ the eigenvector of **R** corresponding to the eigenvalue λ_j, then

$$\mathbf{R}\, \mathbf{u}_j = \lambda_j\, \mathbf{u}_j \Leftrightarrow (\, \mathbf{A}\mathbf{R_S}\mathbf{A}^H + p_N\, \mathbf{I}\,)\, \mathbf{u}_j = (\tilde{\lambda}_j + p_N)\, \mathbf{u}_j \qquad (18)$$

so that $\lambda_j = \tilde{\lambda}_j + p_N$. This means that the eigenvalues of the array data covariance matrix **R** are

$$\lambda_i = \begin{cases} \tilde{\lambda}_i + p_N & i = 1, 2, ..., K \\ p_N & i = K+1, ..., M \end{cases} \qquad (19)$$

We have thus found that $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_K > \lambda_{K+1} = \lambda_{K+2} \ldots = \lambda_M = p_N$. In fact, the eigenvalues of **R** ca be split into two groups, one containing the K eigenvalues that are larger than p_N and one containing the M-K eigenvalues that are equal to the noise power p_N.

Let $\mathbf{U}_S$ and $\mathbf{U}_N$ be the matrices whose columns are the orthonormal eigenvectors of **R** corresponding to the two groups respectively. Then the eigenvalue decomposition of R can be partitioned as

$$\mathbf{R} = (\mathbf{U}_S \quad \mathbf{U}_N) \begin{pmatrix} \mathbf{\Lambda}_S & \mathbf{0} \\ \mathbf{0} & \mathbf{\Lambda}_N \end{pmatrix} \begin{pmatrix} \mathbf{U}_S^H \\ \mathbf{U}_N^H \end{pmatrix} \qquad (20)$$

where $\mathbf{\Lambda}_S$ contains the K largest eigenvalues of **R** and $\mathbf{\Lambda}_N$ contains the M-K smallest eigenvalues of **R** which are equal to p_N. Using the orthogonality of $\mathbf{U}_S$ and $\mathbf{U}_N$ we get from (20) that

$$\mathbf{R}\,\mathbf{U}_N = \mathbf{U}_N \boldsymbol{\Lambda}_N = p_N \mathbf{U}_N \quad (21)$$

and from (5) we obtain

$$\mathbf{R}\mathbf{U}_N = \mathbf{A}\mathbf{R}_S\mathbf{A}^H \mathbf{U}_N + p_N \mathbf{U}_N \quad (22)$$

Thus $\mathbf{A}\mathbf{R}_S\mathbf{A}^H \mathbf{U}_N = 0$ and, since the matrix $\mathbf{A}\mathbf{R}_S$ has full rank, we have

$$\mathbf{A}^H \mathbf{U}_N = 0 \quad (23)$$

The columns $\{\mathbf{u}_k\}$ of $\mathbf{U}_N$ belong to the null space of $\mathbf{A}^H$, a fact which is denoted by $\mathbf{u}_k \in N(\mathbf{A}^H)$: $\mathbf{U}_N$ lies in the null space of $\mathbf{A}^H$. Again, since rank($\mathbf{A}$) = K, the dimension of $N(\mathbf{A}^H)$ is equal to M-K which is also the dimension of the range space of $\mathbf{U}_N$. We have $\dim N(\mathbf{A}^H)$ = M-K = $\dim R(\mathbf{U}_N)$ and it follows that $R(\mathbf{U}_N) = N(\mathbf{A}^H)$. The vectors $\{\mathbf{u}_k\}$ span both $R(\mathbf{U}_N)$ and $N(\mathbf{A}^H)$ and due to its orthogonality property with respect to $\mathbf{A}^H$, $R(\mathbf{U}_N)$ is called the noise subspace of $\mathbf{R}$.

Now, since by definition $\mathbf{U}_S^H \mathbf{U}_N = \mathbf{0}$, we also have $R(\mathbf{U}_N) = N(\mathbf{U}_S^H)$; hence, $N(\mathbf{U}_S^H) = N(\mathbf{A}^H)$. Since $R(\mathbf{U}_S)$ and $R(\mathbf{A})$ are orthogonal complements to $N(\mathbf{U}_S^H)$ and $N(\mathbf{A}^H)$, it follows that $R(\mathbf{U}_S) = R(\mathbf{A})$ and $R(\mathbf{U}_S)$ is called the signal subspace of the array data covariance matrix $\mathbf{R}$.

We have shown that the spatial correlation matrix $\mathbf{R}$ has M-K smallest eigenvalues p_N and the associated eigenvectors are orthogonal to the signal direction vectors (see 23). Noting θ_i the arrival angle of the i[th] source we have from equation (23)

$$\mathbf{U}_N^H\ \mathbf{a}(\theta_i) = \mathbf{0} \text{ or } \mathbf{a}^H(\theta_i)\,\mathbf{u}_j = 0 \;;\; i=1,..,K;\; j=K+1,..,M \quad (24)$$

This orthogonality is used in the MUSIC algorithm to estimate the signal directions. The angular spectrum function of the MUSIC algorithm is defined as

$$p_{MUSIC}(\theta) = \frac{1}{\sum_{j=K+1}^{M} \left| \mathbf{a}^H(\theta)\,\mathbf{u}_j \right|^2} \quad (25)$$

This equation can be rewritten in terms of $\mathbf{U}_N$ as

$$p_{MUSIC}(\theta) = \frac{1}{\mathbf{a}^H(\theta)\ \mathbf{U}_N\ \mathbf{U}_N^H\ \mathbf{a}(\theta)} \quad (26)$$

It can be seen that the denominator of the MUSIC spectrum function is the square norm of the projection of the directional mode vector $\mathbf{a}(\theta)$ on to the noise subspace, which will be zero when θ is the signal direction according to the orthogonal relations (23 and 24) and the peaks of the function $p_{MUSIC}(\theta)$ yields the direction of arrival of the signals. Therefore the MUSIC algorithm is also known as a subspace projection algorithm. For equation (25) or (26) to be meaningful, the condition that M>K has to be satisfied. This restricts the minimum number of sensors to be greater than the total number of sources. The initial guess on the source number K then becomes important and a test is required to determine the number of signals presented. However, simulation results show that the MUSIC algorithm as very good resolution, even for a small number of snapshots. Its spectrum is very stable with low sidelobes. Statistical analysis has also shown that the algorithm has a lower SNR resolution threshold. This estimator is also applicable to nonuniform arrays and in contrast with the previously discussed techniques $p_{MUSIC}(\theta)$ has no direct relation to the physical signal power; it simply exhibits sharp peaks at the estimated source locations. We propose a sparse signal reconstruction estimator which has better robustness to low SNR and in a broadband scenario than the Conventional Beamformer.

3.4 The sparse signal reconstruction estimator

We present now a source localization method based on a sparse representation of sensor measurements [5]. The basic idea of enforcing sparsity in overcomplete basis representations is described as follows. We consider an overcomplete representation of the array manifold matrix $\mathbf{A}$ in terms of all possible locations. We introduce a sampling grid of possible source locations of interest $\{\tilde\theta_1, \tilde\theta_2, ...; \tilde\theta_{K'}\}$. The number of potential sources K' is greater than the number of sources K and form the matrix $\mathbf{A}(\tilde\theta)$. The important point is that $\mathbf{A}$ is known and does not depend on the unknown source locations $\{\theta_1, \theta_2, ...; \theta_K\}$ as in (2). We represent the signal field by a K'x1 vector $\tilde{\mathbf{s}}(t)$, where the i[th] element $\tilde{\mathbf{s}}_i(t)$ is nonzero and equal to $s_k(t)$ if source k comes from direction $\tilde\theta_i$ for some k and zero otherwise.

$$\tilde{\mathbf{s}}_i(t) = \begin{cases} s_k(t) & \text{if } \tilde\theta_i = \theta_k \\ 0 & \text{otherwise} \end{cases} \quad (27)$$

Then the problem takes the form

$$\mathbf{x}(t) = \mathbf{A}(\tilde\theta)\,\tilde{\mathbf{s}}(t) + \mathbf{n}(t) \quad (28)$$

and the important points are that $\mathbf{A}$ is known and that the source localizations are now encoded by the non zero indices of $\tilde{\mathbf{s}}(t)$. We have transformed the problem from finding a point estimate of $\boldsymbol{\theta}$, to estimating the spatial spectrum of $\tilde{\mathbf{s}}(t)$, which has to exhibit sharp peaks at the correct source locations.

In principle, one can use the overcomplete basis methodology to solve a signal representation problem at each time instant. However, this leads to a significant computational load and to sensitivity to noise. Instead, we would like to use all the sensor time data in synergy using an SVD approach. To this end, the data $\mathbf{x}(t)$ are viewed as a cloud of N points lying in a K dimensional subspace. Instead of keeping every time sample, we can represent the cloud using its K largest singular vectors corresponding to the signal subspace. Let $\mathbf{X} = [\mathbf{x}(t_1), \mathbf{x}(t_2), \ldots, \mathbf{x}(t_N)]$ and define $\mathbf{S}$ and $\mathbf{N}$ similarly. Then we have

$$\mathbf{X} = \mathbf{A}(\tilde\theta)\,\mathbf{S} + \mathbf{N} \quad (29)$$

Take the singular value decomposition of the (MxN) data matrix: $\mathbf{X} = \mathbf{WLV}^T$ and keep a reduced (MxK) dimensional matrix containing most of the signal power

$$\mathbf{X}_{SV} = \mathbf{WLD}_K = \mathbf{XVD}_K \qquad (30)$$

where $\mathbf{D}_K = [\,\mathbf{I}_K\ \mathbf{0}]^T$, here I_K is a KxK identity matrix and $\mathbf{0}$ is Kx(N-K) matrix of zeros. Also, consider $\mathbf{S}_{SV} = \mathbf{SVD}_K$ and $\mathbf{N}_{SV} = \mathbf{NVD}_K$. We obtain $\mathbf{X}_{SV} = \mathbf{A}(\tilde{\theta})\ \mathbf{S}_{SV} + \mathbf{N}_{SV}$. The matrix $\mathbf{S}_{SV}$ is indexed by i in the spatial dimension and by k in terms of the singular vector and we want to impose sparsity in $\mathbf{S}_{SV}$ only spatially. This can be done by first computing the l_2 norm of all $\mathbf{S}_{SV}$ values for a particular spatial index i

$$\tilde{s}_i^{(l2)} = \left\| s_i^{SV}(1), s_i^{SV}(2), \ldots, s_i^{SV}(K) \right\|_2 \quad , i=1,2,\ldots,\ N_\theta \qquad (31)$$

The sparsity of the resulting N_θ x1 vector $\tilde{\mathbf{s}}^{(l2)}$ corresponds to the sparsity of the spatial spectrum and we can find the spatial spectrum of $\tilde{\mathbf{s}}$ by minimizing the cost function associated with the Frobenius norm

$$\left\| \mathbf{X}_{SV} - \mathbf{A}(\tilde{\theta})\mathbf{S}_{SV} \right\|_f^2 + \lambda \left\| \tilde{\mathbf{s}}^{(l2)} \right\|_1 \qquad (32)$$

Now we have an objective function containing the term $\left\| \tilde{\mathbf{s}}^{(l2)} \right\|_1$ which is neither linear nor quadratic to minimize. We turn to second order cone (SOC) programming which is a suitable framework for optimizing functions [6].

In the case of wideband source localization we work in the frequency domain: the time samples are grouped into several snapshots and transformed into the frequency domain

$$\mathbf{x}^{(n)}(f) = \mathbf{A}(f)\mathbf{s}^{(n)}(f) + \mathbf{n}^{(n)}(f) \quad ; n = 1,\ldots N_s \qquad (33)$$

The signal spectrum is separated into several narrowband regions, each of which yields to narrowband processing. For each frequency f we have N_s snapshots and we are interested in a 2-D power spectrum as a function of both direction of arrival θ and frequency f.

4 Simulation results

We present an example where a linear array of 10 sensors is used to localize 3 narrowband sources with DOAs 50°, 75° and 100°. A SNR of 20 dB is considered and 1000 temporal samples used in the algorithms. Figure 1 shows the localization of these three sources with the conventional beamforming estimator and two high resolution estimators presented in the paper.Using the conventional beamforming we cannot separate the sources. Figure 2 shows the wideband scenario: three wideband signals consisting of or two harmonics are present. At DOA 75° there are two harmonics with frequencies 200 and 550 Hz, at direction of arrival 110° there are again two harmonics with frequencies 200 and 400 Hz and at DOA 85° there is a single harmonic with frequency 550Hz. Figure 2 represents this scenario and we can see the weakness of the conventional beamforming estimator.

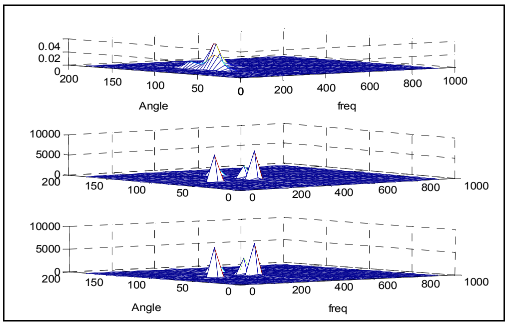

Fig.1 Three sources localization

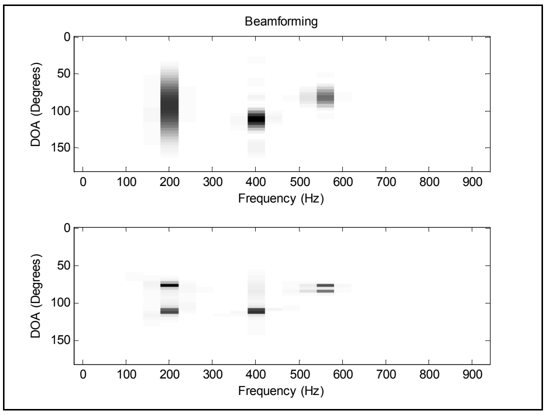

Fig.2 Five sources localization

5 Conclusion

We have presented four methods to source localization and compared these methods with the conventional beamformer. We are exploiting the robustness of the sparse signal reconstruction estimator and compared it with MV and MUSIC estimators.

6 References

[1] P. Stoica, R. Moses, *Introduction to Spectral Analysis* , Prentice Hall (1997)

[2] J. Lardies, ''Power estimation of multiple sources by an acoustical array'', *Journal of Sound and Vibration 145*, 309-320 (1991)

[3] J.Li, P.Stoica, X.Zheng, J.Ward, ''Beampattern synthesis via a matrix approach for signal power estimation'', *IEEE. Trans. on Signal Processing 55*, 5643-5657 (2007)

[4] R.O. Schmidt, '' Multiple emitter location and signal parameter estimation, IEEE Trans. On Antennas and Propagation 34, 276-280 (1986)

[5] D. Malioutov, M. Cetin, A.S. Willsky, A S, ''A Sparse Signal Reconstruction Perspective for Source Localization with Sensor Arrays'', *IEEE Trans. on Signal Processing 53*, 3010-3022 (2005)

[6] J.S. Sturm, *Using DeDuMi 1.02 a Toolbox for Optimization over Symmetric Cones*, Tilburg University, Netherlands (2001)