Ultrasonic behaviour of high pressure air filled porous media

Stephane Griffiths and Christophe Ayrault

LAUM, CNRS, Université du Maine, Av. O. Messiaen, 72085 Le Mans, France
christophe.ayrault@univ-lemans.fr

This paper deals with the acoustic behaviour of porous media when the saturating fluid is high pressured. These observations are performed by ultrasonic transmission through a porous sample with variations of the static pressure of the saturating fluid. In order to characterize high damping materials, measurements are performed for high static pressure (up to 18 bars). It is shown that the behaviour of transmission coefficient and speed with pressure follow the Biot's theory. Moreover, measurements are strongly dependant on temperature, which is not visible in modelizations with Biot's model, although this parameter is taken into account in thermodynamical parameters. It is therefore assumed that mechanical characteristics vary with temperature and pressure. An estimation of mechanical parameters is then performed by minimization between the Biot's model and experimental data. First results, obtained in suitable cases for which measurements quality is good and minimization process converges correctly, show that mechanical parameters follow the evolution with frequency described by Pritz at low frequencies. Further researches are still necessary to determine dependance to temperature.

1 Introduction

The acoustic behaviour of a porous material can be described with two sets of parameters. Parameters linked to the saturating fluid are named in this paper acoustical parameters. They correspond to porosity (φ), tortuosity (α_∞), viscothermal lengths (Λ, Λ') and flow resistivity (σ). Other parameters, linked to the structure, are called here mechanical parameters. They correspond to Young modulus (E), Poisson's ratio (ν_p) and loss factor (η). A precise knowledge of these parameters lead to a good understanding of the porous medium's behaviour submitted to an acoustical or mechanical stimulation.
Many experimental works have proposed diverse methods in order to measure these parameters. Each of them owns its proper advantages and limits and does not allow the simultaneous measurement of all parameters describing the medium. They are then complementary. Beyond all methods available, in order to characterize highly damping materials, a specific ultrasonic method, based on first works by Nagy [1], similar to the frequency method by transmission [2] but using variations of static pressure of the saturating fluid has been developed [3, 4]. With the densification of the saturating fluid, leading to a better coupling between transducers and fluid and a decrease of viscothermal skin depths, this method allows a better transmission through media. Thus it permits to characterize highly absorbant or damping porous media, even in the presence of weak scattering [5].
Previous works have shown that transmission coefficient logarithm $|ln|T||$ and propagation index n_r^2, follow, for a given frequency, a linear behaviour with the variable $1/\sqrt{P_0}$, according to the asymptotic high frequency Johnson-Allard model. Moussatov and al. have shown that this linear behaviour is satisfied for static pressures from 0.2 to 6 bars for different types of media [4].
In order to characterize highly damping materials, a new experimental set-up was developed to realize measurements for higher static pressures included in the range [1-18] bars. In this high pressure range, the behaviour of n_r^2 and $|ln|T||$ differs from the equivalent fluid model one, and is highly sensitive to temperature.
After a short description of the experimental set-up used in this work in a first section of this paper, measurements performed on two different porous media (weakly and highly air-resistive) are presented. It is shown that temperature has a strong effect on measurements. In the following section, limp and Biot's models, with the effect of temperature included, are compared to measurements. Finally, the last section presents first results of mechanical parameter estimation by minimization between the Biot's model and experimental data.

2 Experimental set-up

The experimental set-up is composed of a classical transmission measurement set-up by ultrasounds. A sample of material is placed between a pair of ultrasonic transducers. Measurements in transmission are performed at normal incidence inside a barometric chamber which allows a variation of the saturating fluid static pressure. For each pressure value, decreasing from 19 to 1 bar, measurements of signals propagating in air and through the porous sample are performed. Propagation index and transmission coefficient are deducted from delay and attenuation of the wave which propagates through the porous sample in relation to the one in air. Movement between different positions of the sample is provided by an externally controlled rotation motor. A compressor is used for variations of pressure from 1 to 19 bars. Lastly, an electronic emptying gate allows a controlled emptying of the chamber bar by bar. A dryer drain air from humidity with an hydrophile system which allows us to consider the air in the chamber as dry. Hygrometry is thus not considered for experimentations. A waveform of burst type with 6 sinusoidal periods is chosen to avoid multiple reflections inside the sample and to consider a measurement at a given frequency. Transducers are wide band and allow measurements from 100 to 400 kHz. The barometric chamber also contains temperature and pressure sensors. The experimental set-up is completely automated.

3 Acoustic behaviour at high pressure

3.1 Case of a weakly resistive foam

This section presents the behaviour of a porous foam (M1) for pressure included in the [1-18] bars range. This foam is considered as a reference one, because its acoustical parameters are well known. They are reported in table 1. When static pressure increases, figure 1 shows an apparent linear behaviour up to 7 bars, as an equivalent fluid one. Beyond this pressure value, an increase of the transmission coefficient logarithm $|ln|T||$ (corresponding to a decrease of the transmission) and

Parameters	M1 values	M2 values
Porosity φ	0,95	0.8
Resistivity σ $(N.m^{-4}.s)$	2850	94200
Tortuosity α_∞	1,06	1.11
Λ (μm)	300	11
Density ρ $(kg.m^{-3})$	30.9	29.0
L (mm)	30	6
Young modulus E (kPa)	675	377
Loss factor η	0.27	0.21

Table 1: *Parameters measured with classical methods for the foams M1 and M2.*

a decrease of the propagation index n_r^2 (corresponding to an increase of speed) is observed. Such a behaviour shows clearly that the medium doesn't behave any more as an equivalent fluid in this high pressure range. However, linear regression stemmed from measurements presented in figure 1 are associated to pressure range for which measurement's evolution seems relatively asymptotic (from 1 to 7 bars). Therefore, assuming the equivalent fluid model is valid, acoustical parameters (α_∞ and Λ) can be estimated. Tortuosity is determined from the origin coordinate of the linear regression stemmed from measurements of n_r^2 and viscous characteristic lengths from the slope of linear regressions stemmed from n_r^2 or $|ln|T||$. Measured tortuosity is then equal to 1.07 (reference value is 1.06 with classical measurements). The characteristic viscous length is here estimated from speed greater than 600 μm, namely twice more than the value estimated with classical methods. The estimation of Λ stemmed from transmission is greater than 800 μm which is wildly over estimated. Finally, the apparent linear behaviour at weak pressure ([1-7] bars) can not be described by the equivalent fluid model. The equivalent fluid model, represented on figure 1 with optimized values ($\Lambda_{|ln|T||} = 350\mu m$, $\Lambda_{n_r^2} = 500\mu m$) for best fit, diverges quickly from measurements when P_0 increases.

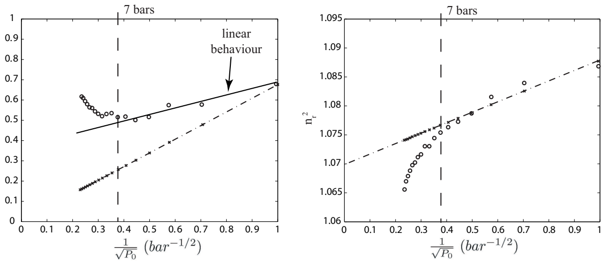

Figure 1: *Transmission coefficient logarithm $|ln|T(P_0)||$ (a) and propagation index $n_r^2(P_0)$ (b) for the foam M1: measurements for 100 kHz (○), asymptotic equivalent fluid model (-· × ·-). Linear regression performed on the first 7 bars (—).*

Moreover, repetition of these measurements highlights that the behaviour observed for $|ln|T||$ on figure 1 is greatly influenced by the temperature. This is described in the next section.

3.2 Influence of temperature

Influence of temperature is mainly observable on transmission coefficient, so measurements of n_r^2 are not reported here. Figure 2 presents the evolution of the transmission coefficient logarithm at 100 kHz versus pressure for 11 different measurements and the evolution of the corresponding temperature. It appears a large disparity in $|ln|T||$ measurement's curves. However, it can be noticed, in spite of this disparity, that 4 groups can be separated containing curves which have the same absolute position. A parallel can be done between these absolute position disparities seen on figure 2 a) and temperatures measured for each transmission coefficient measurements (fig. 2 b)). For one group of $|ln|T||$ measurements, their temperatures are similar, leading to similar transmission coefficient measurements.

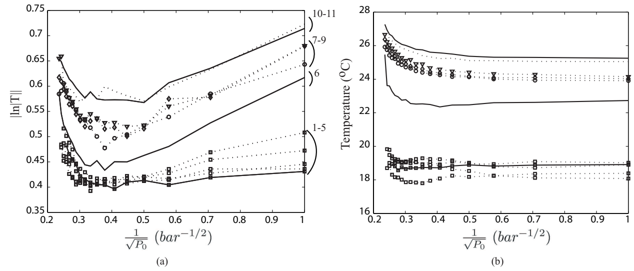

Figure 2: *Repeatability of transmission coefficient at 100 kHz (a), and corresponding temperatures (b).*

It is very difficult to evaluate the measurement repeatability, this one being sensitive to temperature variations inside the barometric chamber and to atmospheric temperature in the room. Indeed, experimentation's room is not air-conditioned, and the temperature evolves differently with the pressure decreasing process, in spite of identical measurements conditions, probably due to external temperature variations. However, it is possible to conclude that in a similar temperature range (with differences lower than one degree), measurement is repeatable.

3.3 Case of a highly resistive foam

In order to apply this method to highly damping media, measurements of transmission coefficient at high pressure have been performed for a highly resistive foam (M2). Moreover its Young modulus is quite twice less than the M1's one, which means that this foam is softer than the previous one. Its known parameters, measured with classical methods, are reported in table 1.

Let's notice the thickness of this foam which is small. Such a thickness, facilitating the transmission, allows the use of classical method to determine acoustical parameters, but leads to a decrease of the measurements precision (as for Λ for instance).

This material presents a behaviour of transmission coefficient at 100 kHz (figure 3) very different from the one observed for the foam M1. Indeed, the increase of $|ln|T||$ observed at high pressure for M1 doesn't exist anymore here, but a weak decrease appears. The equivalent fluid model simulated with reference values parameters can not describe the experimental behaviour of $|ln|T||$ for this foam M2. To sum up, several behaviour of $|ln|T||$

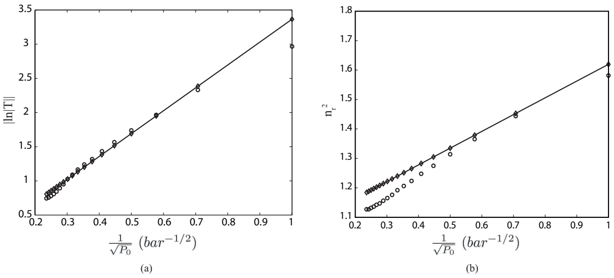

Figure 3: *Transmission coefficient $|ln|T(P_0)||$ (a) and propagation index $n_r^2(P_0)$ (b) for the foam M2. Measurements for 100 kHz (○) from 1 to 18 bars and comparison to Johnson-Allard model (-◇-).*

and n_r^2 have been observed for two different foams according to their air resistivity and their Young modulus. The temperature has also a great influence on measurements. In order to model the behaviour of such media, mechanical parameters and temperature must be taken into account in the model. Next section presents two analytical models liable to explain the observed behaviour and how temperature is taken into account in these models.

4 Analytical description

If the assumption that the internal geometry of the porous medium doesn't vary is taken into account, several assumptions can already be done on the vibratory behaviour of the medium. First of them is that, due to pressure increase, the density of air around the internal structures of the medium become sufficient to make structures vibrate at the frequency of the ultrasound wave. The medium's skeleton begins thus to vibrate and the Biot's model is used. The model used in this work is written according to solid and total displacements $[u^s,u^t]$ of the medium [6]. It presents the advantage of being a simplification of Biot's model without adding additional assumptions, and being an analytic formulation, which is much easier to handle than a numerical one for inversion problems.
A second assumption is to consider a movement of the whole medium under the acoustic wave's effect, and neither a movement of the skeleton led by the vibration of each internal structure of the medium and the limp model is used. In this case, the stiffness of the medium is null. The strain energy of the solid phase is negligible compared to those of the other mechanisms of the propagation [6]. Thus, limp model seems to be a simple solution to model the behaviour of porous medium for such pressure. It takes into account the density of the medium and is principally used for negligible stiffness media. It means that, compared to air viscosity and density which increases under effect of pressure, the medium is considered as a soft medium. As the equivalent fluid model, this model considers only one compressive wave. However, solid structure is not motionless and the skeleton's inertia is taken into account. The limp model is thus a Biot's model simplification where porous medium's stiffness is neglected.
A few calculations and simplifications of the Biot's model leading to these two models are fully described in [6], and are not expressed here.
Furthermore, a strong correlation between measurements and temperature has been shown in previous section. Temperature is then taken into account in models with the use of a real gas state law where the expressions of specific heat for constant pressure and volume are expressed according to pressure and temperature [7]. The temperature is also taken into account in the expression of the air dynamic viscosity with a linear relation [8]. Other thermodynamical parameters used in both models don't depend on temperature and/or pressure in the range used in this work.
The influence of temperature is then studied with the Biot's analytical model $[u^s,\ u^t]$. Simulations of $|ln|T||$ made for temperatures between 20 and 29 degrees indicate that the effect of temperature in thermodynamical parameters is negligible on transmission coefficient in the considered pressure range. It is then possible to consider the assumption that mechanical parameters themselves vary with temperature. This was already discussed in the literature at low frequencies [9] and could explain the experimental behaviour observed for $|ln|T||$.

4.1 Comparison to measurements

Figure 4 presents for the foam M1 (reference foam) a comparison between experiments and asymptotic Johnson-Allard, limp and analytical Biot's models for the transmission coefficient logarithm and the propagation index. First, with same optimized parameters used in fig-

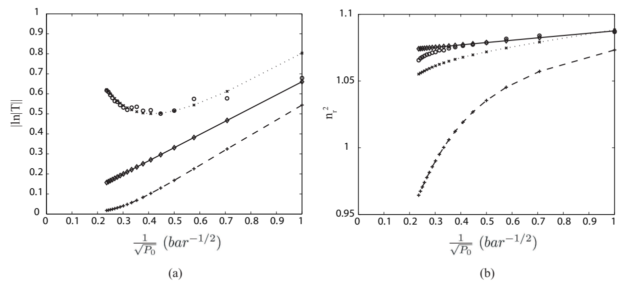

Figure 4: *Transmission coefficient logarithm $|ln|T(P_0)||$ (a) and propagation index $n_r^2(P_0)$ (b) for the foam M1 at 100 kHz: comparison between experimental data (○) and simulations with Johnson-Allard model (—◇—), limp model (- -+- -), and analytical model ($\cdots\times\cdots$).*

ure 1, it appears a large difference between asymptotic Johnson-Allard model and limp model on the whole pressure range. The limp model presents a behaviour looking like the experimental behaviour of the medium. A weak increase of the modelized transmission coefficient logarithm and a decrease of the propagation index can be observed. However, the increase at high pressure, in the case of transmission coefficient logarithm, is not as important as the experimental one. The range of parameters used for fitting with this model doesn't allow a better fit to the experimental data.
On the other hand, Biot's analytical model is well fitted

on experimental data. In this case, acoustical parameters of the foam M1 have been measured with classical methods and are kept constant. The fit is carried out manually by the variation of mechanical parameters (E, η and ν_p) from their quasi-static measured values for E and η (table 1) and with a variable value of ν_p between 0 and 0.5. It appears clearly that Biot's model is well suited to describe the acoustic behaviour of porous media with static pressure. Therefore, it should be possible to estimate acoustical and mechanical parameters from such measurements by an inversion process. The next section presents first results of such mechanical parameters estimation by minimization.

5 Mechanical parameters estimation

Direct simulations have shown that mechanical parameters have a predominant influence at high pressure. Moreover, using too many parameters for the minimization yields divergence problems in the process. Therefore, in this work, minimization is processed only on mechanical parameters (acoustical parameters are kept fixed to their reference values).

5.1 Application to a weak resistive medium

Figure 5 a) presents, for the foam M1, the minimization for one measurement (curve 6 on figure 2, obtained at $23°C$) for pressure in the range [4-18] bars. Estimated

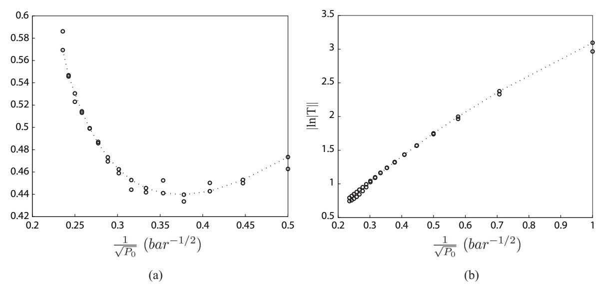

Figure 5: *Fit of the Biot's analytical model on measurements of the transmission coefficient logarithm at 23° C for the foam M1 a) and for the foam M2 at 23° C b).*

values of the Young modulus and of the viscous damping for this fit at 100 kHz can be considered as acceptable in comparison to quasi-static measurements (tab. 2). The Young modulus value is higher than quasi-static values which is in agreement with the evolution of this parameter with frequency at low frequency [10]. Such values of E and η are obtained in the pressure range [1-18] bars for a temperature which is not constant all over this pressure range (it varies more than $1°C$ for pressure greater than 15 bars). Secondly, such a fit was possible for a value of ν_p equal to 0.47. This estimated value of the Poisson's coefficient is very high but is difficult to estimate in practice even at low frequencies.
Although the variation of temperature and the overestimated value of the Poisson's coefficient, the minimization can thus be considered as successful. However, due to the instability of minimization, it was not possible to extract correctly these mechanical parameters at different temperatures and to study their evolution with temperature (see section 5.3).

Parameters	at 5 Hz	at 100 Hz	at 100 kHz
E (kPa)	513	675	830
η	0,16	0,27	0.2

Table 2: Mechanical characteristics measured for the foam M1. Quasi-static method at 5 and 100 Hz and minimization from $|ln|T(P_0)||$ at 100 kHz.

5.2 Application to a high resistive medium

The same procedure is applied to the high resistive foam M2. Figure 5b presents the minimization of the analytical Biot's model with $|ln|T||$ measurements of the foam M2. Estimated values of the Young modulus and of viscous damping can be considered as acceptable in comparison to quasi-static measurements (tab. 3). Such

Parameter	at 5 Hz	at 100 Hz	at 100 kHz
E (kPa)	327	377	594
η	0,12	0,21	0.28

Table 3: Mechanical characteristics measured for foam M2. Quasi-static method at 5 and 100 Hz and minimization from $|ln|T(P_0)||$ at 100 kHz.

value of E and η are obtained in the pressure range [1-18] bars for a temperature which is not constant all over this pressure range (with a variation of 1 or 2°C). Such a fit was possible for a value of ν_p equal to 0.47, which is probably very high.
The increase of the Young modulus value with frequency observed in table (3) agree with Pritz research [10]. Moreover, the estimated value of structural damping can be considered as acceptable, in regard to the dependence on frequency of this parameter. Finally, except for the Poisson's coefficient which has a very high estimated value and the temperature variation, the minimization gives coherent and acceptable values.

5.3 Discussion

The extraction of mechanical parameters by the fit of the Biot's analytical model on experimental data is a method, at the present time, complex. Indeed, the strong interdependence between parameters leads to different sets of parameters for a good fit, leading difficult the convergence to a global minimum. Starting value of parameters is a capital factor for minimization and this value should be chosen nearby quasi-static measurements. Consequently, at the present time of the experimentation, it is not possible to conclude on the evolution of mechanical parameters with temperature. Fluctuations of room's temperature, added to temperature evolution due to pressure variation, lead to temperature gaps reaching 3 degrees for one measurement. Mechanical parameters depending *a priori* on this variable at studied frequencies, their values change during one measurement, and the estimated value from minimization are probably averaged values of these parameters. The presence of an air conditioning allowing to stabilize the temperature in the room or in the chamber is thus necessary.

6 Conclusion

With this transmission ultrasonic method at variable pressure, measurements of transmission and speed can be proceeded up to 19 bars. Three models have been tried to modelize the acoustic behaviour at high pressure. Only Biot's model, which takes into account the mechanical parameters fits well on the experimental data. Moreover, it appears with repetition of transmission coefficient measurements, that the temperature has a strong effect on the behaviour of the medium. Taken into account into the Biot's analytical model, used here, temperature does not explain this behaviour. Then, mechanical parameters should be sensitive to temperature. A minimization between Biot's analytical model and experimental data is proceeded to estimate the mechanical parameters and to study their possible sensitivity to temperature. The Young modulus and the loss factors are well estimated in suitable cases and seem acceptable but the Poisson's coefficient is over estimated and the temperature is not constant all over the pressure range. In order to realize good inversion process and to study the evolution of mechanical parameters with temperature, the experiment has to be controlled in temperature and the algorithm of minimization has to be optimized.

References

[1] P. B. Nagy, and D. L. Johnson, Appl. Phys. Lett., **68(26)**, pp. 3707-3709, 1996.

[2] P. Leclaire, L. Kelders, W. Lauriks, C. Glorieux, and J.Thoen, J. Acoust. Soc. Am. **99 (4)**, pp. 1944-1948, 1996.

[3] C. Ayrault, A. Moussatov, B. Castagnède and D. Lafarge, Appl. Phys. Let., **74(21)**, pp. 3224-3226, 1999.

[4] A. Moussatov, C. Ayrault and B. Castagněde, Ultrasonics, **39(3)**, pp. 195-202, 2001.

[5] C. Ayrault and S. Griffiths, Ultrasonics, **45(1-4)**, pp. 40-49, 2006.

[6] O. Dazel, B. Brouard, C. Depollier and S. Griffiths, J. Acoust. Soc. Am., **121(6)**, pp. 3509-3516, 2007.

[7] O. Cramer, J. Acoust. Soc. Am., **93(5)**, pp. 2510-2516, 1993.

[8] D.R. Lide and H.V. Kehiaian, *CRC Handbook of thermophysical and thermochemical data*, CRC Press Inc, 1994.

[9] M. Etchessahar, PhDthesis, Université du Maine, Le Mans, France, 2002.

[10] T. Pritz, J. of Sound and Vibration, **214(1)**, pp. 83-104, 1998.