The modelling of vibration transmission through plate/beam structures typical of lightweight buildings

Laurent Galbrun

Heriot-Watt University, School of the Built Environment, EH14 4AS Edinburgh, UK
l.g.u.galbrun@hw.ac.uk

Vibration transmission through plate/beam structures typical of lightweight buildings is examined in this paper. Modelling difficulties are analysed and related to the several modelling choices available, including connection type, beam modelling, modal properties and material properties. A number of key experiments carried out on structures made of one or two parallel plates attached with screws to a single beam are discussed, and the applicability and limitations of fundamental theories are identified through the use of Statistical Energy Analysis (SEA) models. It is shown that the interrelation between the various modelling factors can lead to complex models. The structures and frequency ranges for which simple point models can be used are identified, as well as those for which more complex models need to be used.

1 Introduction

Most of the basic theories used to predict vibration transmission through plate/beam structures are well established. However, modelling transmission through a simple system made of a single plate and a single beam is still far from being an easy task, because of the numerous modelling factors to consider. These factors include connection type, beam modelling options, beam modal properties and anisotropic characteristics of materials.

Applications found in the literature underline some of the difficulties related to modelling, but the applicability and limitations of existing theories are not well known as only few structures have been tested to date. The aim of this paper is therefore not to predict the performance of particular systems by developing new complex models, but rather to use examples as a means of understanding the difficulties involved in modelling and the consequences of different approaches. This is particularly relevant for the plate/beam systems examined, in which real junctions are often far from the idealised conditions considered by fundamental theories.

The structures tested in this paper consist of a timber beam connected to either one or two parallel plasterboard plates, as found in lightweight buildings (Fig. 1). Although simple structures, these are fairly complex systems because of the non-rigid nature of junctions and orthotropic properties of timber. This has allowed examining the importance of the modelling factors mentioned above for lightweight constructions.

Statistical Energy Analysis (SEA) is the framework of analysis that has been used for modelling, but it should be noted that the fundamental problems discussed are independent from SEA and are related to boundary conditions and other fundamental modelling options (i.e. problems would be identical if, for example, finite element modelling had been used).

The paper begins by briefly describing the background theory of vibration transmission in plate/beam systems and then uses a series of experiments to highlight the modelling difficulties for these structures and to identify systems for which existing models are valid and systems for which more complex models are needed.

2 Background theory

There are several options available for the modelling of structure-borne sound through plate/beam systems. These include connection type, beam and plate modelling options, finite or infinite length modelling for the beam, and material characteristics. These are briefly outlined below and no references are made to equations, as details of the relevant theories can be found in the literature (e.g. [1, 2]).

2.1 Connection type

In timber-frame constructions the plates and beams are typically attached together with screws or nails. Depending on the spacing between screws, it has been shown that the coupling occurs along a line (screws close to each others) or at points (screws widely spaced). Craik and Smith [3] found that an appropriate transition frequency between the line and point theories occurs where half bending wavelength on the plate fits between the nails or screws that connect the plate to the frame.

Semi-infinite plates are commonly used to describe line junctions, while power transmission and impedances (or mobilities) are commonly used to calculate sound transmission through point connected structures [2]. More complex point models have also been developed using the wave approach for coupled infinite plates connected by a narrow tie beam [4] and coupled semi-infinite plates connected by elastic interlayers [5]. The model developed by Bosmans and Vermeir [5] is the most accurate, as it can take into account the distance between points, as well as the influence of the density and the width of point connections, and can therefore cover the transition between line and point behaviours. In this model, plates are connected by a mass-less junction beam. It is appropriate to describe point connected plates, but not plate/beam systems. For point connected models, sophisticated mobility measures including the effective contact area can be found in references [6, 7].

2.2 Beam modelling

Three factors need to be considered when modelling a beam. Firstly, a beam can be modelled as a one-dimensional system (a beam), a two-dimensional system (a plate) or a three-dimensional solid. Secondly, thin or thick theories can be used, the latter taking into account rotatory inertia and shear deformation which can be important at high frequencies. Thirdly, a beam can be modelled either as a subsystem (if statistical energy analysis is used) or as an element. If the beam is modelled as a SEA subsystem, its motion is determined by free waves and the vibration field is assumed to be diffuse. If it is modelled as an element, its motion supports forced waves and the vibration field is not diffuse. The complexity increases from thin to thick theories, as well as from SEA subsystem to element modelling. For plate/beam systems, several applications of these modelling options can be found in the literature [2].

2.3 Plate modelling

In a plate/beam assembly, the plate can be modelled either as a thin or thick plate. For lightweight structures, the plates are typically made of plasterboard, plywood or chipboard, for which the thickness is small (as a rule of thumb, the limit of applicability of thin theories is where the wavelength is less than six times the thickness [8]). Thin plate theory is then acceptable even at high frequencies and thick plate theory does not need to be used. However, this might not be the case for systems where plates are thicker.

2.4 Beam length (finite vs. infinite)

Beams used in lightweight partitions are generally only few metres long. They can then exhibit large fluctuations in response due to their low mode count and modal overlap, and these modal properties can be modelled by taking into account the finite length of the beam [8]. Craik and Galbrun [9] showed that details of the fluctuations can be determined for laboratory situations and that limiting bands can be determined for field results of timber frames. However, for line connected systems, where the beam is modelled as a plate element, the use of a finite length plate element would significantly increase complexity.

2.5 Material characteristics

In lightweight buildings, beams are made of timber which is a non homogeneous and orthotropic material. Properties of timber vary with direction (anisotropy) and position (non homogeneity) across the width, depth and length of the beam. The Young's modulus can be measured in three orthogonal directions (axial, radial and tangential) which are relative to timber rings. Depending on the direction in which sound waves travel, these orthotropic properties can then be taken into account by models. For transmission through a timber frame [9], waves travel along the beam's length and the axial Young's modulus is then used. For transmission through plate/beam systems, and depending on the connection type, waves can travel along the beam's length (axial Young's modulus) or across its depth (in which cases averages of the axial/radial/tangential values are generally considered [3]). The complex variation of properties with direction and position is difficult to model, and that is why simplifications are normally introduced in models.

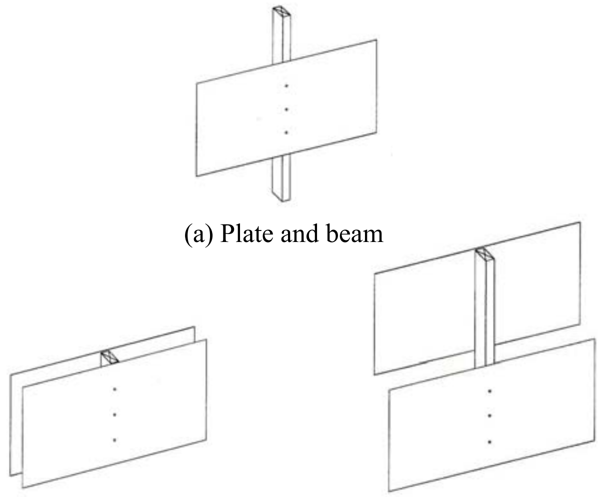
2.6 Discussion

Most of the problem with plate/beam systems is in deciding how to model the beam, choice that relates in turn to the connection type. This choice can therefore vary depending on whether the system is point connected or line connected.

For point connected systems, the beam is normally modelled as a beam (i.e. one-dimensional system), but for lightweight structures there is no experimental evidence to prove that this is the best way to model a beam. Additional experiments to those made by Craik and Smith [3] are discussed in this paper in the attempt to provide a better view of applicability and limitations of modelling a beam as a beam in point connected plate/beam systems.

3 Results

The structures examined are illustrated in Fig.1 and material properties are given in Table 1. Through the use of SEA models, several of the modelling factors previously described are tested and discussed below.



(a) Plate and beam

(b) Two plates opposite (c) Two plates offset

Fig.1 Test structures (drawings not to scale).

	Plates			Beams		
Dimensions (m)	L_x	L_y	L_z	L_x	L_y	L_z
	2.4	1.2	0.0125	0.05	-	0.1
Density	680 kg/m^3			480 kg/m^3		
Young's modulus	2.4×10^9 N/m^2			9.0×10^9 N/m^2		
Poisson's ratio	0.2			0.3		
Damping (ILF)	0.008			0.010		

Table 1 Material properties of the structures tested (Young's modulus measured in the axial direction of the beam). All values were measured except Poisson's ratio that was assumed. The beam length L_y is specified in the text for each of the structures.

3.1 Coupling from a plate to a beam

In order to obtain a proper understanding of transmission mechanisms, it is important to initially examine transmission between a plate and a single beam (Fig.1(a)). In this example the plate is attached with either 1, 3 or 7 screws to a 2.3 m long beam.

For the 1 screw case, this simple system can be properly described as point connected. For the 3 and 7 screws cases, it behaves as point connected over most of the frequency range (the transition between line and point connected behaviour is calculated by using the half wavelength rule of thumb given by Craik and Smith [3]). The beam can be described as a beam, and only bending waves need to be considered for the excitation used. The SEA model has then only two subsystems and the coupling can be computed using the theory for point coupling as [3]

$$\eta_{12} = \frac{r\,\mathrm{Re}(Y_2)}{\omega m_1 \left|Y_1 + Y_2\right|^2} \quad (1)$$

where Y is the mobility (inverse of impedance), m is the mass of the respective subsystems and r is the number of point connections.

The beam and plate point force mobilities used are [8],

$$Y_b = \frac{1}{2\rho_l \left[\frac{B_b \omega^2}{\rho_l}\right]^{1/4} (1+i)} \quad \text{(beam – centre excited)} \quad (2)$$

$$Y_p = \frac{1}{8\sqrt{B_p \rho_S}} \quad \text{(plate - centre excited)} \quad (3)$$

where ρ_s is the surface density of the plate, ρ_l is the mass per unit length of the beam, B_p is the plate bending stiffness per unit width and B_b is the beam bending stiffness.

In the experiment the plate was space average excited by a plastic headed hammer and the response of the plate and beam were measured, from which the level difference was computed for comparison with the theory. As screw connections can be a bit variable, the experiment was repeated with washers between the plate and the beam, to ensure that there was no physical contact other than at the screw points. It can be noted that the difference between the experiments with and without washers is significant only above 2 kHz (Fig.2), regardless of the number of screws used. However, no differences are found below 2 kHz, suggesting that the use of spacers is not effective at reducing transmission at low frequencies. Nightingale *et al.* [10] already pointed out the importance of the contact area for transmission through plate/beam systems.

The beam exhibits significant modal behaviour resulting in large fluctuations in the level difference, due to the low number of modes present in the beam (low mode count and modal overlap). If the beam is modelled with a finite length, these fluctuations can be reproduced by using the corrected coupling loss factor [9]

$$\eta_{12} = \eta_{12\infty} \frac{\mathrm{Re}(Y_2)}{\mathrm{Re}(Y_{2\infty})} \quad (4)$$

where η_{12} is the actual coupling loss factor, $\eta_{12\infty}$ is the standard coupling loss factor usually calculated from infinitely extended subsystems, Y_2 is the mobility of the receiving subsystem and $Y_{2\infty}$ is the mobility of the receiving subsystem assuming it to be infinitely extended. This equation can be used at all frequencies as $\mathrm{Re}(Y_2)/\mathrm{Re}(Y_{2\infty})$ tends to unity as the frequency increases.

It can be seen in Fig.2 that up to 1 kHz there is good agreement between the measured and predicted level difference. The low frequency correction accurately predicts both the magnitude of the fluctuations and the frequency where these occur (the modal frequencies were computed for free-free boundary conditions of the beam edges). The fluctuations in the level difference decrease as the number of connections increases and this is correctly predicted by the theory.

According to the rule of thumb given by Craik and Smith [3], the transition between line and point connected behaviours occurs at 125 Hz for the 3 screws case and at 400 Hz for the 7 screws case. However, even at frequencies where line behaviour is expected, point connection theory works well at predicting the level difference.

At high frequencies it can be seen that the level difference decreases sharply with a large dip at about 4 kHz. It is believed that this is because the timber beam is no longer behaving as a one-dimensional system (a beam) but is supporting different wave types [2]. Volumetric deformation is also likely to occur across the beam, as it is not incompressible [10]. The Timoshenko wave equation for tick beams, instead of thin beam theory, can increase the modal density, and for the system considered changes in the order of 3 dB can be expected at 10 kHz [8]. This difference is however too small to justify the 10 dB difference observed between the measured and predicted level differences of Fig.2. In addition, at these high frequencies even the Timoshenko equation is not valid [8].

These results suggest that the beam might actually be better modelled as a three-dimensional solid, which would however increase significantly the complexity of the model.

This experiment has shown that the simple system of one plate attached to one beam with few screws can be properly modelled as a point connected system in which the beam is modelled as a one-dimensional system (i.e. a beam), except at high frequencies where it is suggested that additional wave types are present. It has also shown that modal fluctuations of the beam can be accurately predicted.

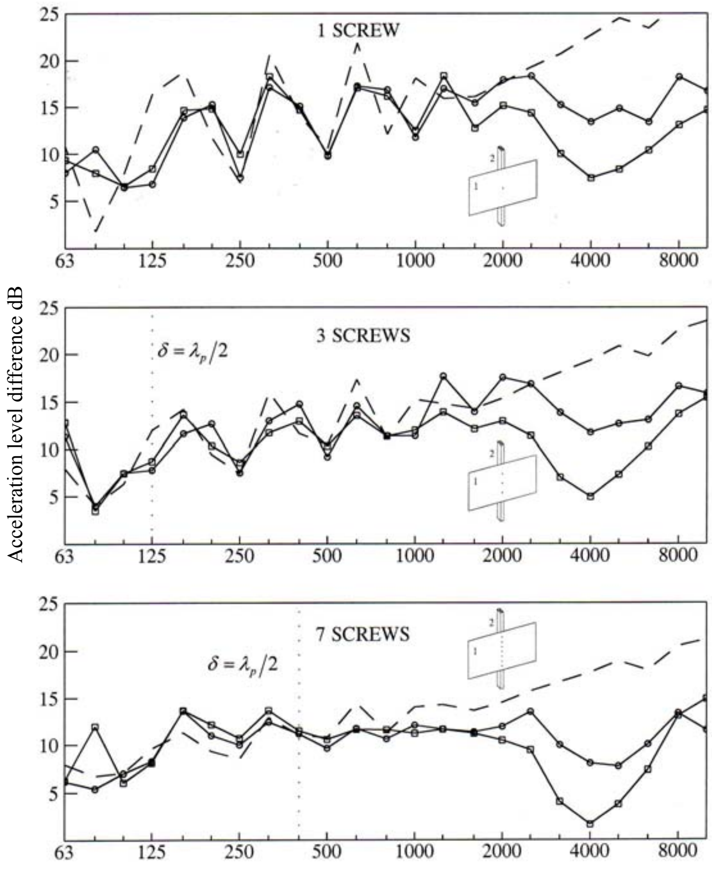


Fig.2 Measured and predicted acceleration level difference between plate and beam attached with 1, 3 or 7 screws. —□—, measured with plate screwed directly to beam; —○—, measured with spacers between plate and beam; – – – –, predicted with low frequency correction; · · · · · · ·, frequency at which half bending wavelength on the plate fits between the screws.

3.2 Coupling between two plates through a beam

A natural development of the previous experiment is to examine transmission between two plates connected to a beam (Fig.1(b)). This experiment was carried out with the screws directly opposite and not opposite. Unlike the previous experiment, no washers were used and the beam was 1.2 m long.

Two coupling formulae were used for predictions, the first assuming that screws connecting the parallel plates are not opposite (Eq.(1)), and the second one assuming that plates (subsystems 1 and 3) are connected through the beam (subsystem 2) with screws opposite, in which case the coupling is given by [3]

$$\eta_{13} = \frac{r}{\omega m_1} \mathrm{Re}\left(\frac{1}{Y_3}\right) \frac{|Y_e|^2}{|Y_1 + Y_e|^2} \tag{5}$$

where Y_e is the combined effective mobility of the plate and beam and is given by

$$\frac{1}{Y_e} = \frac{1}{Y_2} + \frac{1}{Y_3} \tag{6}$$

It can be seen in Fig.3 that the standard SEA theory (no mode correction) predicts that if the screws are directly opposite, then the level difference will be slightly reduced and this is reflected in the measured results. There is good agreement between the measured and predicted results not including mode correction, though again there is a significant dip in the level difference above 2 kHz. This provides more evidence to the hypothesis previously mentioned for which this dip is related to the presence of additional wave types. As no spacers were used, this dip is expected to be more pronounced than if spacers had been introduced in the system, as was shown in Fig.2.

When mode correction is applied to the beam using Eq.(4), it can be seen that, for the 1 screw case, fluctuations predicted are significantly larger than measured data and the frequencies at which the dips occur are not predicted accurately. Predicted fluctuations are larger for screws not opposite, damping being lower. Comparison with measured results suggests that the offset considered (approximately 200 mm) might then not be sufficient to consider screws independent (i.e. not opposite) when no spacers are used. Reasonable estimates are obtained for the 3 screws case, fluctuations being less pronounced (increased damping).

For both cases, uncertainties in the modal frequencies can probably be attributed to the interaction between the plates and beam, boundary conditions along the connected sides not being taken into account by the mode correction.

Tests were repeated with a 2.6 m long beam and results obtained were similar, suggesting that modal properties of the beam do not influence transmission significantly.

Transmission through offset parallel plates was then examined. The offset was taken large enough to have the plates not facing each others (Fig.1(c)). The aim of this test was to verify the validity of point models, theory assuming that for screws not opposite results are independent of the amount of offset (Eq.(1)). No washers were used and the beam was 2.6 m long.

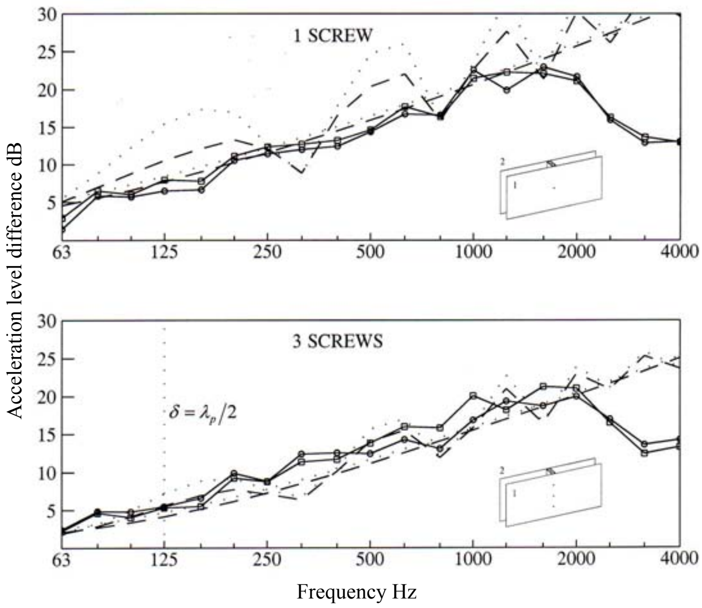


Fig.3 Measured and predicted (standard SEA theory and with low frequency correction) acceleration level difference between opposite parallel plates attached with 1 or 3 screws. —□—, measured with screws not opposite; —○—, measured with screws opposite; – – – –, predicted with screws opposite; · · · · · ·, predicted with screws not opposite.

Results displayed in Fig.4 show that there is a significant difference between the opposite and offset plates measured results, difference that varies approximately between 5 and 10 dB for the 1 screw case and is slightly smaller for the 3 screws case. For the 1 screw case, this reduced transmission can be partly justified by introducing mode correction, but the extent of fluctuations is again larger than what is observed and the frequency dips are not predicted accurately. Unlike the 1 screw case, mode correction does not reduce transmission significantly when 3 screws are used.

Transmission between plate 1 and the beam was examined, in order to verify if the same problem was present. Measurements were taken between plate 1 (source) and the lower part of the beam (opposite plate 1) and between plate 1 and the upper part of the beam (opposite plate 2). This time, results were close for most of the frequency range, and the significant difference observed for opposite and offset plates was not present [2].

The plate/plate results and plate/beam results suggest that the mechanism of transmission between offset plates is more complicated than simple offset points, so that transmission can then not be predicted by Eq.(1). Results suggest that the amount of offset considered can influence transmission. Boundary conditions between the plates and beam might vary and forced waves might be present in the beam, in addition to free waves.

However, it should be noted that if the beam is modelled as an element (forced waves) and not as a beam (free waves), the element would normally be infinitely long so that no boundary conditions are present at the edges and no modal properties are associated with reflections at the two edges. It is then not possible to predict fluctuations due to modal behaviour of the beam, as was made for the single plate and beam structure.

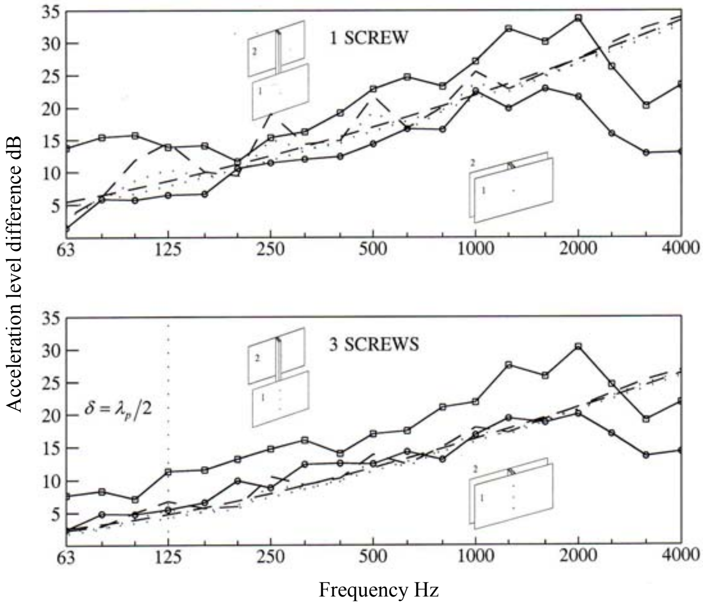


Fig.4 Measured and predicted (standard SEA theory and with low frequency correction) acceleration level difference between parallel plates offset or opposite, attached with 1 or 3 screws. —□—, measured with plates offset; —○—, measured with plates and screws opposite; – – – –, predicted with screws not opposite; · · · · · ·, predicted with screws opposite.

If the connection element is to be modelled with a finite length, in order to reproduce modal behaviour, considerable complications will arise in the modelling due to the forced waves present at the edges.

The two experiments discussed in this section have shown that modelling the beam as a one-dimensional subsystem and using point theories produces good results for opposite plates but not for offset plates, raising the question of how to alternatively model the junction and beam. Results have confirmed the presence of a high frequency dip, as observed in the single plate and beam structure, and fluctuations in response observed were smaller than what was expected from modal fluctuations of the beam.

4 Discussion

Vibration transmission through plate/beam lightweight structures has been examined through a number of key experiments. For simple point systems such as plate/beam and parallel opposite plates, it was shown that the beam can be properly modelled as a one-dimensional subsystem (i.e. a beam), with the exception of a high frequency dip probably related to the presence of waves in the beam other than bending waves. At these high frequencies, the beam might be better modelled as a three-dimensional solid.

For offset plates, point models could not represent transmission properly and the beam did not appear to behave as a simple one-dimensional system. Forced waves might be present in the beam, which might then be better modelled as a junction element.

It is also interesting to note that if the beam behaves as a one-dimensional system, modal behaviour and large fluctuations in responses are expected, but this is not always what is observed in practice. Large fluctuations were present in the plate/beam system, but not in parallel plate systems. When large fluctuations were present (i.e. plate/beam system), a low frequency correction was included in the models which could accurately predict both the magnitude of the fluctuations and the frequency where these occur.

5 Conclusions

All the results discussed suggest that, for plate/beam structures typical of lightweight buildings, the behaviour of the beam varies. Sometimes it behaves as an independent beam (with modal properties), sometimes as a junction element and sometimes as a three-dimensional solid. Plate/beam theories based on mobilities can be used for simple point connected systems at low and mid frequencies, but results suggest that more complex models are needed to be able to predict transmission through the variety of structures found in lightweight buildings.

The analysis presented was limited to point connected systems, but it can be noted that similar problems are found in line connected structures [2].

References

[1] R.J.M. Craik, "Sound transmission through buildings using statistical energy analysis", *Gower*, UK (1996)

[2] L. Galbrun, "Vibration transmission in timber-framed buildings", Ph.D. thesis, Heriot-Watt University, Edinburgh, UK (2004)

[3] R.J.M. Craik, R.S. Smith, "Sound transmission through lightweight parallel plates. Part II: structure-borne sound", *Applied Acoustics*, 61 (2), 247-269 (2000)

[4] M.C. Bhattacharya, K.A. Mulholland, M.J. Crocker, "Propagation of sound energy by vibration transmission via structural junctions", *Journal of Sound and Vibration*, 18 (2), 221-234 (1971)

[5] I. Bosmans, G. Vermeir, "Diffuse transmission of structure-borne sound at periodic junctions of semi-infinite plates", *J. Acoust. Soc. Am.*, 101 (6), 3443-3456 (1997)

[6] B.A.T. Petersson, M. Heckl, "Concentrated excitation of structures", *Journal of Sound and Vibration*, 196(3), 295-321 (1996)

[7] B.A.T. Petersson, "Concentrated force excitation of deep beams", *Journal of Sound and Vibration*, 224(2), 243-266 (1999)

[8] L. Cremer, M. Heckl, E.E. Ungar, "Structure-borne sound", *Springer-Verlag*, 2nd edition, Berlin (1988)

[9] R.J.M. Craik, L. Galbrun, "Vibration transmission through a frame typical of timber-framed buildings", *Journal of Sound and Vibration*, 281(3-5), 763-782 (2005)

[10] T.R.T. Nightingale, K. Kohler, J. Rohlfing, "On predicting structure borne sound transmission from wood studs to direct-attached gypsum board", *Proceedings of the International Congress on Acoustics 2004,* Kyoto, Japan, April 4-9 (2004)