Acoustics'08 Paris
June 29-July 4, 2008
www.acoustics08-paris.org

Measurement of total sound energy density in enclosures at low frequencies

Finn Jacobsen

Acoustic Technology Department, Technical University of Denmark, Ørsted Plads, B352, DK-2800 Lyngby, Denmark
fja@oersted.dtu.dk

Many acoustic measurements, e.g. measurement of sound power and transmission loss, rely on determining the total sound energy in an enclosure. This quantity is usually estimated by measuring the mean square pressure at a number of discrete positions. The idea of measuring the total energy density instead on the assumption that this quantity varies less with the position goes back to the 1930s. However, until recently measurement of the total sound energy density has required an elaborate arrangement based on finite different approximations using at least four matched pressure microphones; therefore the method has never come into use. With the advent of a three-dimensional particle velocity transducer, 'Microflown', it has become somewhat easier to measure total rather than only potential energy density in a sound field. This paper examines the spatial variation of potential, kinetic and total sound energy density in enclosures theoretically and experimentally.

1 Introduction

Many acoustic measurements rely on determining the sound energy in an enclosure. Examples include standardised measurements of sound power and transmission loss in reverberation rooms. The total sound energy is usually estimated by measuring the mean square pressure (that is, the potential sound energy density) either at a number of discrete positions or using a moving microphone, and much effort has been spent on developing efficient averaging procedures [1, 2]. The idea of measuring the total energy density rather than the potential energy density on the assumption that the former quantity varies less with the position than the latter goes back to the 1930s [3] and has occasionally been discussed in the literature [4]. In the late 1970s the phenomenon was analysed using a stochastic interference model of a diffuse sound field [5], and in the late 1980s the matter was examined experimentally for the first time [6]. However, until recently measurement of the total sound energy density has required an elaborate arrangement based on finite difference approximations using at least four pressure microphones [6-9]. The four (or six) microphones should be amplitude and phase matched very well, and the signal-to-noise ratio is poor because the finite difference signals should be time integrated [10]; therefore the method has never been much used in practice. However, with the advent of a three-dimensional particle velocity transducer, 'Microflown' [11], it has become somewhat easier to measure total rather than only potential energy density in a sound field, as demonstrated by a recent investigation [12]. This paper examines the spatial variation of potential, kinetic and total sound energy density in enclosures theoretically and experimentally.

2 The spatial statistics of potential, kinetic and total energy density

2.1 Above the Schroeder frequency

At frequencies above the Schroeder frequency [13] the problem seems to have been solved. On the basis on the following stochastic pure-tone diffuse-field interference model, originally developed by Waterhouse [14],

$$p(t,\mathbf{r}) = \lim_{N\to\infty} \frac{1}{\sqrt{N}} \sum_{n=1}^{N} A_n \cos(\omega t + \varphi_n + \mathbf{k}_n \cdot \mathbf{r}) , \qquad (1)$$

where the phase angles φ_n are uniformly distributed between 0 and 2π, the wavenumber vectors $\mathbf{k}_n$ are uniformly distributed over all angles of incidence (corresponding to a sinusoidal distribution of the polar angles) and the amplitudes A_n have an arbitrary distribution, Jacobsen showed i) that the normalised spatial variance of the potential energy density ($\varepsilon^2\left\{w_{\mathrm{pot}}\right\}$) equals one (as shown earlier by Waterhouse [14] and Lubman [15]); ii) that the three Cartesian mean square particle velocity components are statistically independent and also have a normalised spatial variance of one; iii) that the normalised spatial variance of the kinetic energy density is $1/3$; and that the normalised spatial variance of the total energy density is also $1/3$ [5]. If the room is driven with noise rather than a pure tone the spatial variances should be multiplied by a factor of $3\ln(10)/\left(BT_{60}\right)$, where B is the bandwidth and T_{60} is the reverberation time. The effect of the finite bandwidth of the signal follows from expressions derived by Lubman [15] and Schroeder [16]. In short, measuring the total sound energy density at one position gives the same information as measuring the potential energy density at three statistically independent positions.

These results have recently been confirmed by a numerical study [12], which also demonstrated larger spatial standard deviations of potential, kinetic and total energy density below the Schroeder frequency.

The diffuse-field model has also been used successfully for predicting spatial correlation functions [5, 17] and the spatial statistics of active and reactive sound intensity [18].

2.2 Low frequencies; the modal approach

Below the Schroeder frequency one would perhaps not expect the diffuse-field theory to be accurate. So far nobody has examined the spatial statistics of kinetic and total energy density in this frequency range, except in the extreme case where one single mode is dominating. For this case Cook and Schade found that the normalised spatial variance of the total energy density in an axial mode in a rectangular room is zero; that the corresponding variance compared with that of potential energy density in a tangential mode is reduced by a factor of up to five; and that the variance compared with that of potential energy density is reduced by a factor of up to $19/6$ (which is very close to three) in an oblique mode [4]. There is no equivalent to axial and tangential modes in enclosures of a more complicated shape, and in practice the vanishing variance of total energy density in axial modes is not very important. All in all it does not seem unreasonable to expect the normalised spatial variance of the total energy density to be about one third of that of potential energy density in the general case, also at low frequencies.

Many authors have examined the spatial variation of potential energy density in rectangular rooms below the Schroeder frequency. Employing a theoretical method that involves replacing modal sums with integrals and assuming that the modal frequencies have a Poisson distribution (i.e., are completely randomly distributed), Lyon derived the following expression for the normalised variance of the potential energy density in a rectangular reverberation room [19],

$$\varepsilon^2\left\{w_{\text{pot}}\right\} = 1 + \left(\frac{3}{2}\right)^6 \frac{1}{2M}, \tag{2}$$

where

$$M = \frac{6\pi \ln(10) V f^2}{T_{60} c^3} \tag{3}$$

is the modal overlap (the product of the noise bandwidth of a mode and the modal density). Note that the normalised variance approaches one at high modal overlap, in agreement with the diffuse-field theory. A similar but slightly different method was used by Jacobsen about ten years later [20], but whereas Lyon in effect studied the ensemble statistics, Jacobsen (following Bodlund [21]) distinguished between the spatial variance and the room variance. The former can be observed when a microphone is moved about in a given room; the latter is associated with an ensemble of rooms. A few years later Davy derived a more general version of Lyon's expression in the form of the variance of a transmission function [22].

One problem with the approach used by these authors is that the results are fairly sensitive to the assumed distribution of modal frequencies; therefore both Lyon and Davy also examined the 'nearest neighbour distribution', which is supposed to reflect the observation that the modal frequencies seem to tend to repel each other, rather than being statistically independent, i.e., Poisson distributed. This phenomenon, which means that the distribution of the modal frequencies is closer to the average density that one would have thought, is sometimes referred to as the 'spectral rigidity' [23]; and at present it seems to be generally accepted that the modal frequencies have a Gaussian orthogonal ensemble distribution [23-26]. This distribution does not have an elementary function representation, but it can be approximated by the Rayleigh distribution [24]. Under the assumption that the modal frequencies have a Gaussian orthogonal ensemble distribution Lyon's expression for the normalised variance of potential energy density (Eq. (2)) should be modified [24, 27] to

$$\varepsilon^2\left\{w_{\text{pot}}\right\} = 1 + \frac{1}{2M}\left(\left(\frac{3}{2}\right)^6 - 3\right). \tag{4}$$

There is an interesting coupling between the statistics of energy density and the statistics of the sound power emitted by the source that generates the sound field. In sound power measurements in reverberation rooms one usually combines averaging over source positions and receiver positions, and reciprocity considerations lead to the conclusion that, say, the combination of N source positions and L receiver positions must result in the same uncertainty as L source positions and N receiver positions. Davy's expression satisfies this relation [22, 24, 27].

If the modal frequencies have a Gaussian orthogonal ensemble distribution the normalised spatial variance of the sound power emitted by a monopole in a rectangular room is, according to Davy [26],

$$\varepsilon^2\left\{P_{\text{a}}\right\} = \frac{1}{2M}\left(\left(\frac{3}{2}\right)^3 - 1\right). \tag{5}$$

2.3 Low frequencies; the diffuse-field approach

The normalised variance of the sound power emitted by a pure-tone monopole moved about in a reverberant sound field has also been estimated with the diffuse-field approach based on Eq. (1) [5, 28]. This approach is far and away simpler than the modal approach and does not involve any assumptions about the shape of the room or the distribution of the modal frequencies. The result is

$$\varepsilon^2\left\{P_{\text{a}}\right\} = \frac{1}{2M}, \tag{6}$$

which is small (less than 0.1) above the Schroeder frequency, but important at low frequencies. The reciprocity considerations mentioned in Sect. 2.2 now lead to the conclusion that the normalised spatial variance of the mean square pressure at low frequencies will be increased by the source position variance, that is,

$$\varepsilon^2\left\{w_{\text{pot}}\right\} = 1 + \frac{1}{2M}. \tag{7}$$

The explanation is that the amplitudes A_n in Eq. (1) depend on the sound power emitted by the source. Note the strong similarities between Eqs. (4) and (7) (and between Eqs. (5) and (6)). It follows that if the diffuse-field model is applied below the Schroeder frequency then, to first order, one can expect an increase of the normalised spatial variance of potential energy density of the order of $1/2M$. The corresponding normalised variance of the total energy density can be expected to be three times less,

$$\varepsilon^2\left\{w_{\text{tot}}\right\} = \left(1 + \frac{1}{2M}\right)\Big/3. \tag{8}$$

For noise excitation one may expect Lubman's and Schroeder's expression to give a good approximation [15, 16], and thus Eqs. (7) and (8) to become

$$\varepsilon^2\left\{w_{\text{pot}}\right\} = \left(1 + \frac{1}{2M}\right)\frac{1}{1 + BT_{60}/3\ln(10)} \tag{9}$$

and

$$\varepsilon^2\left\{w_{\text{tot}}\right\} = \frac{1}{3}\left(1 + \frac{1}{2M}\right)\frac{1}{1 + BT_{60}/3\ln(10)} \tag{10}$$

3 Experimental results

To test the validity of the foregoing considerations some experiments have been carried out in a small reverberation chamber of about 2 m^3 (a model (in scale 1:5) of DTU's large reverberation rooms). In the frequency range of concern the reverberation time of this chamber is about 1 s. The chamber was driven by a loudspeaker, and the mean square sound pressure and mean square particle velocity components in three perpendicular directions were measured at sixteen random positions using a Microflown 3D pressure-velocity sound intensity probe, 'USP' (calibrated as described in Ref. [29]), combined with a Brüel & Kjær 'PULSE' analyser in the FFT mode. The loudspeaker was driven by synchronised pseudo-random noise, corresponding to analysing the sound field with 6400 independent pure tones. The Schroeder frequency of the chamber was estimated to be about 1.3 kHz.

Sixteen positions is not much for determining the variance of a random variable, and therefore the estimated spatial standard deviations vary significantly from frequency to frequency. However, when the normalised spatial standard deviations are averaged over frequency bands the fluctuations are smoothed out. Figure 1 shows such band average values of the normalised spatial standard deviation of the rms value of the sound pressure and of the three Cartesian particle velocity components. According to the diffuse-field theory all these quantities have a Rayleigh distribution, the normalised standard deviation of which is $\sqrt{(4-\pi)/\pi} = 0.52$. This is in good agreement with the results above the Schroeder frequency. Below this frequency the standard deviation is slightly larger.

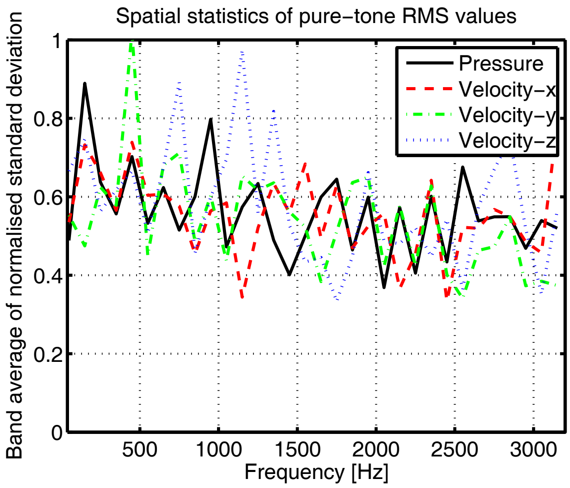

Fig. 1. Normalised spatial standard deviation of rms pressure and particle velocity components for pure tones, averaged over 100 Hz bands.

Figure 2 shows similar band average values of the normalised spatial standard deviation of potential, kinetic and total energy density compared with the predicted values (Eqs. (7) and (8)). Above the Schroeder frequency the results fluctuate about the predicted values. Below the Schroeder frequency the theory seems to overestimate the standard deviations. However, note that the modal theory (Eq. (4)) would overestimate even more (a factor of 2.9). It is certainly confirmed in the entire frequency range that kinetic and total energy density varies less with the position than potential energy density does.

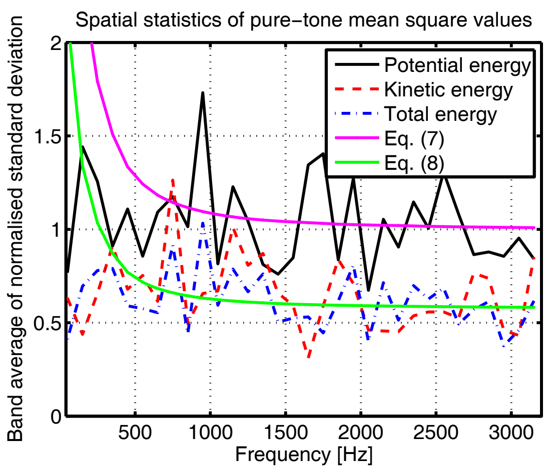

Fig. 2. Normalised spatial standard deviation of potential, kinetic and total energy density for pure tones, averaged over 100 Hz bands.

Figure 3 show the results of averaging the mean square values of the pressure and the three particle velocity components in 100 Hz frequency bands. This corresponds to driving the chamber with noise and measuring the spatial standard deviations in such frequency bands. The measured spatial standard deviations are compared with the predicted values (Eqs. (9) and (10)). As can be seen there is reasonable agreement with the theory.

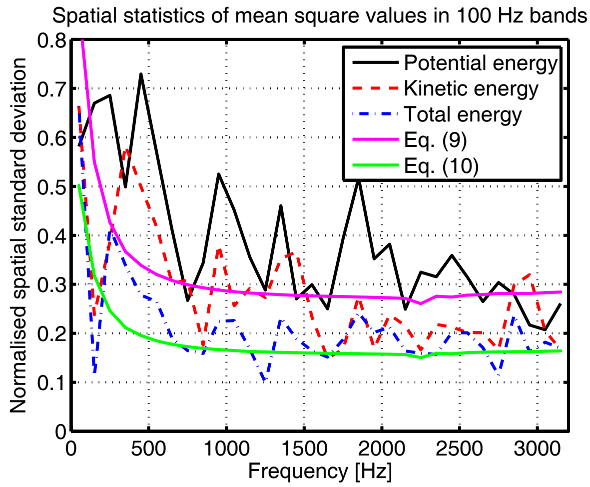

Fig. 3. Normalised spatial standard deviation of potential, kinetic and total energy density in 100 Hz bands.

Finally Fig. 4 shows the results of another kind of statistical analysis. In this case potential, kinetic and total energy density has been analysed statistically in 100 Hz wide bands along he frequency axis at one single position. Above the Schroeder frequency one should expect the same statistics with respect to frequency as with respect to position [30, 31]; and this is confirmed. Below the Schroeder frequency this ergodicity is not self evident, but it seems to be confirmed.

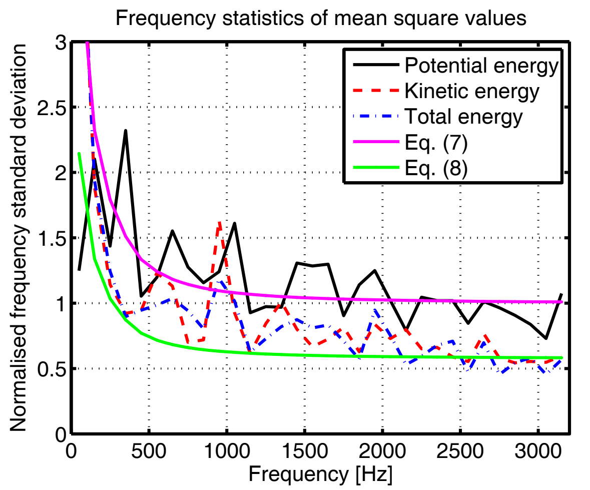

Fig. 4. Normalised standard deviation of potential, kinetic and total energy density with respect to frequency at one single position, calculated in 100 Hz bands.

4 Conclusions

The diffuse-field theory can be extended to the frequency range below the Schroeder frequency, where it gives predictions that are similar to the predictions of the modal theory. The diffuse-field theory is far simpler than the modal theory and requires much less information; it is only necessary to know the reverberation time of the room and its modal density. According to this theory it is three times more efficient to measure the total sound energy density in a reverberant sound field than to measure the potential sound energy density; and this is confirmed by experimental results.

References

[1] R.V. Waterhouse and D. Lubman, 'Discrete versus continuous space averaging in a reverberant sound field,' *Journal of the Acoustical Society of America* **48**, 1-5 (1970)

[2] D. Lubman, R.V. Waterhouse and C. Chien, 'Effectiveness of continuous averaging in a diffuse sound field,' *Journal of the Acoustical Society of America* **53**, 650-659 (1973)

[3] I. Wolff and F. Massa, 'Direct measurement of sound energy density and sound energy flux in a complex sound field,' *Journal of the Acoustical Society of America* **3**, 317-318 (1932)

[4] R.K. Cook and P.A. Schade, 'New method for measurement of the total energy density of sound waves,' *Proceedings of Inter-Noise 74*, Washington, DC, USA, pp. 101-106 (1974)

[5] F. Jacobsen, 'The diffuse sound field,' The Acoustics Laboratory, Technical University of Denmark, *Report no* 27 (1979)

[6] J.A. Moryl and E.L. Hixson, 'A total acoustic energy density sensor with applications to energy density measurement in a reverberation room,' *Proceedings of Inter-Noise 87*, Beijing, China, pp. 1195-1198 (1987)

[7] J.W. Parkins, S.D. Sommerfeldt and J. Tichy, 'Error analysis of a practical energy density sensor,' *Journal of the Acoustical Society of America* **108**, 211-222 (2000)

[8] B.S. Cazzolato and J. Ghan, 'Frequency domain expressions for the estimation of time-averaged acoustic energy density,' *Journal of the Acoustical Society of America* **117**, 3750-3756 (2005)

[9] J.-C. Pascal and J.-F. Li, 'A systematic method to obtain 3D finite difference formulations for acoustic intensity and other energy quantities,' *Journal of Sound and Vibration* **310**, 1093-1111 (2008)

[10] F. Jacobsen, 'A note on finite difference estimation of acoustic particle velocity,' *Journal of Sound and Vibration* **256**, 849-859 (2002)

[11] D.R. Ynterma, W.F. Druyvesteyn and M. Elwenspoek, 'A four particle velocity sensor device,' *Journal of the Acoustical Society of America* **119**, 943-951 (2006)

[12] D.B. Nutter, T.W. Leishman, S.D. Sommerfeldt and J.D. Blotter, 'Measurement of sound power and absorption in reverberation chambers using energy density,' *Journal of the Acoustical Society of America* **121**, 2700-2710 (2007)

[13] M.R. Schroeder and K.H. Kuttruff, 'On frequency response curves in rooms. Comparison of experimental, theoretical and Monte Carlo results for the average frequency spacing between maxima,' *J. Acoust. Soc. Am.* **34**, 76-80 (1962)

[14] R.V. Waterhouse, 'Statistical properties of reverberant sound fields,' *Journal of the Acoustical Society of America* **43**, 1436-1444 (1968)

[15] D. Lubman, 'Fluctuation of sound with position in a reverberation room,' *Journal of the Acoustical Society of America* **44**, 1491-1502 (1968)

[16] M.R. Schroeder, 'Effect of frequency and space averaging on the transmission responses of multimode media,' *Journal of the Acoustical Society of America* **46**, 277-283 (1969)

[17] R.H. Lyon, 'Statistical analysis of power injection and response in structures and rooms,' *Journal of the Acoustical Society of America* **45**, 545-565 (1967)

[18] F. Jacobsen and T. Roisin, 'The coherence of reverberant sound fields,' *Journal of the Acoustical Society of America* **108**, 204-210 (2000)

[19] F. Jacobsen, 'Active and reactive sound intensity in a reverberant sound field,' *Journal of Sound and Vibration* **143**, 231-240 (1990)

[20] F. Jacobsen, 'Sound power determination in reverberation rooms,' The Acoustics Laboratory, Technical University of Denmark, *Report no* 26 (1979)

[21] K. Bodlund, 'A normal mode analysis of the sound power injection in reverberation chambers at low frequencies and the effects of some response averaging methods. *Journal of Sound and Vibration* **55**, 563-590 (1977)

[22] J.L. Davy, 'The relative variance of the transmission function of a reverberation room,' *Journal of Sound and Vibration* **77**, 455-479 (1981)

[23] R.L. Weaver, 'On the ensemble variance of reverberation room transmission functions, the effect of spectral rigidity,' *Journal of Sound and Vibration* **130**, 487-491 (1989)

[24] J.L. Davy, 'The distribution of modal frequencies in a reverberation room,' *Proceedings of Inter-Noise 90*, Stockholm, Sweden, pp. 159-164 (1990)

[25] O. Legrand, F. Mortessagne and D. Sornette, 'Sprectral rigidity in the large modal overlap regime: beyond the Ericson-Schroeder hypothesis,' *Journal de Physique I* **5**, 1003-1010 (1995)

[26] R.S. Langley, 'On the diffuse field reciprocity relationship and vibrational energy variance in a random subsystem at high frequencies,' *Journal of the Acoustical Society of America* **121**, 913-921 (2007)

[27] J.L. Davy, 'The variance of pure-tone reverberant sound power measurements,' *Proceedings of Fifth International Congress on Sound Vibration*, Adelaide, South Australia, 1997.

[28] A.D. Pierce, *Acoustics. An Introduction to Its Physical Principles and Applications*. McGraw-Hill, New York (1981). See section 6.3.

[29] F. Jacobsen and V. Jaud, 'A note on the calibration of pressure-velocity sound intensity probes,' *Journal of the Acoustical Society of America* **120**, 830-837 (2006)

[30] M. Schröder, 'Die statistichen Parameter der Frequenzkurven von Grossen Räumen,' *Acustica* **4**, 594-600 (1954)

[31] H.G. Andres, 'Über ein Gesetz der räumlichen Zufallsschwankung von Rauschpegeln in Räumen und seine Anwendung auf Schalleistungsmessungen,' *Acustica* **16**, 279-294 (1965/66)