Modeling of piano sounds using FEM simulation of soundboard vibration

Luis Ortiz-Berenguer, Francisco Casajus-Quiros, Elena Blanco-Martin and David Ibanez-Cuenca

Universidad Politecnica de Madrid, Ctra.Valencia km7, 28031 Madrid, Spain
lortiz@diac.upm.es

Pattern-matching methods for polyphonic transcription of piano sounds require a set of patterns that can be obtained by modeling the piano-sound spectra. The modeling should take into account not only the string stiffness but also the effect of the soundboard impedance on the string vibration. Studies on that effect corresponding to a wide range of impedance values have previously been carried out by the authors. However, actual impedance values for real pianos must be used in the model. Although the impedance value of a few grand-pianos have been measured by the authors, these results are not significative enough to create a model. Thus, a FEM simulation of soundboard vibration is proposed to obtain nearly-actual impedance values. The simulation considers several cases of vibrating plates from the simplest rectangular one and increasing the similarity to real piano soundboards. The quality of the simulation is verified comparing the obtained results with either recognized theoretical results for the simplest cases or measured values for the more complex ones. The complexity of the simulated soundboard is limited to the case that produces only slight variations in the modeled spectrum. This work has been supported by Spanish National Project TEC2006-13067-C03- 01/TCM.

1 Introduction

The goal of this research is to improve the accuracy of a synthesized piano spectrum. In our case, the aim of the spectrum synthesis is to obtain a set of spectral patterns to be utilized for a transcription system based on a pattern-matching method.

Two basic vibrational aspects affect the piano note spectrum. First, the stiffness of the piano strings produces every partial to be slightly above the harmonic frequency. Second, the fact that one of the string ends is fixed to a moving support (i.e., the bridge). The effect of the string stiffness is taken into account by using the inharmonicity coefficient and the Fletcher's equation [1].

$$f_n \;=\; n\, f_0 \sqrt{1 \,+\, n^2\, B} \qquad (1)$$

Where n is the partial order, f_0 is the fundamental frequency in the case of fixed elastic string and B is the inharmonicity factor due to the string stiffness.

The moving support turns out to behave as a finite mechanical impedance with both real and imaginary parts. The effect on the spectrum is a deviation of the frequency respect to the expected value for the case of fixed ends. This deviation might be positive or negative, leading to a higher or a lower frequency. This deviation is a sort of inharmonicity and it can be called 'Soundboard induced inharmonicity'.

Solving the vibration frequencies of a string with a mechanical impedance (with both real and imaginary parts) at one end has been carried out by Morse [2] and by Ortiz et al. [3][4]. Morse obtained an approximated expression and stated that it was correct only for small deviations.

$$f_n \;\simeq\; n\,\frac{c}{2\,L} \;+\; \frac{c}{2\,\pi\,L}\,\rho_L\,c\,\frac{X_{SB}}{R_{SB}^2 \,+\, X_{SB}^2} \qquad (2)$$

Where n is the partial order, c is the propagation velocity, L is string length, ρ_L is density per length unit and R_{SB} and X_{SB} are the real and imaginary part of the soundboard impedance.

Ortiz et al. obtained the vibration frequencies by solving numerically the equation of the modes for a string with finite impedance at one end. The results were tabulated as a function of both real and imaginary parts of the soundboard impedance. Their results were very close to those obtained by Morse's equation even for larger deviations. Unlike Morse, Ortiz et al. calculated their results as a deviation value expressed in cents, therefore any vibration frequency could be expressed as:

$$f_n \;=\; 1200\,log_2\left(\frac{n\cdot f_0}{27.5}\right) + I_{SB}\,\left(n, f_0, R_{SB}, X_{SB}\right) \quad (3)$$

Where 27.5 Hz has been selected as a reference for the cents-scale because it is the equal-tempered frequency for the first piano note (i.e., A0).

This is a good way to express deviation in musical environment giving a better estimation of whether the deviation is of significance or not. Figure 1 shows the deviation values obtained for the first partial of 28th note.

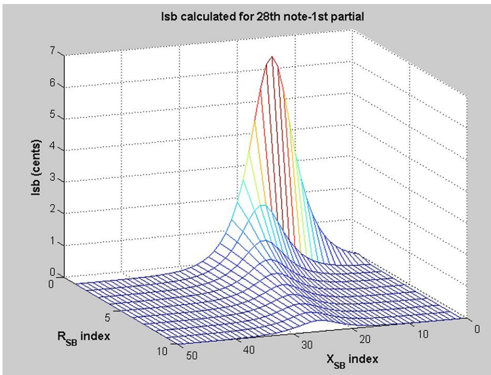

Figure 1: I_{SB} deviation calculated by Ortiz et al.[5] for the first partial of 28-th note as a function of a range of R_{SB} and X_{SB} values, represented by indexes. R_{SB} indexes correspond to a range linearly spaced from 100 to 1900 kg/s. X_{SB} indexes correspond to a range logarithmically spaced from 0 to $10^{+6.5}$ kg/s.

It turned out that the I_{SB} deviation value for n-th partial is n times the deviation value for the fundamental if the impedance were the same at both frequencies.

The limitation of both methods is they calculate the solutions for a flexible string rather than a stiff string. Authors are currently using FEM methods to evaluate the validity of those solutions when applied to stiff strings.

Despite of the method used to calculate the deviation due to the soundboard impedance, a remaining problem is to know the real value of that impedance.

2 Measured soundboard impedance

The obvious way to know the actual value of the soundboard impedance is to measure it. Some of the already published articles including soundboard impedance measurements are those from Giordano [6], Berthaut[7] and Suzuki [8]. But all of them lack from the fact that present the modulus of impedance, but they do not separate the real and imaginary parts, which is necessary to evaluate the frequency deviation due to that impedance.

In some case (e.g., Giordano) it is possible to derive a range for the values of real and imaginary parts using the also indicated phase of the impedance. The obtained range of values is approximately: 200 to 2000 kg/s for R_{SB} and 0 to ± 10000 for X_{SB} [4].

In this section we present some of the values obtained by measuring the impedance of a Yamaha C7 grand piano (2.27m long).

The measures were carried out in a complete piano without dismounting any piece. The impact measurement method was utilized. The impact hammer, a B&K8202, was connected to a charge-amplifier (B&K2635) to adapt the accelerometer and amplify its signal. The resulting vibration was measured with a Polytech laser vibrometer at a point only a few millimeters away from the excitation point (to prevent the hammer interrupting the laser light). Figure 2 shows a photo of the main used elements.

Both force and velocity signals were recorded synchronously in a laptop using a 2-channel audio interface from TASCAM. The audio interface and the audio processing software were previously level-calibrated by introducing a test signal of known amplitude from a Hewlett-Packard signal generator.

Figure 2: Photo showing the C7 piano with the B&K impact-hammer and the Polytech laser-head utilized during the measuring sessions.

Digitalization was done at 44.1 kHz and 16 bits for both channels to preserve most information. Several impacts were recorded for every excitation point. An audio editing software allowed to select every part of the recording with both signals (force an velocity) available synchronously. Data were read and calculations were done to obtain mobility and impedance by using a software application written by the authors using Matlab.

Figure 3 shows the measured values for the point of the bass bridge corresponding to 24-th note. Values of R_{SB} range from slightly below 100 to about 4000. On the other hand, X_{SB} range mainly between $\pm$ 1000. Velocity spectrum shows clearly the first modes. Table 3 includes these measured modes-frequencies along with simulated ones.

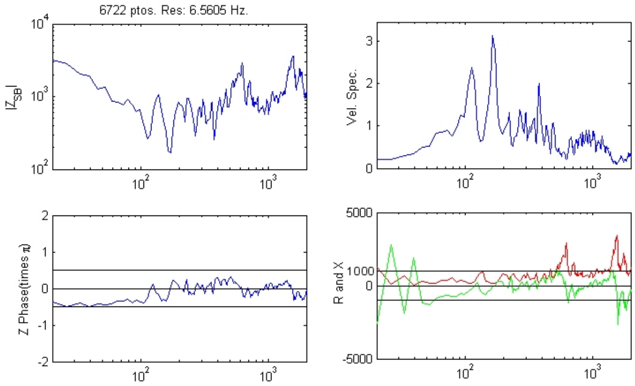

Figure 3: Measurement results when exciting the point of 24-th note in the bass bridge. The modulus and phase of Z_{SB} are shown in the left side. The upper-right plot shows the velocity spectrum and its peaks indicate the modes frequencies. The lower-right plot shows both the real (always positive valued) and imaginary parts of the measured impedance.

3 Modeling soundboards using FEM

Simulating the vibration of plates has several benefits. Mainly, it is possible to estimate the impedance for several plate sizes and several excitation points without the need of having several pianos and repeating the measuring process many times.

However, it is important to be sure that the simulation model is realistic enough. The present section shows the results of FEM simulations for several kinds of plates and how the more complete model leads to the more realistic results. Trying to model a piano soundboard as a simple plate is not realistic enough. It is necessary to include in the FEM model, besides the piano irregular-shape-orthotropic plate, the ribs and the bridges.

To verify whether the FEM model leads to good results, these must be compared to the existing measures. The first element to be verified to decide whether a simulation is good enough, is the frequency values of the vibration modes. These values must be coincident (or at least nearly coincident) with the measured ones.

A recently published article by Mamou-Mani et al. [9] shows that FEM modeling including the downbearing force of the strings over the soundboard leads to a significant modification of the vibration modes frequencies. As far as downbearing has not been considered in our FEM model, our results are allowed to be slightly different than the actual ones.

The simulated plates have been the following:

1. Rectangular orthotropic with the wood grain fol-

lowing the piano longitudinal axis.

2. Rectangular orthotropic with the wood grain following its real direction (i.e., about 40 degrees counterclockwise of the longitudinal piano axis). Therefore, the plate in the FEM model was rotated 40 degrees clockwise for the wood-grain axis being coincident with FEM y-axis. The longitudinal piano axis was not the y-axis anymore. Longitudinal wood axis (i.e., wood-grain axis) was coincident with FEM y-axis, which allowed the correct orthotropic-material modeling in the used FEM software.
3. Trapezoid shaped orthotropic plate. It was obtained form the previous rectangular, performing a cut in its right side and obtaining nearly the Cristofori's original pianoforte soundboard shape.
4. Current-piano shaped orthotropic plate.
5. Piano shaped orthotropic plate with hard-wood ribs glued below the plate at a perpendicular direction to the wood grain of the plate.
6. The previous but having added the two hard-wood bridges on the upper face of the plate. These bridges resembled the shape established in the Steinway's patent for a overstrung pianoforte[10].

The FEM simulation has been carried out using the program ANSYS. The mechanical parameters for the plate, the ribs and the bridge have been obtained from the Wood Handbook published by the U.S. Department of Agriculture-Forest Products Laboratory [11]. Sizes for ribs and bridge have been obtained from the Steinway's classic patent on the overstrung piano-forte [10]. Table 1 summarizes the values and figure 4 shows the physical details of the models.

Regarding FEM elements, shell elements were used for the simplest plates, whereas solid elements were used for the more detailed. A simple test proved that for a 1cm thick rectangular plate, mode frequencies were the same despite the element used. Due to software constrains, shell elements could not be used when meshing non-rectangular plates. Figure 4 shows the designed soundboard utilized for FEM calculations.

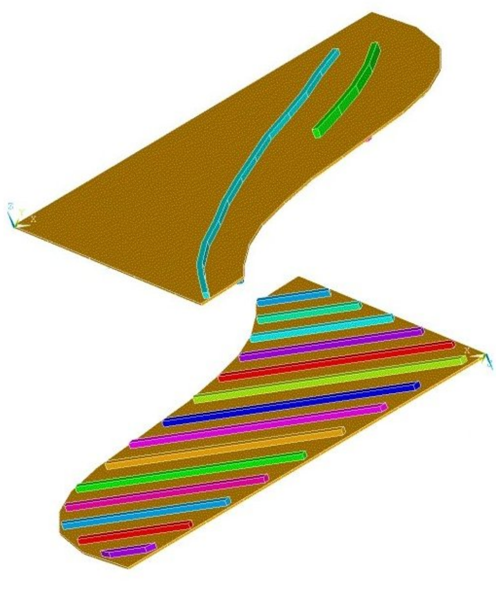

Figure 4: The designed model of the soundboard with ribs and bridges used in FEM calculations. The shapes of soundboard and bridges follow the Steinway's patent.

	Soundboard		Ribs and Bridges
E_t (GPa)	0.469		
E_l (GPa)	10.9	E	$12GPa$
E_r (GPa)	0.85		
G_{tl} (GPa)	0.665		
G_{lr} (GPa)	0.698		
G_{tr} (GPa)	0.033		
σ_{tl}	0.025		
σ_{lr}	0.372	σ	0.4
σ_{tr}	0.245		
ρ (kg/m^3)	450	ρ	$500kg/m^3$

Table 1: Wood parameters used in FEM simulation [11]. E is Modulus of Elasticity, G is Modulus of Rigidity, σ is Poisson's ratio and ρ is density. Sitka Spruce soundboard has been considered orthotropic and values for longitudinal, tangential and radial axis of wood are given. Hard-wood ribs and bridges have been considered isotropic to reduce complexity.

Regarding boundary conditions, in a piano, the soundboard is fitted into the rim gluing its perimeter, thus all the soundboard surfaces in contact with the rim are not allowed to move. In figure 5 showing the displacement of the plates it can be seen that all their edges are nodes presenting no vibration at all.

The soundboard modeled in ANSYS has been rotated in order to make both the wood-longitudinal and graphical-y axis coincident.

3.1 Vibration modes

The first parameter under study has been the modes and their frequencies. Table 2 summarizes the obtained values for the seven modeled plates. It can be seen that all of the cases presented different values of frequencies, even though in some cases they were similar. Only the last two plates showed a somehow realistic value for the first mode of a piano soundboard. Therefore, it should be considered that simpler plate models are not adequate for soundboard impedance estimation.

Another interesting difference was that obtained modes did not appear in the same order for all plates. Modes can be referred to by using two dimensional indexing (i.e., $M_{i,j}$), where i and j indicates the number of peaks and valleys in the transversal and longitudinal piano-axis. If modes are ordered by increasing frequency, some of them do not appear in the same order (see fig 5, the 5th and 8th modes). Moreover, it was difficult to

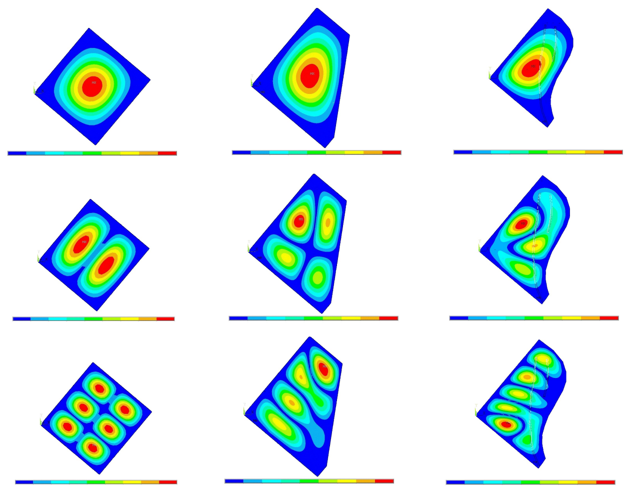

Figure 5: Modes 1st, 5th and 8th of three types of plates. Left, rectangular orthotropic. Center, trapezoid shaped orthotropic. Right, piano shaped orthotropic with ribs and bridges. Plots are not equally scaled. All of them show motionless edges, which corresponds to boundary conditions.

identify $M_{i,j}$ modes in the trapezoid and piano shaped plates, because mode-shapes were not regular (see fig 5 the 5th mode of the third column).

It turns to be clear that almost every mechanical property or detail of the modeled plate affects the obtained vibration modes. The size, the shape, the wood-grain orientation respect to the piano keyboard axis, the ribs (including number, size and material) and the bridges.

Only the more complete model leads to a realistic FEM simulation as it is analyzed in the following subsection.

	1	2	3	4	5	6
1	22.7	25.1	23.1	25.0	58.4	96.4
2	28.3	29.7	41.2	38.8	81.8	141.5
3	39.1	38.8	49.5	53.3	115.6	204.3
4	55.4	52.8	63.2	60.3	144.5	249.7
5	59.9	67.0	76.7	75.3	163.2	289.3
6	64.8	71.2	84.9	82.4	181.0	311.1
7	73.9	71.6	90.8	92.3	208.6	353.9
8	76.8	78.9	105.7	103.6	225.4	406.5
9	88.1	91.0	118.6	108.6	254.4	430.7
10	103.2	95.1	123.2	121.8	277.2	488.2

Table 2: Frequencies of the first ten modes obtained by FEM calculation for the six simulated plates. The change of shape and direction of wood grain modified the frequency of some modes. However, including ribs and even bridges modified a lot the modes frequencies. Only the last two plates showed a realistic value for the first mode of a piano soundboard.

4 Simulation compared to measures

If only the mode frequencies have to be compared, table 3 shows the simulated values and the measured ones.

It is clear that only the more complete FEM model approaches the measured values. It must be notice that the size of the modeled soundboard was a little longer than the size of the measured Yamaha C7 piano. The model was 1.75x1.46 whereas the real piano was 1.55x1.46. This can explain the fact that all the measured values are somehow above the simulated ones, specially those modes more related to the longitudinal size. It also has to be taken into consideration the effect of down-bearing as it was discussed previously.

A more complete analysis requires to make use of the mobility or its inverse, the impedance. FEM simulation requires taking into consideration a damping factor in order to obtain realistic mobility curves. The articles from Chaigne [12] and Lambourg [13] present a study on damping plates, including Spruce wood plates (aimed to guitar soundboards). Their results show an increasing decay factor with frequency, which is consistent with the fact that bass notes are longer than treble ones. In our case, measurements indicate that resistive part of soundboard impedance, which is related to decay factor, also increases with frequency. Furthermore, velocity spectrum (see fig 3) of the first modes present nearly equal bandwidth, which leads to Q increasing with frequency. Thus, damping ratio decreases with frequency. The last means that FEM modeling using ANSYS requires the use of the named α-damping.

Obtaining a simulated results nearly equal as a measured value is difficult. Figure 6 shows the simulated velocity when exciting a point in the bass-bridge corresponding to about 24-th note. In this result, the plate has been considered undamped, thus the peaks are narrow. It can be seen that the envelope or levels distribution is similar to the measured one (see also figure 3 to compare).

	Measured	3	4	5	6
1	115	23.1	25.0	58.4	96.4
2	165	41.2	38.8	81.8	141.5
3	215	49.5	53.3	115.6	204.3
4	265	63.2	60.3	144.5	249.7
5	298	76.7	75.3	163.2	289.3
6	338	84.9	82.4	181.0	311.1
7	382	90.8	92.3	208.6	353.9
8	445	105.7	103.6	225.4	406.5
9	490	118.6	108.6	254.4	430.7
10	540	123.2	121.8	277.2	488.2

Table 3: Mode frequencies (Hz): simulated and measured ones

For a more detailed simulation, the selected α-damping coefficient was 20 and the spectral resolution was forced to 6 Hz (as in the case of measured values). Only the first ten modes have been used for calculations in the range 20-600 Hz. Results are shown in figure 7. They exhibit a certain resemblance of the measures.

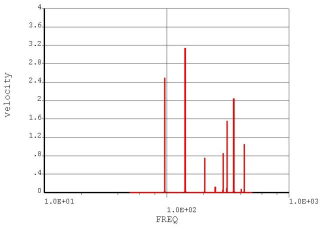

Figure 6: Velocity simulated at about 24-th note point, considering an undamped piano plate. The distribution of levels is similar to the measured ones.

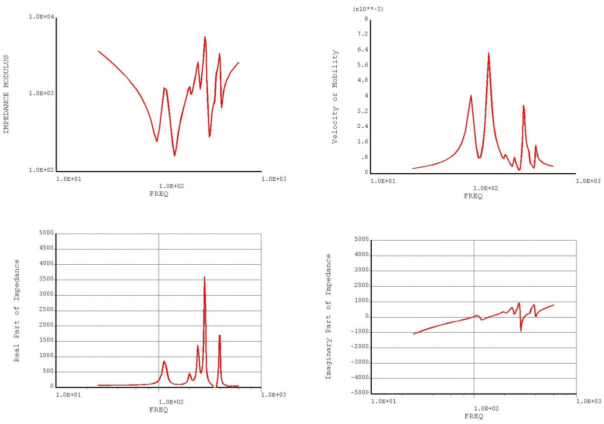

Figure 7: The complete simulation results. A perfect resemblance of the measures is impossible, but the similarity is good enough. Impedance measures includes the effect of the strings impedances, which produce 'small irregularities' in the measured curves. Velocity curve is also the mobility curve because simulation was carried out by exciting the point with a frequency-sweeping-unity-amplitude sinusoidal force.

5 Conclusions

Simulation using FEM allows to estimate the values of both the real and imaginary parts of the soundboard impedance. Even though this estimation is not perfect, it permits to know the range of possible values, which is enough to evaluate the range of likely deviation due to I_{SB}. The best estimation requires the use of a very detailed model of the soundboard, not being the plate alone enough. Considering ribs and bridges is necessary to obtain a good enough approximation to real modes, what is the first step to a good impedance estimation.

Acknowledgments

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