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The structural acoustic properties of stiffened shells

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Plates stiffened with ribs can be modelled as homogeneous isotropic or orthotropic plates, and modeling such an equivalent plate numerically with, say, the finite element method is, of course, far more economical in terms of computer resources than modeling the complete, stiffened plate, and this is important when a number of stiffened plates are combined in a complicated structure composed of many plates. However, whereas the equivalent plate theory is well established there is no similar established theory for stiffened shells. This paper investigates the mechanical and structural acoustic properties of curved shells with stiffening ribs. Finite element simulations and analytical data are compared and discussed.

1 Introduction

Stiffeners are efficient for enhancing the stiffness of a plate or shell without adding too much mass. However, it normally takes long time to find the acoustic properties of a designed stiffened structure. A coarse but faster method is to smear the stiffeners to the base plate or shell. The theory of a smeared stiffened plate with an effective torsional rigidity has been latest summarized by Szilard [1]. However, there is no similar established theory for doubly curved stiffened shells.

During the last thirty years, researchers have paid much attention to the dynamic behavior of stiffened shells. Works have been done on cylindrical shells [2-12] and on conical shells [13-15]. Since doubly curved shells need more degrees of freedom to be analyzed, researchers mostly use finite element method (FEM) to deal with such cases. The application of the FEM to the vibration analysis of a stiffened shell allows for discrete stiffeners, variable curvature and irregular geometry. However, FEM calculations may be very time consuming.

The aim of the present work is to present a smearing method for finding natural frequencies of a slightly curved shell with periodic small stiffeners. In order to derive at a simple method, assumptions have to be made, such as Donnell-Mushtari-Vlasor's simplification and an infinitesimal distance assumption. Because of these assumptions, the present method is limited to slightly curved thin shells with small stiffeners.

Although the application of this method is somewhat limited, it is quite useful for making a fast estimate in its working range. Nowadays, engineers usually draw a new design structure in a 3D program, and later simulate its dynamic properties in a FEM program. The drawing and the FEM calculation may take days or even weeks for a relatively simple structure. Furthermore, it is often necessary to make modifications to the structure and that require new FEM calculations. In all it may be very time consuming. This calls for a coarse but fast method for estimating the natural frequencies at the beginning of the design before detailed drawings have made. The present method is developed for this purpose, and the equations are implemented in a MATLAB computer program.

In the following, the natural frequencies of a simply supported rectangular plate with cross-stiffened ribs will be presented first, and the obtained equation will be used in the case of a curved plate. Later on, the analytical results will be compared to FEM simulated data. Moreover, the effects of the stiffeners and the curvature will be discussed. Note that plates, shells and stiffeners are taken to be of the same material in this presentation.

2 Smeared stiffened plate

It has long been recognized that the lower modes of vibration of stiffened plates may be estimated by "smearing" the mass and stiffening effects of the stiffeners over the surface of the plate. The theory are latest summarized by Szilard [1]. The results in this section are mainly based on Szilard's presentation.

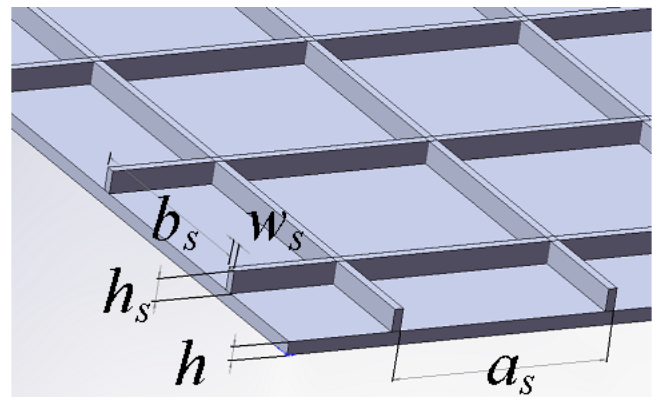


Fig.1 A cross-stiffened plate.

In the following, we will determine the natural frequencies of a rectangular plate with cross-stiffeners. The plate is simply supported along all edges, and has the length dimension a in the x direction and b in the y direction. Other dimensions of the plate are shown in Fig. 1, where w_s is the width of the stiffeners, a_s is the distance between stiffeners in the x direction, b_s is the distance between stiffeners in the y direction, h_s is the height of the stiffeners, and h is the thickness of the plate.

The bending stiffness in the x direction, D_x , can be computed by the product of the Young's modulus, E , of the material and the area moment of inertia in the x direction, I_x . Here, the stiffeners in the y direction have nearly no effect on the bending stiffness in the x direction. Therefore, only stiffeners in the x direction are taken into account in I_x .

$$I_x = I_p + \left(d - \frac{h}{2}\right)^2 \cdot h + \frac{I_s}{a_s} + \left(h_s + h - d - \frac{h_s}{2}\right)^2 \cdot (w_s \cdot h_s) \quad (1)$$

In Eq. (1), $I_p = h^3/[12(1-\nu^2)]$ and $I_s = w_s h_s^3/12$ are the area moment of inertia of the plate and the stiffeners, respectively; ν is the Poisson's ratio; d is the distance between the plate's bottom surface and the neutral axis of the stiffened plate. The second term of the right hand side is the shifting of the moment of inertia of the plate, and the last term is the shifting of the moment of inertia of the stiffeners.

Similarly, the bending stiffness in the y direction, D_y , can be determined from the same equations using b_s instead of a_s .

When the stiffeners are smeared and spread on top of the plate, the thickness of the equivalent plate becomes

$$h_e = h + h_s w_s \left(\frac{1}{a_s} + \frac{1}{b_s} \right) - \frac{h_s w_s^2}{a_s b_s}. \quad (2)$$

The equation for the natural frequencies of this simply supported orthotropic plate is [1]

$$f_{m,n,p} = \frac{1}{2\pi} \sqrt{\frac{1}{\rho_e} \sqrt{D_x \left(\frac{m\pi}{a} \right)^4 + 2H \left(\frac{m\pi}{a} \right)^2 \left(\frac{n\pi}{b} \right)^2 + D_y \left(\frac{n\pi}{b} \right)^4}}, \quad (3)$$

where $\rho_e = \rho h_e$ is the total smeared mass per unit area; ρ is the density of the material, and H is an empirical approximate formula for the equivalent torsional rigidity of a cross-stiffened plate [16].

3 Smeared stiffened shell

In this section, we will find an equation for the natural frequencies of a simply supported cross-stiffened rectangular shell.

3.1 Natural frequencies of a slightly curved rectangular shell

Soedel [17] studied a simply supported rectangular shell, which is slightly curved. Assumptions like Donnel-Mushtari-Vlasor's simplification and the infinitesimal distance assumption are used in his derivation.

The first basic assumption of Donnel-Mushtari-Vlasor's simplification is that contributions of in-plane deflections can be neglected in the bending strain expressions but not in the membrane strain expressions. The second assumption is that the influence of inertia in the in-plane direction is neglected. Both assumptions introduce a considerable error in the estimation of the fundamental natural frequency [18].

Thirdly, the infinitesimal distance assumption is

$$(ds)^2 \cong (dx)^2 + (dy)^2, \quad (4)$$

where ds is the magnitude of the differential change [17]. This assumption limits the applicability of the method to only slightly curved shells.

With these assumptions, the equation of motion for free vibration of a homogenous shell is therefore obtained to be [18]

$$D\nabla^8 U_3 + Eh\nabla_k^4 - \rho'' \omega^2 \nabla^4 U_3 = 0, \quad (5)$$

where

$$D = \frac{Eh^3}{12(1-\nu^2)}, \quad (6)$$

$$\nabla^2(\cdot) = \frac{\partial^2(\cdot)}{\partial x^2} + \frac{\partial^2(\cdot)}{\partial y^2}, \quad (7)$$

$$\nabla_k^2(\cdot) = \frac{1}{R_x} \frac{\partial^2(\cdot)}{\partial x^2} + \frac{1}{R_y} \frac{\partial^2(\cdot)}{\partial y^2}, \quad (8)$$

and $\rho'' = \rho h$ is the mass per unit area. For the doubly curved, simply supported rectangular shell, the deflection is expressed by a double sine series with terms of the form

$$U_3 = A_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}. \quad (9)$$

Substituting Eq.(9) into Eq.(5) gives,

$$D \left[\left(\frac{m\pi}{a} \right)^2 + \left(\frac{n\pi}{b} \right)^2 \right]^4 + Eh \left[\frac{1}{R_x} \left(\frac{m\pi}{a} \right)^2 + \frac{1}{R_y} \left(\frac{n\pi}{b} \right)^2 \right]^2 - \rho'' \omega^2 \left[\left(\frac{m\pi}{a} \right)^2 + \left(\frac{n\pi}{b} \right)^2 \right]^2 = 0 \quad (10)$$

The natural frequencies are therefore obtained to be

$$f_{mnc}^2 = \frac{1}{4\pi^2} \left[\left(\frac{m\pi}{a} \right)^2 + \left(\frac{n\pi}{b} \right)^2 \right]^2 \frac{D}{\rho''} + \frac{\left[\frac{1}{R_x} \left(\frac{m\pi}{a} \right)^2 + \frac{1}{R_y} \left(\frac{n\pi}{b} \right)^2 \right]^2}{4\pi^2 \left[\left(\frac{m\pi}{a} \right)^2 + \left(\frac{n\pi}{b} \right)^2 \right]^2} \cdot \frac{Eh}{\rho''}, \quad (11)$$

where a and b are the length of the edge in the x direction and y direction, respectively, h the thickness of the shell, D the bending stiffness, R_x the radius of curvature in the x direction, R_y the radius in the y direction, E the Young's modulus.

The first term of Eq. (11) is the natural frequency for an equivalent flat plate that has the same dimensions as the shell; the second term is accounting for the curvature.

3.2 Natural frequencies of a stiffened and slightly curved rectangular shell

This section presents the natural frequencies of a stiffened and slightly curved rectangular shell. Both a physical explanation and an analytical derivation are offered.

3.2.1 Physical explanation

A doubly curved shell can be made by bending a flat plate in the x and y directions. Equation (11) shows that its natural frequencies are composed by two terms, the natural frequencies of the flat plate and a curvature term.

It is possible to use this theory for an equivalent smeared flat plate mentioned in section 2, when it is bended to an equivalent smeared shell. An equivalent smeared plate of a stiffened plate is a flat plate with equivalent bending stiffness, torsional rigidity, total smeared mass per unit area, and thickness. If the equivalent plate is curved to a shell, its natural frequencies can be found by adding the second term of Eq.(11) to the contents of Eq.(3). Therefore, the natural frequencies of a stiffened shell become

$$f_{mnc,p}^2 = f_{m,n,p}^2 + \frac{\left[\frac{1}{R_x} \left(\frac{m\pi}{a} \right)^2 + \frac{1}{R_y} \left(\frac{n\pi}{b} \right)^2 \right]^2}{4\pi^2 \left[\left(\frac{m\pi}{a} \right)^2 + \left(\frac{n\pi}{b} \right)^2 \right]^2} \cdot \frac{Eh_e}{\rho_e''}. \quad (12)$$

Since the current shell is an equivalent smeared shell made from the equivalent smeared flat plate, its thickness and the mass per unit area should be h_e and ρ_e , respectively.

Considering the assumptions in section 3.1, it is expected that Eq.(12) only works on simply supported slightly curved shells, which have small stiffeners.

3.2.2 Analytical derivation

Another way to obtain Eq.(12) is to use the equation of motion. In Eq.(5), the bending stiffness D is independent of the curvature. It is actually the bending stiffness of an isotropic plate which is similar to the shell. It is possible to find the equivalent bending stiffness of a stiffened shell, and substitute this into Eq.(5) to obtain the equation of motion of the stiffened shell.

It is clear that the second ‘‘curvature’’ of Eq.(12) is independent of the bending stiffness of the shell, and that the first term of Eq.(12) corresponds to a flat plate. In other words, the bending stiffness is independent of the curvature of the slightly curved shell. Therefore, we assume that a stiffened and slightly curved rectangular shell has the same bending stiffness as a similar stiffened plate. Now the work is to find the equivalent bending stiffness of the stiffened plate. Moreover, using the smeared theory from section 2, the equivalent flat plate can be found. However, the smeared equivalent plate is an orthotropic plate with different bending stiffness in different directions and a torsional rigidity.

We assume that we have found an isotropic equivalent plate, which has the same natural frequencies as the orthotropic equivalent plate. The natural frequencies of an equivalent isotropic rectangular plate is

$$f_{mn,e} = \frac{1}{2\pi} \left[\left(\frac{m\pi}{a} \right)^2 + \left(\frac{n\pi}{b} \right)^2 \right] \sqrt{\frac{D_e}{\rho_e}}, \quad (13)$$

where D_e is the bending stiffness of the equivalent isotropic rectangular plate. The assumption that the natural frequencies are identical gives

$$f_{mn,e} = f_{mn,p}. \quad (14)$$

Substituting Eq.(3) and Eq.(13) into Eq.(14) gives an expression for the bending stiffness of the assumed isotropic equivalent plate as,

$$D_e = \frac{D_x \left(\frac{m\pi}{a} \right)^4 + 2H \left(\frac{m\pi}{a} \right)^2 \left(\frac{n\pi}{b} \right)^2 + D_y \left(\frac{n\pi}{b} \right)^4}{\left[\left(\frac{m\pi}{a} \right)^2 + \left(\frac{n\pi}{b} \right)^2 \right]^2}. \quad (15)$$

This yields the wanted bending stiffness. Now, we can substitute Eq.(15), h_e and ρ_e into Eq.(5) to obtain the equation of motion of the stiffened shell,

$$D_e \nabla^8 U_3 + Eh_e \nabla_k^4 - \rho_e \omega^2 \nabla^4 U_3 = 0, \quad (16)$$

Also, h_e is used instead of h , and ρ_e is used instead of ρ , since h_e and ρ_e are the thickness and mass per unit area of the current shell, respectively.

Next, by solving Eq.(16), the natural frequencies of the slightly curved, simply supported rectangular shell are obtained to be

$$f_{mnc,p}^2 = \frac{1}{4\pi^2} \frac{1}{\rho_e} \left[D_x \left(\frac{m\pi}{a} \right)^4 + 2H \left(\frac{m\pi}{a} \right)^2 \left(\frac{n\pi}{b} \right)^2 + D_y \left(\frac{n\pi}{b} \right)^4 \right] + \frac{\left[\frac{1}{R_x} \left(\frac{m\pi}{a} \right)^2 + \frac{1}{R_y} \left(\frac{n\pi}{b} \right)^2 \right]^2}{4\pi^2 \left[\left(\frac{m\pi}{a} \right)^2 + \left(\frac{n\pi}{b} \right)^2 \right]^2} \cdot \frac{Eh_e}{\rho_e}. \quad (17)$$

Note here that the first term equals $f_{mn,p}^2$ from Eq.(3). It is therefore seen that Eq.(17) is exactly equal to Eq.(12).

4 Comparison

4.1 Analytical and simulated data

A MATLAB code is developed for a simply supported slightly curved stiffened shell based on the mentioned theories using Eq.(17). ANSYS simulations are also made for the same models and compared with the analytical results. The finite element named SHELL93 is used in ANSYS.

A series of models of slightly curved stiffened rectangular shell are examined. The material, which is taken to be the same for all models, has a Young’s modulus of $2.1 \cdot 10^9$ N/m², Poisson’s ratio of 0.38, and density of 1030 kg/m³. Some dimensions of the models are the same. The edge of the shell in the x direction is 1m, and in the y direction is 0.7 m; the thickness of the shell is 6 mm; the distance between the stiffeners in the x and y directions are 0.2 m and 0.1 m, respectively. Other dimensions of the models are listed in table 1.

Model	w_s	h_s	R_x	R_y
1	3mm	9mm	5m	2m
2	3mm	12mm	5m	2m
3	3mm	18mm	5m	2m
4	4mm	9mm	5m	2m
5	5mm	9mm	5m	2m
6	9mm	9mm	5m	2m

Table 1 Dimensions of models 1 to 6.

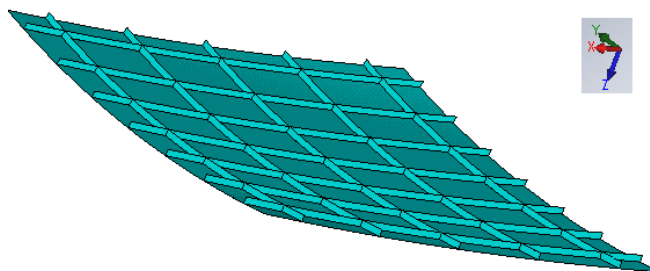


Fig.2 Model 1: Middle surface of the shell with stiffeners.

The results of the analytical method and the FEM simulation are listed in table 2. As mentioned earlier, the fundamental natural frequency is inaccurate. However, for mode numbers higher than (1,1), the deviations of the analytical and the simulated results are seen to be within 10% for model 1.

Although the analytical results have up to 10% deviation in comparison to the FEM simulated results, the former is quite useful for a coarse estimate of the natural frequencies of the structure.

m	n	MATLAB	ANSYS	Deviation
1	1	67	125	-46%
1	2	65	61	6%
1	3	98	88	10%
1	4	157	159	-1%
1	5	237	242	-2%
1	6	336	341	-1%
2	1	91	100	-9%
2	2	84	88	-5%
2	3	112	125	-10%
2	4	169	172	-2%
2	5	248	253	-2%
2	6	347	351	-1%
3	1	109	117	-7%
3	2	107	114	-6%
3	3	135	142	-5%
3	4	190	193	-2%
3	5	268	272	-1%
3	6	367	368	0%
4	1	129	142	-9%
4	2	135	146	-8%
4	3	165	172	-4%
4	4	219	224	-2%
4	5	297	299	-1%
4	6	394	393	0%

Table 2 Comparison of the analytical and simulated results for model 1.

Models 2 and 3 are for shells with increasing height of the stiffeners: 12mm and 18mm. The maximum deviation of model 2 is found to be 13% and for model 3 it is up to 21%.

It is obvious that the theory is limited to shells with stiffeners of small height.

Models 4 to 6 are for increased width of the stiffeners, namely 4mm, 5mm and 9mm. The maximum deviation of model 4 is 11%, model 5 is 12% and model 6 is 15%. In model 6, the width of the stiffeners takes its limiting value, where it equals to the height of the stiffener. This is because the empirical formula for the equivalent torsional rigidity H , only is valid for the ratio of h_s/w_s lying between one and infinity [16]. That means that no empirical formula is available for cases where the width of the stiffeners is larger than there height.

In the following, we will study the effect of the stiffeners and the effect of the curvatures with the present method. Model 6 is considered.

4.2 The effect of the stiffeners

The natural frequencies of model 6 are compared to an un-stiffened shell, which is the shell part of model 6 without stiffeners. The results are shown in Fig. 3 except for the fundamental frequency of mode (1, 1). The y axis shows the difference of the natural frequencies in percentage.

Fig. 3 shows that the natural frequencies of the stiffened shell in most cases are higher than the un-stiffened shell. The higher the mode numbers the higher the differences. Since the stiffeners are relatively small, the increase of the natural frequencies is only moderate.

4.3 The effect of the curvature

The natural frequencies of model 6 are also compared to those of a stiffened plate that has the same dimensions as model 6 but without curvatures. Figure 4 shows the natural frequencies of the modes except for the fundamental frequency of mode (1, 1).

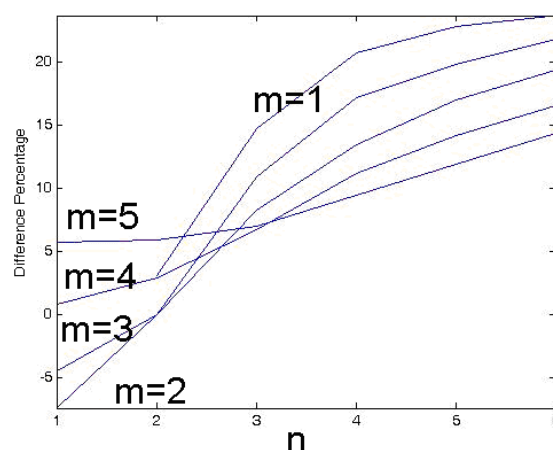


Fig.3 Comparison of the natural frequencies of the stiffened shell and an un-stiffened shell.

Although model 6 is only slightly curved, the curvatures obviously increase the natural frequencies especially for the lower modes. The difference reduces to a small value as m and n increase.

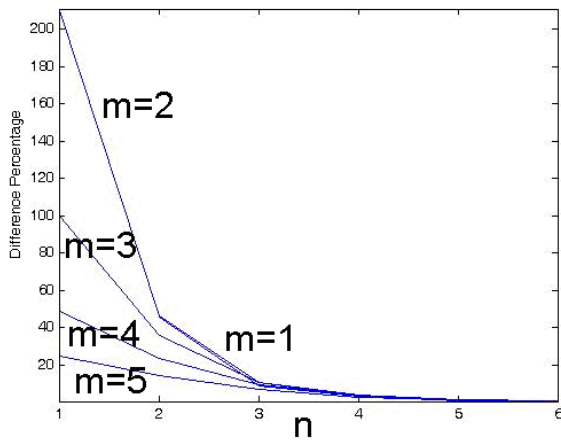


Fig.4 Comparison of the natural frequencies of the stiffened shell and a stiffened plate.

5 Conclusion

A simple method for calculating the natural frequencies of a simply supported, stiffened and slightly curved rectangular shell has been presented in this paper. For the cases examined the results show that a reasonable engineering accuracy can be obtained with little computational effort for weakly double-curved panels with small cross-stiffeners.

It is expected that it is possible to improve this estimation method to a wider range of structures by adding a correction factor, as mentioned in Ref. [18], to the Donnell-Mushtari-Vlasor's shell equations.

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