Nonlinear focal shift in focused ultrasonic transducers and its dependence on the Fresnel number

Yuri Makov[a], Victor Sánchez-Morcillo[b], Francisco Camarena[b] and Víctor Espinosa[b]

[a]Moscow State University, Dept. of Acoustics, 46730 Moscow, Russian Federation
[b]IGIC - Universitat Politècnica de València, Cra. Nazaret-Oliva S/N, E-46730 Gandia, Spain
fracafe@fis.upv.es

A systematic study of the on-axis location of the maximum pressure and intensity points in the field radiated by a focused transducer is presented as well as the motion of these characteristic points as the transducer voltage is increased. Different initial distributions, ranging from uniform to Gaussian cases, are considered.
Experimental and numerical results, based on the solutions of the KZK equation, with different initial conditions are analyzed. An analytical expression of the initial (linear) shift of the maximum pressure position is given. This expression, and the results of the numerical simulations, shows a good agreement with the experimental data. As a main conclusion, we demonstrate that the axial range of the nonlinear shift of pressure is larger for strong initial focal shifts, occurring for small Fresnel number transducers. Theoretical and numerical predictions of the focal shift effect in Gaussian beams are also presented. In this work we have established the relation between the focal shift (both in linear and nonlinear regime) with the Fresnel number of the transducer.

1 Introduction

Sound beams have attached an increasing attention during the last six decades [1,2], being the object of numerous theoretical and experimental investigations, mainly motivated by their extended use as multifunctional instruments in many applied technologies like in medical therapy [3] and non-destructive testing [4]. Nevertheless, the interplay among factors like diffraction, nonlinearity, and focusing in the acoustic field propagation still keep some open questions. An example of that is the case of the on-axis maximum pressure position of a beam. It differs with respect to the geometrical focus (linear shift), specially for low Fresnel number transducers (those with weak focusing), and also it moves towards focal point when the driving transducer voltage increase, and backward, to the transducer, if we reach highly nonlinear regimes (nonlinear shift). The range of this movement can be very appreciable and several references [5,6] have tried to explain this behavior with different interpretations.

The aim of this work is to provide a systematic study of the linear and nonlinear shift in pressure and intensity for different transducers (low and high focalization) as well as for different initial conditions (Uniform and Gaussian amplitude distribution in the transducer).

First, an analytical expression of the initial (linear) shift of the maximum pressure position is given that improve the previously reported ones [7] and match with the experimental data. After that, a theoretical study of the nonlinear shift is provided for Gaussian transducers. In the third and fourth sections the experimental dispositive and the numerical simulation code are exposed. In the next section, the experimental results for the nonlinear behaviour of the on-axis pressure and intensity maxima are presented, as well as the numerical results obtained for the nonlinear shift in uniform transducers with different Fresnel number (different focusing) and its relation with the linear shift. In this section it will be discussed also the numerical results obtained for nonlinear shift in Gaussian transducers.

2 Theoretical Predictions

2.1 Linear focal shift

In the linear regime it is clear that the on-axis pressure/intensity main maximum position and the geometrical focus (the shift of the maximum position from geometrical focus towards the transducer) do not coincide. Really, the diffraction, inherent to any beam, is the widening factor accumulated along the axis and it expands the focal waist (therefore decrease the field maximum in waist) and moves the waist (and maximum) position to the side of the lesser diffractional beam expansion, i.e. toward transducer. In our investigation this linear effect is the initial situation for the study of the influence of the nonlinear beam propagation regimes on the evolution of focal shift. Therefore the exact numeric data about the value of the linear focal shift is important for us. This data for beam can be given from the spatial distribution of the pressure $p(r,z,t) = A(r,z)e^{ikz-i\omega t}$, whose complex amplitude $A(r,z)$ is the solution of the ordinary wave equation in the parabolic approximation. This solution is (see, for example [7]):

$$A(r,z) = -\frac{ik}{z}\exp\left(\frac{ik}{2z}r^2\right)\int_0^\infty \exp\left(\frac{ik}{2z}r'^2\right)J_0\left(\frac{k}{z}rr'\right)A(r',0)r'dr' \quad (1)$$

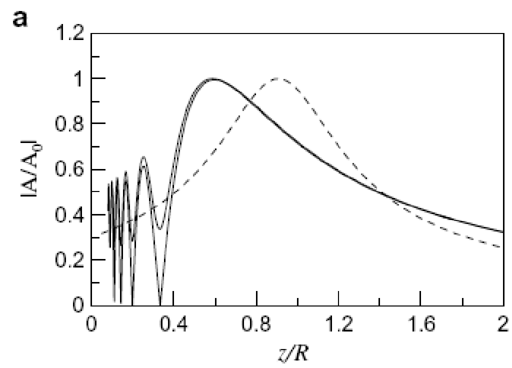
where z is the longitude coordinate along beam axis, r the radial (transverse) coordinate, k the wave number and $A(r, 0)$ is determined by the initial condition.

As a boundary condition, we assume an initial distribution of pressure along the transducer in the form of a truncated Gaussian function, with a parabolic phase profile accounting for the focusing effect

$$A(r,0) = A_0 \exp\left(-\frac{r^2}{a_G} - \frac{ik}{2R}r^2\right) \quad , 0 \le r \le a \quad (2)$$

and zero otherwise.

For this initial condition the on-axis pressure-amplitude distribution can be calculated from Eq. (1). Figures 1 and 2 show some of the results:



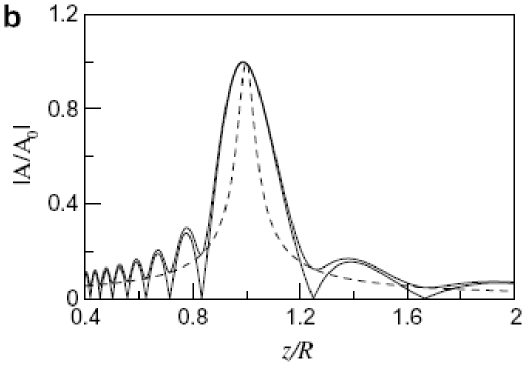


Fig. 1. Theoretical on-axis pressure distributions in linear regime for a gaussian beam (dashed line), truncated gaussian beam (upper solid line), and a beam with uniform initial condition (lower solid line). (a) and (b) corresponds to the cases of low (N_F = 1) and high (N_F = 10) Fresnel number.

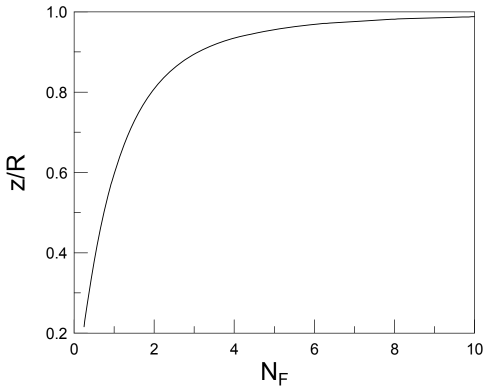


Fig 2. Dependence of the position of the on-axis main pressure maximum on Fresnel number N_F. Distance in dimensionless units.

2.2 Nonlinear focal shift in Gaussian transducers

For the analysis of the nonlinear behaviour of Gaussian beams we make use of the analytical pressure distributions obtained in [8] for the paraxial region. There, analytical solutions of the Khokhlo-Zabolotskaya equation were obtained in the form $P = f^{-1} \sin(\theta_0 + \eta) + \Delta$, where $P = p / p_0$ is the acoustic pressure normalized to its peak value at the source, $\theta_0 = \tau + g \sin(\theta_0 + \Phi) + \delta$ is an implicit function of time, and f, η, Δ, Φ, δ are functions of the gain G, the nonlinearity parameter N and the normalized axial distance $\tilde{z} = z / R$, defined by Eqs. (38)-(43) in [8].

The pressure and intensity distributions (Fig. 3) can be obtained from these expressions and depend only on the parameters G and N (defining the initial conditions), and the axial coordinate $\tilde{z}$.

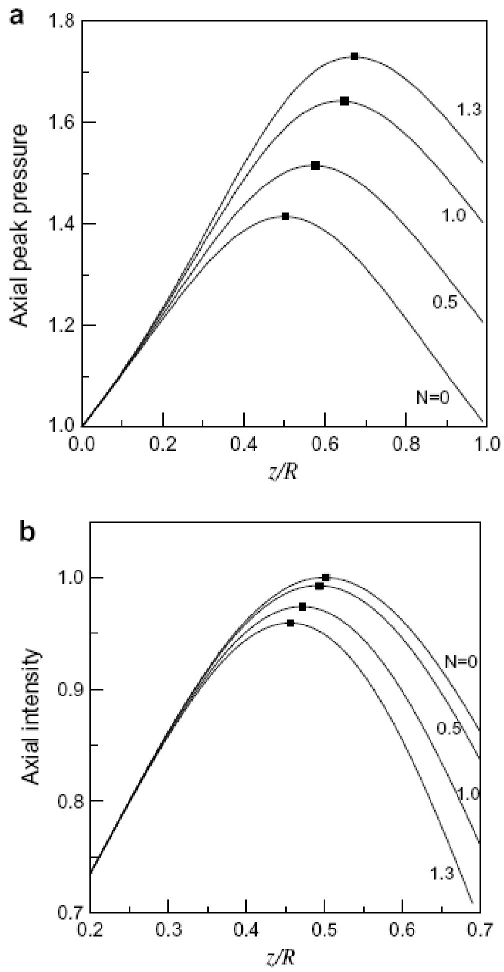


Fig. 3. On-axis peak pressure (a) and intensity (b) as follows from the analytical solutions of the KZK equation with an initial Gaussian distribution, for G = 1 and different values of the nonlinearity.

3 Experimental set-up

The experimental setup follows the classical scheme of confronted emitting transducer and receiving hydrophone in a water tank. We use a Valpey-Fisher focused transducer based on a piezoelectric spherical shell of curvature radius of 11.7 cm and diameter of 3 cm working at 1 MHz. The transducer is driven by the signal provided by a programmable Agilent 33220 function generator, amplified by a broadband rf power amplifier which permits to deliver voltage amplitudes at the transducer terminals up to 750 Vpp without distortion. The emitted signal is detected by a calibrated membrane hydrophone NTR/Onda Corp. MH2000B, with a sensing aperture of 0.2 mm and a flat frequency response between 1 and 20 MHz, which allows for the registration of the wave-form profile confidently.

4 Numerical Simulation

The propagation of high intensity focused ultrasonic beam can be modeled with great accuracy by the Khokhlov-Zabolotskaya-Kuznetsov (KZK) equation, which for axisymmetric beams is:

$$\frac{\partial^2 p}{\partial t'\partial z} = \frac{c_0}{2}\left(\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r}\right) + \frac{\delta}{2c_0^3}\frac{\partial^3 p}{\partial t'^3} + \frac{\beta}{2\rho_0 c_0^3}\frac{\partial^3 p^2}{\partial t'^2} \tag{3}$$

where $t' = t - z/c_0$ is a retarded time, c_0 the propagation speed, δ the sound diffusivity, β the coefficient of nonlinearity, and ρ_0 the ambient density of the medium.

Equation 3 is valid in the paraxial approximation (*ka*<<1) and takes into account nonlinearity, diffraction, and thermoviscous absorption by its corresponding terms, and the focusing through initial condition.

Several numerical schemes have been proposed for solving Eq. 3. We have used the time domain algorithm described in [9] with uniform or Gaussian amplitude distributions as boundary conditions.

5 Results

The experiment and the numerical simulations tried to confirm the theoretical results obtained in section 2, as well as to study the nonlinear shift in the case of uniform pressure distribution in the transducers, where we have not analytical solutions for the KZK equation.

5.1 Uniform pressure distribution in the transducer

The next aim is to inspect systematically in experiments how the nonlinearity influences the pressure and intensity maxima. In section 2.1 it was shown that the low-Fresnel-number beams (transducers) are characterized by a large initial linear focal shift what gives the possibility for its appreciable change in nonlinear regime. According to this fact, a Valpey–Fisher focused transducer with a = 1.5 cm and R = 11.7 cm, operating at a frequency f = 1 MHz, was used in our experiments. Under these parameters of the transducer the Fresnel number is equal to 1.28 and, on the basis of the model of uniform pressure distribution on transducer, the linear focal shift is equal to 0.33 R = 3.9 cm (accordingly z_{max} = 0.67 R = 7.8 cm).

The full experimental information is collected and presented in Fig. 4, where the experimental on-axis pressure distributions (dashed curves) and the corresponding calculated on-axis intensity distributions (solid curves) under increasing input voltages in the range 200–500 V_{pp} are given.

We have numerically solved the KZK equation (see details in section 4) We simulated the propagation in water of the beams radiated by focused transducers with different Fresnel number, from low to moderate values, concretely N_F=0.6, 1.3, 2.2, and 3.8. The initial distribution was assumed to be uniform. The results are shown in Fig. 5, where the position of the axial pressure maximum is represented, for each transducer, for increasing driving amplitudes. The Fresnel number of the transducers increases from left to right. We clearly observe that the magnitude of the shift (distance from the initial focus position to the return point) is larger for the transducer with smaller N_F, and decreases monotonically with N_F.

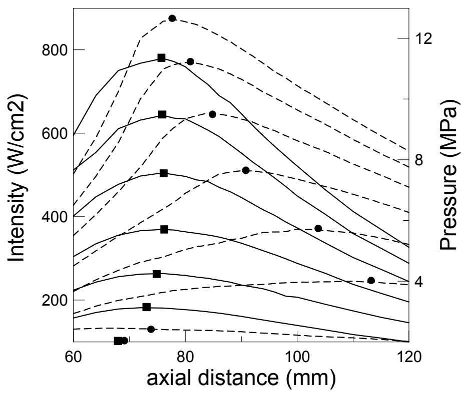


Fig. 4. On-axis peak pressure (dashed lines, right axis) and intensity (continuous lines, left axis) curves. Only the neighbourhood of the maxima is plotted. Maximum values are marked with symbols. Input values are 200, 250, 300, 350, 400, 450 and 500 Vpp from bottom to top.

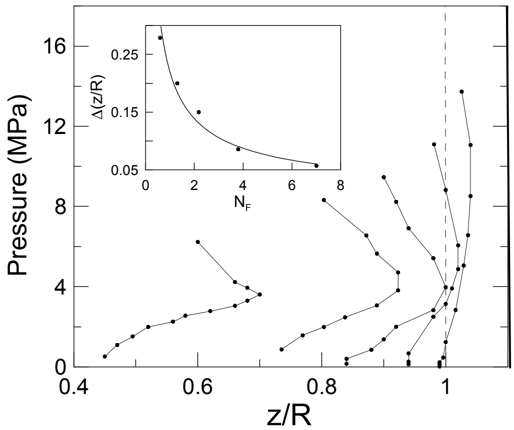


Fig. 5. On-axis maximum pressure for different Fresnel number transducers working from linear to nonlinear regime.

5.2 Gaussian pressure distribution in the transducer

We have numerically solved the KZK equation using a Gaussian amplitude distribution as boundary condition.

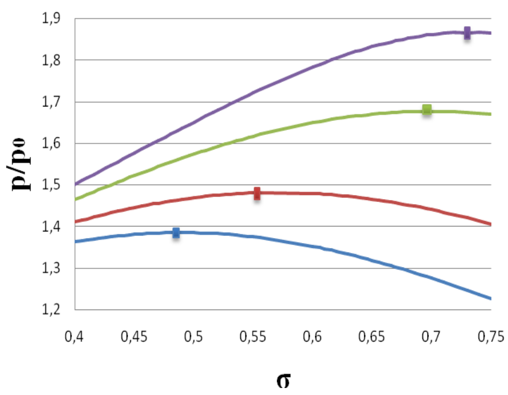


Fig. 6. On-axis pressure for G=1 and different values of the nonlinearity parameter, N=0, 0.5, 1.0 and 1.3.

Acknowledgments

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