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A stochastic source model for turbulent noise prediction including sweeping time dynamics

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Temporal changes in turbulent flows have a substantial influence on noise production and are the dominant noise source in free flows. We introduce a stochastic model including the effect of sweeping, i.e. the advection of inertial range structures by the energy containing large scales, which has been identified as a main source for temporal changes in turbulent flows. The resulting properties of the spatial-temporal correlations are in agreement with experimental findings. Due to the use of recursive filters the method is efficient in producing synthetic turbulence with realistic temporal properties. An application to the sound production of a jet is presented. The resulting spectra agree well with experimental data.

1 Introduction

Aeroacoustics comprises both, acoustical and aerodynamical aspects which cause some difficulties due to the disparities of the length scales and energy levels involved. Lighthill cuts back aeroacoustics to aerodynamics and acoustics by using an acoustic analogy. This allows to describe the flow separately as acoustic sources. Similar approaches based on linearized Euler equations or perturbation equations split the problem into an acoustic part and sources containing the influence of the flow. In hybrid methods the flow is computed separately from the acoustic part and is afterwards connected to the aeroacoustic calculations. It is still a challenge to provide the correct turbulent field containing all properties which are important for acoustics. In complex flow situation it can be provided by high fidelity simulations, e.g. a direct numerical simulation (DNS) or a large eddy simulation (LES), but the time effort is huge if the acoustic spectrum is of interest. An alternative approach is to set up stochastic models for the small scale turbulent fluctuations. The flow is split according to a Reynolds decomposition in a mean flow, which can be provided by a RANS simulation, and a fluctuating part, which has to be modeled stochastically. Such a separation of mean flow and stochastic model was used in different works [8, 1, 18, 10, 4].

Often, the description of turbulent fluctuations is based on a *spatial* consideration, e.g. the wave number spectrum or spatial structure functions. For the noise production however, the *frequency* spectrum is essential. If the mean flow velocity is large in comparison to the fluctuations, Taylor's hypothesis of frozen turbulence can be applied and the frequency spectrum can be calculated directly from the spatial one. It is however known that the neglected temporal changes intensify the noise [7, 18] and the acoustic spectrum can differ from the one calculated from Kolmogorov's theory of turbulence [9, 20, 14]. The consideration of realistic temporal changes is thus essential for aeroacoustic problems.

During the last years there have been controversial discussions about the origin of temporal changes in turbulent flows. Possible candidates are dynamical effects due to nonlinear interactions, vortex stretching, pressure fluctuations, viscosity effects or sweeping. They result in different turbulent frequency spectra. In the more recent debate, sweeping has been considered as the dominant effect in the decorrelation of small scales [20, 3, 11, 12, 14].

Sweeping causes a statistical dependence on the small scales of the larger ones and a Eulerian frequency spectrum which cannot be explained by Kolmogorov's traditional theory of turbulence [19]. Because the noise production is mainly affected by the Eulerian frequency spectrum, sweeping should have an important effect on the noise field as well [15]. In two workshops held in 2007 on synthetic turbulence models, the absence of 'sweeping effects' in present conventional kinematic simulation versions was considered as a major drawback for aeroacoustics [16].

The objective of this study is to incorporate sweeping in a synthetic turbulent model based on a random particle method introduced by Ewert [4] and to examine its influence onto the noise production.

First, we describe the formulation of the synthetic velocity field. The basis of this method has been introduced by Ewert [5]. In contrast to many other methods, it is formulated in real space and allows, due to the Lagrangian description, the correct convection by the velocity field. Then we describe the jet noise model and its acoustic source term. We introduce via a feedback mechanism temporal changes of the source by taking into account sweeping of the velocity field. Because the source is convected by the velocity field it takes over its temporal properties. In order to show that the dynamic is indeed dominated by sweeping, we study the spatial-temporal correlation function. We implement the stochastic model into the CAA solver PIANO developed by the German Aerospace Center and apply it to a jet noise problem. The acoustic spectra are discussed and compared to experiments.

2 Modeling the velocity field

In the following we describe the model of the fluctuating part $\mathbf{u}^t = \mathbf{u} - \mathbf{u}_0$, $\langle \mathbf{u}^t \rangle = 0$, of the turbulent velocity field $\mathbf{u}$. The main idea is to reconstruct the nonstationary turbulent fluctuations from a RANS simulation in order to use them as a source-term in a CAA simulation. The RANS calculation gives the isotropic one-point mean quantities: the mean velocity $\mathbf{u}_0$, the kinetic energy k and the dissipation rate ϵ (ω for the k-ω model). The correct acoustic source term has to contain however, in addition the two-point statistics of the velocity field. Thus, the aim is to generate a nonstationary stochastic field, which is compatible with the solution of the RANS equation and furthermore contains two-point stochastic properties, i.e. the distribution of the velocity fluctuations $p(\mathbf{u})$ as well as the spatial-temporal correlations $\langle u_i(\mathbf{x}+\mathbf{r}, t+\tau) u_j(\mathbf{x}, t) \rangle$. The turbulent field should obey incompressibility, inhomogeneity and convection by the mean flow in order to reproduce important properties necessary for the noise generation.

The stochastic fluctuations are generated as described in the following. We start from a white noise field

$$\langle \eta(x,y) \rangle = 0 \quad (1)$$

$$\langle \eta(x,y)\eta(x',y') \rangle = \delta(x-x')\delta(y-y') \quad (2)$$

$$\frac{D_0}{Dt}\eta(x,y) = 0. \quad (3)$$

Eq. (3) with the substantial time derivative $D_0/Dt \equiv \partial/\partial t + \mathbf{u}_0 \cdot \boldsymbol{\nabla}$ implies that the field is convected by the mean velocity field in such a way that the white field is not changing by following a Lagrangian trajectory in time. This field is convoluted

$$\psi(x,y) = \int\int G(x-x', y-y')\eta(x',y')dx'dy' \quad (4)$$

with the kernel

$$G(x,y) = \hat{R}\exp\left(-\frac{\pi}{2}\frac{(x+y)^2}{l^2}\right) \quad (5)$$

resulting in the stream function ψ. The amplitude of the filter kernel G is given by the kinetic energy k and the length scale l:

$$\hat{R} = l\sqrt{\frac{4k}{3\pi}}. \quad (6)$$

The kinetic energy is directly given by the RANS simulation; the length scale is given by

$$l = c_l \frac{k^{3/2}}{\epsilon} \quad (7)$$

with $c_l = 0.54$ for a k-ϵ turbulence model or by

$$l = \frac{c_l}{C_\nu}\frac{k^{3/2}}{\epsilon} \quad (8)$$

with $c_l/C_\nu \approx 6.0$ for a k-ω turbulence model. From the stream function, the velocity field can be calculated

$$u_i = \epsilon_{ij}\frac{\partial}{\partial x_j}\psi \quad (9)$$

where ϵ_{ij} is the two-dimensional permutation symbol. This field is solenoidal, isotropic and has the longitudinal correlation function

$$f(r) = \exp\left(-\frac{\pi}{4}\frac{r^2}{l^2}\right). \quad (10)$$

Because of the choice of the filter function it is compatible with the kinetic energy distribution and has the correct correlation length. Such a synthetic turbulent field has been used in airframe noise applications as source term for the acoustic equations [4, 5, 6]. In this work we only use it to calculate the temporal dynamics of the source term described below.

The numerical realization of the stochastic field is described in [4]. This 'Random Particle Method' (RPM) is based on a field of particles carrying stochastic values $\boldsymbol{\eta}$ and which is convected by the mean velocity.

3 Sweeping of the velocity field

In the following we look for the possibility to introduce dynamical effects which reproduce correctly sweeping effects. It has been found that these can be incorporated by letting the white field η move on the velocity field: Every point of the white field η is convected according to the differential equation

$$\dot{\mathbf{x}}(t) = \mathbf{u_0}(\mathbf{x}(t)) + \mathbf{u}^t(\mathbf{x}(t), t), \quad (11)$$

where $\mathbf{u}_0$ denotes the mean velocity field. This differential equation has to be solved together with the filtering process and the CAA simulation. In other words, the velocity field carries itself which leads essentially to the above mentioned sweeping effects as we will show below. It can be shown that the introduced feedback mechanism only affects the time properties of the velocity field, all other stochastic properties are unaffected.

4 Spatial-temporal properties

In order to show that the resulting field describes the desired spatial-temporal correlations, let us look at the Eulerian correlations

$$C(r = |\mathbf{r}|, \tau) = \frac{\langle \mathbf{u}(\mathbf{x},t)\cdot\mathbf{u}(\mathbf{x}+\mathbf{r}, t+\tau)\rangle}{\langle \mathbf{u}(\mathbf{x},t)\cdot\mathbf{u}(\mathbf{x}+\mathbf{r}, t)\rangle}. \quad (12)$$

It is well known that, if sweeping is a dominant effect, the Eulerian correlations obey the similarity law

$$C(r,\tau) = \mathcal{C}\left(\frac{u_{\mathrm{rms}}\tau}{r}\right), \quad (13)$$

where the sweeping velocity $u_{\mathrm{rms}}^2 = \langle(\mathbf{u}-\mathbf{u}_0)^2\rangle$ is the mean square of the velocity fluctuations [13].

We perform a simulation on a grid with the edge-length 20 in x-direction and 2.5 in y-direction. The parameters are $l = 0.5$, $k = 0.1$. We record 100.000 samples of the time signal at 8 different locations, giving 7 values for r between 0.5 and 2.0.

Figure 2 shows the correlation functions C in dependence of the normalized time scale $v_{\mathrm{rms}}\tau/r$ for different spatial distances r. The curves collapse into one indicating that the similarity (13) holds. Note that this can not be observed for the case of the decorrelations as proposed by Tam [18]. It is worth noting that experimental data obey the same properties as shown in Fig. 2. Furthermore, the slope of the curve at $\tau = 0$ is zero, which corresponds to a differentiable field. This is in accordance with experiments and a desired property for a source term in a numerical simulation.

5 Modeling the jet noise source

The mixing noise of jets can be attributed to two distinct sound producing mechanisms. In a wide range of Mach numbers, temperatures and nozzle geometries, each source mechanism leads to a universal similar spectrum [17]. The fine scale turbulence produces a broad-band spectrum, the g-spectrum, radiated omnidirectional from the jet. At high jet velocities, a second narrower spectrum, the f-spectrum, arises from Mach waves radiated in down-stream direction between 130° and 150° measured from the nozzle exit.

As jet noise model we take a slightly modified version of the Tam & Auriault jet noise model [18], which was intended as a model for the broad-band noise spectrum, for details see [4]. Different to the Tam & Auriault model, the jet spreading and mean flow gradients

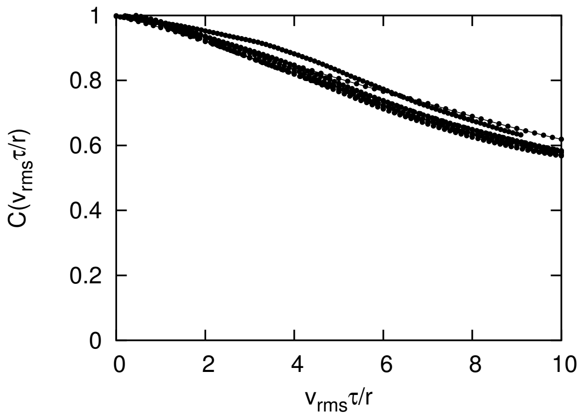

Figure 1: Time-correlation generated by sweeping, see Eq. (11). The axes are scaled according to the sweeping-hypothesis, see Eq. (13). Each curve belongs to a different spatial distance r ranging from 0.5 to 2.0. It can bee seen the curves fall on top of the other.

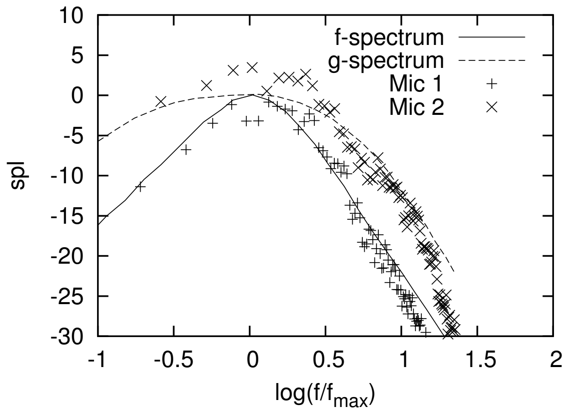

Figure 2: The acoustic spectra of the jet at two different locations in comparison with universal spectra given in the literature [17]. Microphone in direction of the Mach-wave radiation, +, and transverse to the jet-nozzle, ×.

can be included, and the source term is introduced in the momentum equation:

$$\rho_0\left(\frac{\partial \mathbf{u}}{\partial t}+(\mathbf{u}_0\cdot\boldsymbol{\nabla})\mathbf{u}\right)+\boldsymbol{\nabla}p = \mathbf{0} \quad (14)$$

$$\frac{\partial p}{\partial t}+\mathbf{u}_0\cdot\boldsymbol{\nabla}p+\gamma p_0\boldsymbol{\nabla}\cdot\mathbf{u} = \frac{D_0 q_s}{Dt}. \quad (15)$$

Whereas the complete source term $\frac{D_0 q_s}{Dt}$, also including the time dependence, is modeled in the Tam & Auriault model, we use here only a spatial model of the source term. The time-dependence is given by sweeping, i.e. no further parameters or models are needed for it. The source q_s results from a convolution

$$q_s(x,y)=\int\int G(x-x',y-y')\eta(x',y')dx'dy' \quad (16)$$

analogue to the calculation of the stream function with a Gauss-shaped filter kernel but now with the amplitude

$$\hat{R}\to\hat{R}_s=A\frac{2}{3}\frac{\rho_0 k}{l_s}. \quad (17)$$

and the length scale

$$l\to l_s=c_s\frac{k^{3/2}}{\epsilon} \quad (18)$$

with $c_s=0.273$.

The source field q_s is convected by the velocity field and takes over all dynamical properties of it. If the velocity field contains sweeping, so does q_s. In this way we will incorporate temporal changes into the source.

To take advantage of the symmetry of the jet for the aeroacoustic simulation, we introduce cylindrical coordinates and consider only the zero-order mode, i.e. axial symmetry. However this symmetry can not be fulfilled by the turbulent structures because the maximal correlation length of the source is l_s and not the full circumference as implied by the axial symmetry. This would result in an overvalued source amplitude, which can be corrected by setting $\hat{R}_s\to\hat{R}_s l_s/(R\pi+2l_s)$ [2] where R is the radial coordinate.

6 An Example for Jet Noise

To study the effect of sweeping we perform a simulation of a jet whose mean-flow quantities are given by a RANS simulation. The far-field Mach number is 0.039, the jet velocity $u_{\text{jet}}=189.51$ m/s. The resolution of the CAA allows frequencies of up to $f_{\text{max}}=6050$ Hz.

We solve Eq. (14,15) with PIANO and calculate at every time step the stochastic velocity field $\boldsymbol{u}^t$ and the source field q_s. The velocity field is needed only to convect the source field correctly which gives the desired spatial-temporal properties.

We record at two different points the pressure time series. The points are chosen in such a way that they should capture the g-noise and the f-noise spectrum, respectively. Fig. 2 shows the normalized spectra in comparison to the similar-spectra found by Tam [17]. A good accordance can be found. It is remarkable that also Mach waves can be recovered. These waves are not contained in our model but are produced by the dynamics of the linearized Euler equation in conjunction with the source.

7 Conclusion

The objection of this work has been to incorporate sweeping in a stochastic turbulence model and to study the influence of sweeping onto the noise production of a jet. We have added sweeping to a random particle method by a feedback-mechanism between the movement of the white field and the generated velocity field: the velocity field is calculated from the white field and the white field is advected by the velocity field. We have shown that the spatial-temporal correlations collapse by using the scaling (13) indicating that sweeping indeed dominates the dynamics in our model. A Lagrangian approach as used in our model is thus capable to solve the sweeping problem of kinematic simulations. In a next step, we have implemented the model in the CAA solver PIANO and have calculated the noise generation of a jet. The calculated spectra are in accordance with similar spectra found in experiments [17]. This support the arguments of Rubinstein and Zhou [14] that sweeping effects are important for noise generation. It is very interesting that also the f-spectrum of the Mach waves can be reproduced: The Mach waves are radiated from instabilities which are driven by the small-scale turbulence.

Our results suggest that sweeping has to be taken into account in kinematic turbulence models if the Eulerian frequency spectrum is to be of interest.

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