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On the comparison of symmetric and unsymmetric formulations for experimental vibro-acoustic modal analysis

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The classical u-p formulation for vibro-acoustic problems is very convenient for experimental vibro-acoustic modal analysis since the physical variables are directly those which are measured by operators. In this particular context, the objective is to identify from experimental measurements a reduced model which has the same behaviour as the measured one. The complex mode shapes which are identified using this technique must satisfy a properness condition. When they do not verify it, they should be modified to be able to represent the behaviour of a physical system. Some techniques have been proposed in order to develop a strategy to obtain the modified eigenshapes, but this is a quite difficult point because of the unsymmetric topology of the equations. In this paper, a symmetric formulation is used in order to be able to directly apply the classical methodology which has been developed for structural modal analysis to obtain the physical reduced system. The methodology is described and compared with the u-p formulation, in terms of efficiency and precision, in particular when some absorbing devices are considered. All results are first presented on an ideal numerical test-case, and applications on experimental data are finally shown.

1 Introduction

Coupled vibro-acoustics behaviors can either be needed in some cases like musical instruments, or not desired in situations like coupled vibrations between internal fluid domain and body of automotive structures. The topic of this paper is the vibro-acoustical behavior of coupled systems, constituted by a vibrating structure enclosing a fluid domain (internal vibro-acoustics).

Vibro-acoustics modal analysis [2] allows one to experimentally identify coupled modes of the system, following the methodology which is classical for structural dynamics, when the behavior of the vibrating structure is supposed to be decoupled from the acoustic phenomenon. Modal synthesis consists in representing the behavior of the system on a given frequency band using an experimental model, built using a reduced modal basis, constituted by a given number of identified modes. This identification generally leads to the determination of complex modes, that can include errors coming from several physical, environmental or numerical uncertainties. The levels of these errors can be of several magnitude orders depending in particular on the experimental context, on the know-how of the operator, and on the algorithms used for identification, but in all cases, even small errors on complex modes can lead to large differences. In particular, if one is interested by matrices reconstruction, this is a crucial point.

A mathematical property allowing one to insure that the system is able to be represented by a discrete equivalent system is called properness of complex vectors. This condition is very well detailed in ref. [1]. In the cases in which complex vectors do not verify that condition, the author presents a methodology to perform small modifications on the vectors in order that they verify it. In this paper, an extension of this approach is presented for vibro-acoustical modal analysis. A non-symmetric formulation is first presented, before comparing it with a symmetric one.

2 Problem statement and modal decomposition

2.1 Movement equations

Discretizing an internal vibro-acoustical problem using the natural fields for the description of the structure (those which can be directly measured), i.e. displacement for the structure and acoustic pressure for the cavity, leads to the following matrix system [3]:

$$\underbrace{\begin{bmatrix} M_s & 0 \\ L^T & M_a \end{bmatrix}}_{\substack{[M] \\ \{q\}}} \underbrace{\begin{Bmatrix} \ddot{x} \\ \ddot{p} \end{Bmatrix}}_{\{q\}} + \underbrace{\begin{bmatrix} C_s & 0 \\ 0 & C_a \end{bmatrix}}_{[C]} \underbrace{\begin{Bmatrix} \dot{x} \\ \dot{p} \end{Bmatrix}}_{\{q\}} + \underbrace{\begin{bmatrix} K_s & -L \\ 0 & K_f \end{bmatrix}}_{[K]} \underbrace{\begin{Bmatrix} x \\ p \end{Bmatrix}}_{\{q\}} = \underbrace{\begin{Bmatrix} F_s(t) \\ \dot{Q}_a(t) \end{Bmatrix}}_{\{f(t)\}}, \quad (1)$$

in which $\{x\}$ is the vector of generalized displacements of the structure, $\{p\}$ is the vector of acoustic pressures, $[M_s]$ is the mass matrix of the structure, $[M_a]$ is called "mass" matrix of acoustic fluid (its components are not homogeneous to masses, the name is chosen for analogy with structural denomination), $[K_s]$ is the stiffness matrix of the structure, $[K_a]$ is the "stiffness" matrix of fluid domain, $[L]$ is the vibro-acoustic coupling matrix, $[C_s]$ and $[C_a]$ respectively represent structural and acoustic losses. This formulation includes the hypothesis that there is no loss at the coupling between structural and acoustic parts, and that internal losses can be represented using equivalent viscous models. $\{F_s(t)\}$ is the vector representing the generalized forces on the structure, while $\{\dot{Q}_a(t)\}$ is associated to acoustic sources (volume acceleration) in the cavity.

2.2 Complex modes for vibro-acoustics

One way to solve the system (1) for steady-state harmonics is to use modal decomposition. The non-symmetric character of the matrix system implies that right and left modes must be identified. This can be done using the space-state representation of the system:

$$[U] \{\dot{Q}(t)\} - [A] \{Q(t)\} = \{F(t)\} \quad (2)$$

in which:

$$[U] = \begin{bmatrix} C & M \\ M & 0 \end{bmatrix}, [A] = \begin{bmatrix} -K & 0 \\ 0 & M \end{bmatrix} \quad (3)$$

$$\{Q(t)\} = \begin{Bmatrix} q(t) \\ \dot{q}(t) \end{Bmatrix}, \{F(t)\} = \begin{Bmatrix} f(t) \\ 0 \end{Bmatrix}$$

The eigenvalues of this problem can be stored in the spectral matrix Λ :

$$[\Lambda] = \left[\backslash \lambda_j \backslash \right] \quad (4)$$

The j -th eigenvalue is associated to:

- a right eigenvector $\{\theta_{Rj}\}$ such as $(U\lambda_j - A)\{\theta_{Rj}\} = 0$, in which $\{\theta_{Rj}\} = \begin{Bmatrix} \phi_{Rj} \\ \phi_{Rj}\lambda_j \end{Bmatrix}$. Storing the eigenvectors (in the same order as the eigenvalues) in the modal matrix $[\theta_R] = \begin{bmatrix} \phi_R \\ \phi_R\Lambda \end{bmatrix}$, the following relationship is verified:

$$[U][\theta_R][\Lambda] = [A][\theta_R] \quad (5)$$

- a left eigenvector $\{\theta_{Lj}\}$ such as $\{\theta_{Lj}\}^T (U\lambda_j - A) = 0$, in which $\{\theta_{Lj}\} = \begin{Bmatrix} \phi_{Lj} \\ \phi_{Lj}\lambda_j \end{Bmatrix}$. Storing the eigenvectors (in the same order as the eigenvalues) in the modal matrix $[\theta_L] = \begin{bmatrix} \phi_L \\ \phi_L\Lambda \end{bmatrix}$, the following relationships are verified:

$$[U]^T [\theta_L] [\Lambda] = [A]^T [\theta_L] \quad \text{or} \quad [\Lambda] [\theta_L]^T [U] = [\theta_L]^T [A] \quad (6)$$

The orthogonality relationships can be written using $2n$ arbitrary values to build the diagonal matrix $[\xi] = \left[\backslash \xi_j \backslash \right]$:

$$[\theta_L]^T [U] [\theta_R] = [\xi] \quad \text{or} \quad [\theta_L]^T [A] [\theta_R] = [\xi] [\Lambda] \quad (7)$$

The modal decomposition of the permanent harmonic response at frequency ω is finally:

$$\{Q(t)\} = [\theta_R] ([\xi] (j\omega[E_{2n}] - [\Lambda]))^{-1} [\theta_L]^T \{F(\omega)\} e^{i\omega t} \quad (8)$$

in which E_{2n} is the $2n \times 2n$ identity matrix and $F(\omega)$ is the complex amplitude of the harmonic excitation. This relationship can also be written using the n degrees of freedom notations in the frequency domain:

$$\{Q(\omega)\} = [\phi_R] [\Xi] [\phi_L]^T \{f(\omega)\} \quad (9)$$

with:

$$[\Xi] = \left[\backslash \frac{1}{\xi_j(i\omega - \lambda_j)} \backslash \right] \quad (10)$$

For vibro-acoustics, it can easily be shown that there is a relationship between right and left eigenvectors [2]:

$$\text{If } \{\phi_{Rj}\} = \begin{Bmatrix} X_j \\ P_j \end{Bmatrix} \text{ then } \{\phi_{Lj}\} = \begin{Bmatrix} X_j \\ -P_j/\lambda_j^2 \end{Bmatrix}. \quad (11)$$

This means that only the right eigenvectors extraction is necessary to obtain the full modal basis of the system. The previous relation can also be written as:

$$\text{If } [\phi_R] = \begin{bmatrix} X \\ P \end{bmatrix} \text{ then } [\phi_L] = \begin{bmatrix} X \\ -P\Lambda^{-2} \end{bmatrix}. \quad (12)$$

3 Properness of complex modes

3.1 Properness for structural dynamics

The reader is invited to refer to the paper [1] in order to be familiar with the properness condition in structural dynamics.

3.2 Properness for vibro-acoustics

Obtaining the properness condition in the case of a non-self adjoint system is almost instantaneous, starting from orthogonality relationships (7):

$$[U]^{-1} = [\theta_R] [\theta_L]^T \quad (13)$$

or

$$\begin{bmatrix} C & M \\ M & 0 \end{bmatrix}^{-1} = \begin{bmatrix} 0 & M^{-1} \\ M^{-1} & -M^{-1}CM^{-1} \end{bmatrix} \quad (14)$$

$$= \begin{bmatrix} \phi_R\phi_L^T & \phi_R\Lambda\phi_L^T \\ \phi_R\Lambda\phi_L^T & \phi_R\Lambda^2\phi_L^T \end{bmatrix},$$

and

$$[A]^{-1} = [\theta_R] [\Lambda] [\theta_L]^T \quad (15)$$

or

$$\begin{bmatrix} -K & 0 \\ 0 & M \end{bmatrix}^{-1} = \begin{bmatrix} -K^{-1} & 0 \\ 0 & M^{-1} \end{bmatrix} \quad (16)$$

$$= \begin{bmatrix} \phi_R\Lambda^{-1}\phi_L^T & \phi_R\phi_L^T \\ \phi_R\phi_L^T & \phi_R\Lambda\phi_L^T \end{bmatrix}.$$

It is then clear that the properness condition for a non-symmetric second order system can be written as:

$$\phi_R\phi_L^T = 0 \quad (17)$$

Once this relationship is verified, the matrices can be found using the inverse relations:

$$M = (\phi_R\Lambda\phi_L^T)^{-1} \quad (18)$$

$$K = -(\phi_R\Lambda^{-1}\phi_L^T)^{-1} \quad (19)$$

$$C = -M\phi_R\Lambda^2\phi_L^T M \quad (20)$$

For the particular vibro-acoustic case, left eigenvectors are linked to right ones, and the properness condition can be written using only the right complex eigenvectors:

$$\begin{bmatrix} XX^T & -X\Lambda^{-2}P^T \\ PX^T & -P\Lambda^{-2}P^T \end{bmatrix} = 0. \quad (21)$$

3.3 Methodologies for properness enforcement

When the complex modes are available from experimental identification, one can use equations (18) to (20) in order to find the reduced model which is supposed to have the same behavior as the measured one. The fact is that in general, the modes do not verify the properness condition (21). In reference [1], a methodology to enforce properness is proposed. The objective is to find the

approximate complex vectors, which are as close as possible to the identified ones, and that verify the properness condition. It is shown that for structural dynamics, an explicit solution can be found, requiring only to solve a Riccati equation. For vibro-acoustics, one has to solve the following problem:

$$\begin{aligned} & \text{Find } \tilde{X} \text{ and } \tilde{P} \text{ minimizing } \|\tilde{X} - X\| \text{ and } \|\tilde{P} - P\| \\ & \text{while } \begin{cases} \tilde{X}\tilde{X}^T = 0 \\ \tilde{X}\tilde{P}^T = 0 \\ \tilde{X}\Lambda^{-2}\tilde{P}^T = 0 \\ \tilde{P}\Lambda^{-2}\tilde{P}^T = 0, \end{cases} \end{aligned} \quad (22)$$

in which X and P are two given complex rectangular matrices and Λ is a given diagonal complex matrix. This problem can be re-written using 4 Lagrange multipliers matrices δ_i ($i=1$ to 4):

$$\begin{cases} 0 = \begin{Bmatrix} \tilde{X} \\ \tilde{P} \end{Bmatrix} - \begin{Bmatrix} X \\ P \end{Bmatrix} + \frac{1}{2} \begin{bmatrix} \delta_1 + \delta_1^T & \delta_2 \\ \delta_2^T & 0 \end{bmatrix} \begin{Bmatrix} \tilde{X} \\ \tilde{P} \end{Bmatrix} \\ \quad - \frac{1}{2} \begin{bmatrix} 0 & \delta_3 \\ \delta_3^T & \delta_4 + \delta_4^T \end{bmatrix} \begin{Bmatrix} \tilde{X}\Lambda^{-2} \\ \tilde{P}\Lambda^{-2} \end{Bmatrix} \\ 0 = \tilde{X}\tilde{X}^T \\ 0 = \tilde{X}\tilde{P}^T \\ 0 = \tilde{X}\Lambda^{-2}\tilde{P}^T \\ 0 = \tilde{P}\Lambda^{-2}\tilde{P}^T. \end{cases} \quad (23)$$

In this case one can not obtain an explicit solution like in the case of structural dynamics [1], but some approximate solutions can be found [5] that allow one to obtain better results than those corresponding to direct inversion.

3.4 Experimental test-case

An experimental test-case has been performed on a set of vibro-acoustic measurements on a guitar [4]. The so-called A0 and T1 complex modes have been experimentally identified using measured frequency response functions (FRFs), by a classical technique [6], and one wants to obtain a two degrees-of-freedom (dofs) equivalent system that is able to represent the dynamic behavior of the instrument. The figure 1 represents the results obtained using several methodologies. The black continuous curve is the reference one, constituted by the response associated to the identified complex modes (obtained using modal composition). The grey continuous line corresponds to the direct identification of matrices using equations (18) to (20). It is clear in that case that the identified complex vectors do not correspond to those of a 2-dofs equivalent system. The dashed line corresponds to the methodology called "diagonal" in which structural and acoustical parts of the complex vectors are modified independently one from another. This implies the cancellation of diagonal terms of equation (21). Finally, the dotted line corresponds to the methodology called "over-properness", in which the four terms of equation (21) can be canceled, while two other ones are also imposed with zero values, which are not theoretically required [5]. This methodology implies good improvements, compared with direct reconstruction of

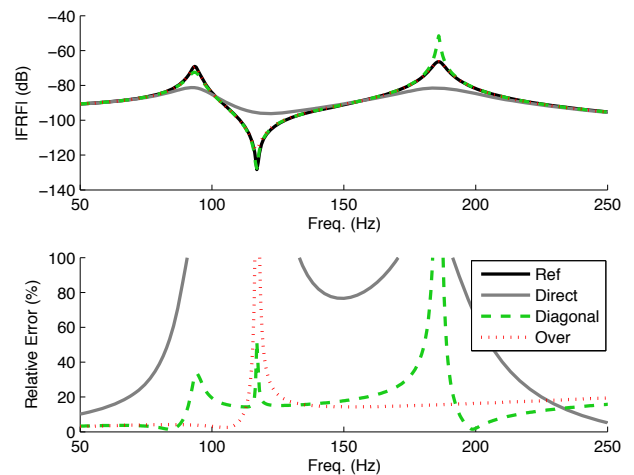


Figure 1: Methodologies for properness enforcement on guitar measurements

matrices, but has some theoretical drawbacks, like the two non-required canceled terms, and the final reconstructed matrices that do not respect the vibro-acoustic topology described by equation 1. The reason is that the proposed solutions are not exact solutions of problem (23), which is not as simple as in the structural case of the non-symmetry of the chosen formulation. Nevertheless, the methodology allows one to find the reduced model, identified from experimental data, using one structural dof and one acoustical dof:

$$\begin{aligned} M &= \begin{bmatrix} 4.71 \times 10^{-2} & -3.84 \times 10^{-9} \\ 5.81 \times 10^{-2} & 2.67 \times 10^{-7} \end{bmatrix}, \\ K &= \begin{bmatrix} 4.29 \times 10^4 & -6.39 \times 10^{-2} \\ -1.78 \times 10^3 & 1.43 \times 10^{-1} \end{bmatrix}, \\ C &= \begin{bmatrix} 3.90 & 3.88 \times 10^{-5} \\ 1.17 & -3.99 \times 10^{-5} \end{bmatrix}. \end{aligned} \quad (24)$$

This set of matrices allows one to have a correct reconstruction of frfs, with coherent stiffness and mass matrices (even if the topology is not correct), but one can observe that there is a physical problem with the damping term associated to the pressure degree of freedom, which is negative.

4 Symmetric formulation for vibro-acoustics

4.1 Matrix formulation

Some alternative vibro-acoustic formulations can be used instead of the one corresponding to equation 1, like explained in references [3, 7] for example. Among them, one can use a modified velocity potential ϑ as the variable describing the fluid behavior, such as $p = -\vartheta$. This change implies that the measured value is no longer the one which is directly in the formulation, which is in principle not a problem since one can easily go back to pressure, but there is a strong advantage, the problem is

now symmetric:

$$\begin{bmatrix} M_s & 0 \\ 0 & -M_a \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{\vartheta} \end{Bmatrix} + \begin{bmatrix} C_s & L \\ L^T & -C_a \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{\vartheta} \end{Bmatrix} + \begin{bmatrix} K_s & 0 \\ 0 & -K_a \end{bmatrix} \begin{Bmatrix} x \\ \vartheta \end{Bmatrix} = 0. \quad (25)$$

The potential problems associated to the non-positivity of the system or to practical force excitation of the acoustical part are avoided since this formulation will be used only for the inversion of the problem, while the classic unsymmetric form can be used for the complex modes identification step.

4.2 Properness for symmetric formulation in vibro-acoustics

Following the same methodology as above, the properness condition can be written using the identified complex modal matrix $[\Psi] = \begin{bmatrix} X \\ \Theta \end{bmatrix}$:

$$\begin{bmatrix} XX^T & X\Theta^T \\ \Theta X^T & \Theta\Theta^T \end{bmatrix} = 0, \quad (26)$$

which is equivalent to:

$$\begin{bmatrix} XX^T & X\Lambda^{-1}P^T \\ P\Lambda^{-1}X^T & P\Lambda^{-2}P^T \end{bmatrix} = 0. \quad (27)$$

This is quite different from (21). This point will be addressed in the next part. Since the formulation is symmetric, one can directly use the results given in reference [1] in order to enforce the properness condition on the identified complex modal matrix Ψ . Denoting $\tilde{\Psi}$ the modified modal matrix verifying the properness condition, its expression is:

$$\tilde{\Psi} = (E_n - \delta\bar{\delta})^{-1} (\Psi - \delta\bar{\Psi}), \quad (28)$$

where the Lagrange multiplier matrix δ is solution of:

$$\Psi\Psi^T - \delta\bar{\Psi}\Psi^T - \Psi\bar{\Psi}^T\delta + \delta\bar{\Psi}\bar{\Psi}^T\delta = 0. \quad (29)$$

4.3 Numerical and experimental test-case

The numerical test-case presented in [5] has been used to test the symmetric version of the procedure used to enforce the properness condition. The figure 2 shows the results associated to this numerical test. The guitar experimental data have been also used to test this version of the enforcement procedure, like shown in figure 3. One can observe that in both cases, no real improvement is added by the symmetric formulation, even if no approximation is done using the enforcement methodology, which was the case when the unsymmetric formulation was used. The reason for that is explained in the next part. Concerning the guitar test-case, the equivalent system is:

$$\begin{bmatrix} 4.71 \times 10^{-2} & 2.90 \times 10^{-6} \\ 2.90 \times 10^{-6} & -2.71 \times 10^{-7} \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{\vartheta} \end{Bmatrix} + \begin{bmatrix} 3.14 & -6.00 \times 10^{-2} \\ -6.00 \times 10^{-2} & -2.77 \times 10^{-6} \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{\vartheta} \end{Bmatrix} + \begin{bmatrix} 4.28 \times 10^4 & 7.57 \times 10^{-1} \\ 7.57 \times 10^{-1} & -1.41 \times 10^{-1} \end{bmatrix} \begin{Bmatrix} x \\ \vartheta \end{Bmatrix} = 0. \quad (30)$$

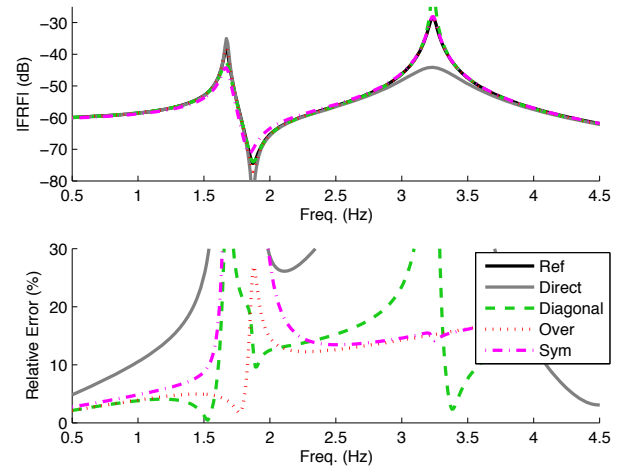


Figure 2: Methodologies for properness enforcement on numerical test-case, including the symmetric formulation

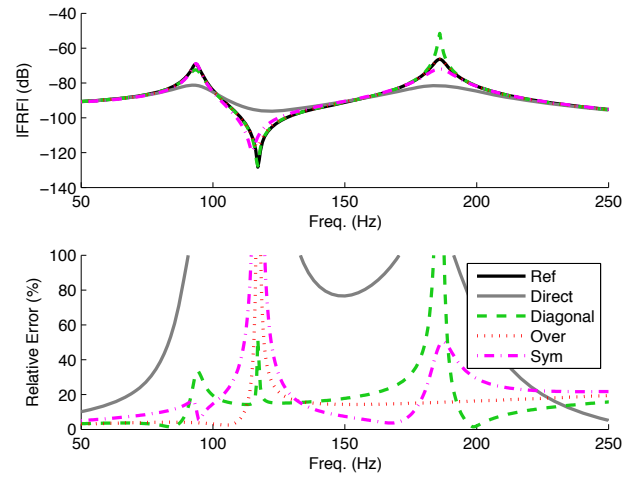


Figure 3: Methodologies for properness enforcement on guitar measurements, including the symmetric formulation

Going back to initial M , K and B matrices is not trivial, since the $(2,1)$ term in the mass matrix (of the (x, ϑ) formulation) would be associate to the third temporal derivative of x (in the (x, p) formulation), which is not included in the initial formulation. The opposite case correspond to the $(2,1)$ term in the stiffness matrix of the (x, p) formulation, that would theoretically be associated to a primitive of the displacement in the (x, ϑ) formulation. A simplified approach is then to chose null values in matrices according to the topology of the initial problem:

$$\begin{aligned} M &= \begin{bmatrix} 4.71 \times 10^{-2} & 0 \\ 6.00 \times 10^{-2} & 2.71 \times 10^{-7} \end{bmatrix}, \\ K &= \begin{bmatrix} 4.28 \times 10^4 & -6.00 \times 10^{-2} \\ -1.78 \times 10^3 & 1.41 \times 10^{-1} \end{bmatrix}, \\ B &= \begin{bmatrix} 3.14 & 0 \\ 0 & 2.77 \times 10^{-6} \end{bmatrix}. \end{aligned} \quad (31)$$

Like observed in figures, this system, even if it has physical values and a coherent topology, is not as efficient as the one obtained in the previous section for the equivalent representation of the vibro-acoustic behavior of the system. The reason why has started to appear in the above lines: the properness condition (21) does not take into account the topology of the vibro-acoustic system (1).

5 Full properness

The equation (21) is not sufficient to insure that the topology of the reconstructed matrices is the same as for the initial vibro-acoustic system (1). The reason is that applying the methodology on the n -dimensional system only permits to have the correct null terms in the state-space representation including $n \times n$ null matrices. If one want to obtain the null matrices of the vibro-acoustic initial formulation, the structural and acoustic blocs must be expressed using the partition of the eigenvectors. One can resume the set of equations using a distinction between the expression of reconstructed matrices:

$$\begin{cases} M_s = (X\Lambda X^T)^{-1} \\ M_a = -(P\Lambda^{-1}P^T)^{-1} \\ L = -M_s X\Lambda P^T M_a \\ K_s = -(X\Lambda^{-1}X^T)^{-1} \\ K_a = (P\Lambda^{-3}P^T)^{-1} \\ C_s = -M_s X\Lambda^2 X^T M_s \\ C_a = M_a P P^T M_a, \end{cases} \quad (32)$$

and the relationships that the eigenvectors and eigenvalues must verify:

$$\begin{cases} XX^T = 0 \\ XP^T = 0 \\ X\Lambda^{-1}P^T = 0 \\ X\Lambda^{-2}P^T = 0 \\ P\Lambda^{-2}P^T = 0 \\ (X\Lambda X^T)^{-1}X\Lambda P^T(P\Lambda^{-1}P^T)^{-1} \\ + (X\Lambda^{-1}X^T)^{-1}X\Lambda^{-3}P^T(P\Lambda^{-3}P^T)^{-1} = 0 \\ P\Lambda X^T(X\Lambda X^T)^{-1}X\Lambda^2 X^T \\ + PP^T(P\Lambda^{-1}P^T)^{-1}P\Lambda X^T - P\Lambda^2 X^T = 0. \end{cases} \quad (33)$$

This full set of equations explains the reason why the symmetric formulation does not improve the results compared with the approximate properness enforcement methodologies proposed with the non-symmetric formulation: in the symmetric case, only three of the above conditions are verified, while four of them are enforced in the non-symmetric case. It is clear that solving this set of equations using the same methodology as the one used in reference [1] will not lead to an explicit solution. Even the search of an approximate solution using all equations is not easy, because of the topology of equations, in which the unknowns are rectangular complex matrices. At this step, the authors propose to use the approximations proposed in the above parts and to perform a choice between the solutions using some a posteriori criterion, like errors on reconstructed FRFs.

6 Conclusion

In this paper, the notion of properness has been extended to the case of vibro-acoustics. Some methodologies for properness enforcement using complex identified vectors have been proposed, based first on a non-symmetric formulation, and on a symmetric one. Both approaches do not lead to the same results because of the particular topology of matrices describing the vibro-acoustic problem. Nevertheless, they allow one to obtain some experimentally identified reduced models and corresponding matrices associated to a set of degrees of freedom. Finally, it is shown that the proposed methodologies only permit to verify a part of the equations among the whole set, which can be enough in some cases.

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