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Excess ultrasonic attenuation due to inhomogeneities in porous media

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Abstract

While Biot's theory seems well adapted to model the acoustical waves propagation in cancellous bone, some of its predictions do not agree with the experimental results. The excess of attenuation of the fast wave is one of these discrepancies. In this paper we propose a modified Biots model which takes into account the fluctuations of the physical parameters and their correlations. As a result of this model, we show that this excess of attenuation is due to several processes: i) classical Biot's attenuation, ii) scattering leading to the extension of the wave path, iii) mode conversion. Some comparison between experimental results and numerical simulations are proposed.

1 Introduction

When an acoustic wave propagates in porous media a part of the energy of the wave is reflected and the other is transmitted or dissipated. To improve the understanding of attenuation processes in porous materials as spongy bone, several theoretical concepts for ultrasound propagation was been adapted or developed,including the Biot theory or the model of the equivalent fluid. The Biot theory of mechanical wave propagation in porous media is now a well accepted model. This model describes the displacements of fluid and structure at the passage of a mechanical disturbance. It takes into account the effects of three couplings between fluid and structure, and it can be shown that the Biot theory is the most general model to describe the propagation linear acoustic waves in porous media saturated.

During the last decade, the Biot model has been proposed to describe theultrasound propagation in the trabecular bones. However, its predictions does not agrees with experimental results. The most important descrepancy is the attenuation coefficient of the fast wave. For that, some authors propose to forsake this model.

In this paper we propose an modified Biot model based on the Müller Gurevich model [1] which explains the deviations between experimental results and theoretical predictions. It is based on the modifications of the propagation due to the inhomogeneities of the porous medium. At the interfaces between inhomogemenous parts, the conversion modes process reallocates energy of the fast and slow waves and because of their quite different damping, it entails an overdamping of the fast wave.

2 The Biot theory

During the last decade, to improve the ultrasound techniques for the diagnosis of osteoporosis, several models where developed with the aim to explain the experimental results about sound propagation through cancellous bone. More specifically, the motivations of these investigations where the dependence of ultrasounds velocity and attenuation on physical parameters of the structure of bones such as density, porosity, bulk moduli.

When the structure of a porous material is not rigid, the wave propagates in solid structure and in fluid filling the pores of the porous medium. The Biot's model is now the most attractive tool to describe these phenomena [2]. One of its main successes is the prediction of three modes of propagation: two longitudinal modes (slow and fast waves) and a transversal mode. Later, Plona confimed the prediction of the slow wave [3]. In addition, it takes into account different couplings between fluid and solid structure: i) inertial coupling, modelled by the dynamical tortuosity of the medium, which is responsible of an additional term of density ii) viscous coupling (due to the viscosity of the fluid) and iii) potential or elastic coupling (due to the reciprocity principle of fluid-structure interactions). So, the scattered acoustical waves (reflected or/and transmitted) by a porous medium contain several informations about the interactions between fluid and solid which are correctly described by the Biot's théory. These reasons argue in favour of use of ultrasound to characterize the state of trabecular bones and to observe the variations of consequences of osteoporosis [4].

2.1 Equation of motion

There are several methods to setup the equations of the motion of the solid and of the fluid in the framework of the Biot's theory. The method of homogenization appears as the most rigorous one and is often quoted as a justification *a posteriori* of the other methods. The Lagrangian formulation given by Johnson in [5] shows that the Biot's model of propagation is the most general theory for a linear description of the interactions fluid structure in diphasic porous media. The Biot's equations of motion of fluid and solid are

$$\begin{aligned} \nabla \cdot \bar{\sigma} &= \rho \ddot{\mathbf{u}} + \rho_f \ddot{\mathbf{w}}, & (1)\\ -\nabla p_f &= \rho_f \ddot{\mathbf{u}} + \tilde{Y}(t) * \dot{\mathbf{w}}, & (2)\\ \tilde{Y}(t) &= m \frac{\partial}{\partial(t)} \delta(t) + \frac{\eta}{k_0} \tilde{F(t)}. & (3) \end{aligned}$$

where σ and p_f is given by :

$$\sigma_{ij} = 2\mu\epsilon_{ij} + (\lambda_c \theta_s - \alpha M \varsigma)\delta_{ij} \quad (4)$$

$$p_f = M(-\alpha_s \theta + \varsigma) \quad (5)$$

with ρ the total density of the porous material, ρ_f is fluid density, u is displacements of the solide, w is the relative fluid-solid displacements $w = \phi(U-u)$, ϕ is the porosity, p_f is the fluid pressure, ς is the variation of the fluid content given by : $\varsigma = -\nabla.w$, $\tilde{Y}(t)$ is the viscodynamic operator, m a mass correction coefficient derived from the microvelocity field reduces to : $m = \frac{\tau_\infty}{\phi}\rho_f$, τ_∞ is the so-called tortuosity $\tau_\infty = \langle v_2 \rangle / \dot{\mathbf{w}}$ and α is the coefficient of Biot-Willis given by : $\alpha = 1 - K_d/K_s$ whereK_s, K_d and K_f are respectively the bulk moduli of the solid, of dry porous solid and fluid. When we substitue the expressions of σ_{ij} and p_f in the Biot equations we obtain the matrix equation:

$$\begin{pmatrix} L_{ij}^{(1)} & L_{ij}^{(2)} \\ L_{ij}^{(3)} & L_{ij}^{(4)} \end{pmatrix} \begin{pmatrix} u_j \\ w_j \end{pmatrix} = 0, \quad (6)$$

where the operators $L^{(n)}$ are given by:

$$\begin{aligned} L_{ik}^{(1)} &= \rho\omega^2\delta_{ik} + \partial_j G\Big(\delta_{jk}\partial_i + \delta_{ik}\partial_j - 2\delta_{ij}\partial_k\Big), && (7) \\ L_{ik}^{(2)} &= \rho_f\omega^2\delta_{ik} + \partial_i C\partial_k, && (8) \\ L_{ik}^{(3)} &= L_{ik}^{(2)}, && (9) \\ L_{ik}^{(4)} &= q\omega^2\delta_{ik} + \partial_i M\partial_k. && (10) \end{aligned}$$

The response of the porous medium to the perturbation $-(F_i^0\delta(r_i-r_i')f_i^0\delta(r_i-r_i'))^t$ constitutes the solution $(u_iw_i)^t$ of this system of equations when the right hand side of the equation (6) is $-(F_i^0\delta(r_i-r_i')f_i^0\delta(r_i-r_i'))^t$. It is obtained by using the Green's tensor ${}^0G$ of the system in the form:

$$\begin{pmatrix} u_i \\ w_i \end{pmatrix} = \begin{pmatrix} {}^0G_{ik}^F & {}^0G_{ik}^f \\ {}^0G_{ik}^f & {}^0G_{ik}^w \end{pmatrix} \begin{pmatrix} F_k \\ f_k \end{pmatrix}. \quad (11)$$

more generally we have :

$$\begin{pmatrix} u_i(\mathbf{r}) \\ w_i(\mathbf{r}) \end{pmatrix} = \quad (12)$$

$$\int d\mathbf{r}' \begin{pmatrix} {}^0G_{ik}^F(\mathbf{r}-\mathbf{r}') & {}^0G_{ik}^f(\mathbf{r}-\mathbf{r}') \\ {}^0G_{ik}^f(\mathbf{r}-\mathbf{r}') & {}^0G_{ik}^w(\mathbf{r}-\mathbf{r}') \end{pmatrix} \begin{pmatrix} F_k(\mathbf{r}') \\ f_k(\mathbf{r}') \end{pmatrix}.$$

2.2 Equations of Biot in random medium

When a wave propagates in a random medium, the coefficients of the wave equation are random variables. So, in such media the wave propagation is described by a stochastic equation. Thus, in (6) we can write the operators $L^{(n)}$ in the following form :

$$L^{(n)} = \bar{L}^{(n)} + \tilde{L}^{(n)} \quad (13)$$

where $\bar{L}^{(n)}$ is the mean value of $L^{(n)}$ and $\tilde{L}^{(n)}$ is the fluctuation of this value acting as a perturbation. From this definition $< \tilde{L}^{(n)} >= 0$. Then, from (6) one has :

$$\begin{pmatrix} \bar{L}_{ij}^{(1)} & \bar{L}_{ij}^{(2)} \\ \bar{L}_{ij}^{(3)} & \bar{L}_{ij}^{(4)} \end{pmatrix} \begin{pmatrix} u_j \\ w_j \end{pmatrix} = -\begin{pmatrix} \tilde{L}_{ij}^{(1)} & \tilde{L}_{ij}^{(2)} \\ \tilde{L}_{ij}^{(3)} & \tilde{L}_{ij}^{(4)} \end{pmatrix} \begin{pmatrix} u_j \\ w_j \end{pmatrix}, \quad (14)$$

the solution of which is given by :

$$\begin{pmatrix} u_i \\ w_i \end{pmatrix} = \begin{pmatrix} u_i^0 \\ w_i^0 \end{pmatrix} \quad (15)$$

$$+\int dV \begin{pmatrix} G_{ik}^F & G_{ik}^f \\ G_{ik}^f & G_{ik}^w \end{pmatrix} \begin{pmatrix} \tilde{L}_{kj}^{(1)} & \tilde{L}_{kj}^{(2)} \\ \tilde{L}_{kj}^{(3)} & \tilde{L}_{kj}^{(4)} \end{pmatrix} \begin{pmatrix} u_j \\ w_j \end{pmatrix}.$$

The Keller method [6] leads to write the Green function G of the random medium in the form:

$$\begin{pmatrix} G_{im}^F & G_{im}^f \\ G_{im}^f & G_{im}^w \end{pmatrix} = \begin{pmatrix} {}^0G_{im}^F & {}^0G_{im}^f \\ {}^0G_{im}^f & {}^0G_{im}^w \end{pmatrix} + \int dV \quad (16)$$

$$\begin{pmatrix} {}^0G_{ij}^F & {}^0G_{ij}^f \\ {}^0G_{ij}^f & {}^0G_{ij}^w \end{pmatrix} \begin{pmatrix} \tilde{L}_{jk}^{(1)} & \tilde{L}_{jk}^{(2)} \\ \tilde{L}_{jk}^{(3)} & \tilde{L}_{jk}^{(4)} \end{pmatrix} \begin{pmatrix} G_{km}^F & G_{km}^f \\ G_{km}^f & G_{km}^w \end{pmatrix}.$$

This relation is more briefly noted:

$$G = G^0 + \int G^0\tilde{L}G. \quad (17)$$

By iteration, this relation becomes :

$$G = G^0 + \int G^0\tilde{L}G^0 + \int\int G^0\tilde{L}G^0\tilde{L}G^0 + \cdots. \quad (18)$$

The averaged Green tensor is given by :

$$\bar{G} = G^0 + \int\int G^0 Q\bar{G}, \quad (19)$$

where the "mass operator" Q is given by the relation

$$Q = \Big\langle \tilde{L}G^0\tilde{L} + \int \tilde{L}G^0\tilde{L}G^0\tilde{L} + \int \cdots \Big\rangle. \quad (20)$$

The equation (19) is the analogue of the equation of Dyson. An approached solution of this equation is obtained by truncating the expression of operator$Q$. For example with the first order one obtains:

$$\bar{G} = G^0 + \int\int G^0\langle\tilde{L}G^0\tilde{L}\rangle\bar{G} = G^0 + \int\int G^0 Q^{(app)}\bar{G}. \quad (21)$$

The matrix $\bar{G}$ is the averaged Green tensor. The matrix $Q^{(app)}$ is the matrix of the operators defined by:

$$Q^{(app)} = \begin{pmatrix} Q_{ij}^{(1)} & Q_{ij}^{(2)} \\ Q_{ij}^{(3)} & Q_{ij}^{(4)} \end{pmatrix} = \Big\langle \tilde{L}G^0\tilde{L} \Big\rangle. \quad (22)$$

This approximation needs only the second order statistics of the random variables. It is thus valid if one assumes weak fluctuations of parameters i.e. if $|\tilde{L}^{(n)}/\bar{L}^{(n)}| \ll 1$. From this result it follows that the effective wave number of the fast wave is given by

$$\bar{k}_p = k_p\Big(1 + \Sigma_2 + \Sigma_1 k_{ps}^2 \int_0^\infty rB(r)e^{ik_{ps}r}dr\Big) \quad (23)$$

were Σ_1 and Σ_2 depend on cross-correlation functions of the parameters of the medium, $B(r)$ is the correlation function of the random variables which is assumed to be the same for each parameter, k_p is the wave number in the homogeneous host medium and $k_{ps} = \sqrt{\imath\omega\eta/\kappa_0 N}$. Here ω is the angular frequency, κ_0 is the permeability of the porous medium and N depend only on the bulk moduli of solid and fluid.

3 Attenuation

When a acoustical wave propagates in a inhomogenous medium the number of wave is modified compared to that of the wave in the host medium, in particular the multiple reflections of the wave on the inhomogeneities are the source of interferences which attenuate its amplitude, in addition, the path of the wave between two points of the medium is modified because of the multiple diffusions, contributing to the increasing of the attenuation and to a change of the phase velocity [3]. In a medium with localized scatterers, the modes conversion transfers the energy from fast wave to slow wave which is strongly attenuated. If we characterize the effective attenuation of the wave by the length l_{eff},we can write in first approximation:

$$\frac{1}{l_{eff}} = \frac{1}{l_{hom}} + \frac{1}{l_{inter}} + \frac{1}{l_{path}} + \frac{1}{l_{conv}} \qquad (24)$$

where l_{hom} is the length characterizing the attenuation in the homogeneous host medium, l_{inter} characterizes the attenuation due to the interferences, l_{path} characterizes the attenuation due to the increase lengthening of the wave path in the porous medium, and l_{conv} characterizes the attenuation due to the mode conversion. It follows from the previous section that the phase velocity is now given by

$$v \approx \frac{\omega}{k_p}\Big(1 - \Sigma_2 + \Sigma_1 \Re\{k_{ps}\}^2 \int_0^\infty rB(r)\sin\left(\Re\{k_{ps}\}r\right)dr\Big).(25)$$

So, in an inhomogeneous porous medium, the changes in the phase velocity of the fast wave are due to i) the medium dispersion proprtional to Σ_1, ii) a shift (Σ_2) which reduces v. In the same way we can express the wave attenuation as the inverse of the quality factor Q.

$$Q^{-1} = 4\Sigma_1 \Re\{k_{ps}\}^2 \int_0^\infty rB(r)\cos\left(\Re\{k_{ps}\}r\right)dr\Big). \quad (26)$$

Numerical simulations were performed from (25) and (26). In a first time we fixe the cross-correlation functions of the medium parametres. The correlation function $B(r)$ is the exponential function $B(r) = \exp\left(|r|/a\right)$, were a the correlation length which is related to the mean distance between inhomogeneities. In Fig.1 and Fig.2 one can see the effects of changes of correlation length on the phase velocity and attenuation. In particular one sees that attenuation is a quite sensible to the correlation, *i.e.* to the inhomogeneities concentration.

Fig.3 and Fig.4 show the variations of phase velocity and attenuation corresponding to a porous medium in which the porosity is a random variable. In trabecular bone, the porosity may be a good indicator of the stage of a disease as osteoposity. As the porosity intervenes in the definition of several bulk moduli through the Gassmann equation, the fluctuations of its value lead to large variations of the phase velocity and attenuation of waves.

4 Conclusion

In this paper we have proposed a modified Biot model to improve the fitting of experimental results and Biot's model theoretical predictions. When correlated inhomogeneities are taken into account, we show that the phase velocity is reduced. This is due to the lengthening of path wave induced by the multiple scattering of the wave. At the same time, the attenuation of waves is increased by sevral processes, the most important being the energy reallocation in the fast and the slow waves by modes conversion.

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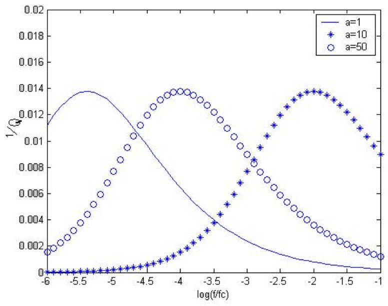

Figure 1: Attenuation as a function of frequency for $a = 1$, $a = 10$ et $a = 50$

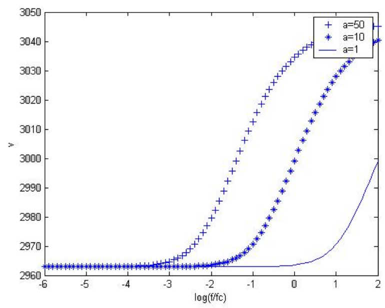

Figure 2: Phase velocity as a function of frequency for $a = 1$, $a = 10$ et $a = 50$.

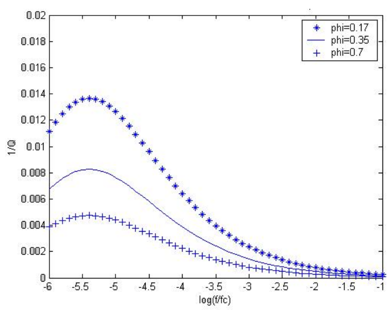

Figure 3: Attenuation vs frequency for $\phi = 0.17$, $\phi = O.35$, $\phi = 0.7$

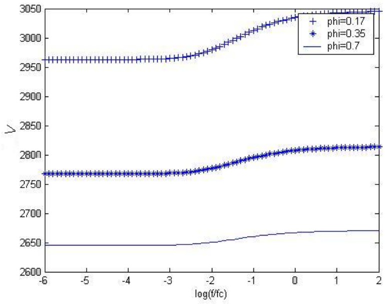

Figure 4: Phase velocity vs frequency for $\phi = 0.17$, $\phi = O.35$, $\phi = 0.7$

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