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Can also diffracted sound be handled as flow of particles? - Some new results of a beam tracing approach based on the uncertainty principle

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In room and city acoustics respectively noise immission prognosis, ray or beam tracing methods are well approved – but the problem of the neglected diffraction is still unsolved in general. The author´s successful approach of 1986 based on Heisenberg's uncertainty principle has now been generalized, embedded in a full ray tracing program and combined with the more efficient beam tracing technique. The results have been compared with Svensson´s exact wave-theoretical secondary edge source model. Reference cases were the semi-infinite screen as well as two parallel wedges forming a slit. For most cases, now also for finite distances, the agreements are again very good (less than 1dB). So, with some restrictions, it seems that indeed diffraction of sound – like light - may be handled as flow of particles, even for higher order diffraction. To avoid the feared explosion of computation time with that, a beam re-unification may now be achieved by Quantized Pyramidal Beam Tracing (QPBT). Higher order diffraction, however, opposite as announced in the abstract to this congress, has not yet been evaluated up to now. This and other results are not presented in this paper but will probably have been presented by the oral presentation on this congress.

1. Introduction

In computational room acoustics as well as in noise immission prognosis ('city acoustics') the mirror image source method (MISM) [1], more efficiently, ray tracing (RT) or beam tracing (BT) are used. A version of RT is the sound particle method [2] which, rather than the $1/r^2$-law, uses the more efficient statistical evaluation of the immitted intensities in detectors crossed by the particles. These are methods for the optical limiting case of short wavelengths. So, their main deficiency is the lack of diffraction simulation. Therefore, it is aimed at to introduce diffraction as a module into ray tracing. The requirements are:

- the 'detour law' [3] should be fulfilled;
- efficient handling of arbitrary diffraction orders and combinations with reflections should be possible (fig.1);
- a pure diffraction module without accounting for flanking walls (reflections are to be handled by another module);
- at least an approximation for short, but not very short wavelengths.

The crucial algorithmic problem is: With any recursive MISM or split-up of rays with diffraction, the number of rays, and hence the computation time, explodes. Much more efficient than the MISM is a straight forward method as RT. But with RT, the computation time still explodes with diffraction. The basic idea for solving this explosion problem is a re-unification of ('similarly running') rays. This is only possible if rays are traced in a quasi-parallel and iterative re-distribution process. Also, rays have to be spatially extended, i.e. rather beams, in order to exploit their overlap, to interpolate and to re-unify them. So, even more convenient is a hybrid method as BT. 'Beams are mirror image sources with built-in visibility limits', so, BT is an efficient version of the MISM.A solution to all these problems is Quantized Pyramidal Beam Tracing (QPBT) by Stephenson [4].

A pre-condition for any effective pyramidal beam tracing is a subdivision of the room into convex sub-rooms. On the transparent dividing 'walls' diffraction events at 'inner edges' may be effectively detected (fig.1).

Due to the use of ray tracing as the framework, basic hypotheses are:

- diffraction happens only near edges (mainly edges that protrude into a room),
- incoherent (energetic) superposition can be used.

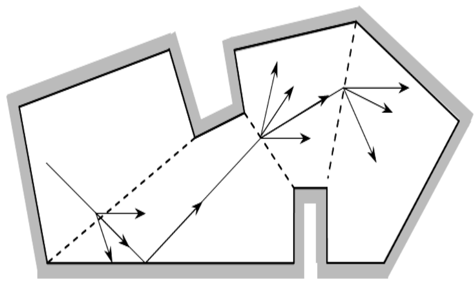

Fig. 1: General model: Multiple diffractions in a (2D) room which is subdivided into convex sub-rooms: 'transparent' dividing walls are dashed; a ray is scattered/diffracted several times on these 'walls' near edges (only one path is drawn)

Known numerical methods appropriate for higher order diffraction simulation are the Geometrical Theory of Diffraction (GTD) [5], or its improvement, the Uniform Theory of Diffraction (UTD) [6]. Both are high frequency approximations that may in principle be combined with the MISM. Funkhouser utilized a very fast version of BT for auralization in room acoustics [7], even including diffraction in form of the UTD [8]. But still, with higher order reflections and diffractions the computation time explodes.
One of the basic ideas for solving the problem of computation time explosion with diffraction is:
- not all combinations and paths of diffracted/ reflected rays or particles are important, only those where particles pass close to edges,
- the bending effect on a sound particle – the diffraction probability- should be the stronger the closer the by-pass-distance.

This idea is inspired by Heisenbergs Uncertainty-Relation (UR), known from quantum mechanics: $\Delta y \cdot \Delta p_y \approx h$ where Δy is the by-pass distance to the edge, interpreted as the 'uncertainty' in *y*, Δp_y is the impulse uncertainty at the point *y* in space and *h* is Planck`s constant/2π. Dividing the UR by *h* (using de Broglie's equation $\Delta p_y = h \cdot \Delta k_y$) yields $\Delta y \cdot \Delta k_y \approx 1$. $\Delta k_y / k$ is then the uncertainty of the direction of the wave vector in the *y*-direction. Analogous equations are valid for the other coordinates. This is valid without any atomic constant. So, the UR should be valid for light and also for sound ray propagation algorithms. This idea has been successfully utilized in numerical methods for light diffraction to optimize optical systems [9, based on 10]. The UR based sound particle diffraction model, however, was found independently by the author [11, now described in much more detail in 12], however in 1986 only for receivers at infinite distance.

This paper describes the results of
- the generalization of the UR based particle diffraction model to more general cases,
- the embedding in a full ray tracing program,
- the more efficient beam tracing

compared with empirical [3] or analytical [13] reference models.

2. The Sound Particle Diffraction Model

There are two basic concepts in the implementation of this method:
-the 'Diffraction angle probability density function' (DAPDF) and
- the 'Edge Diffraction strength' (EDS).

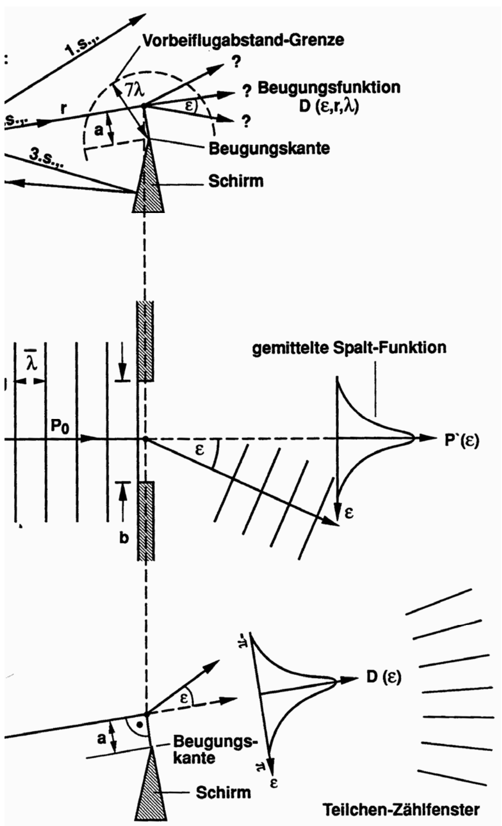

Fig.2: The 'detour into wave theory':

Each moment a particle passes an edge ('Beugungskante') of a screen ('Schirm') at a distance *a* (above), it 'sees' a slit (figure in the middle with the DAPDF on the right hand side). According to the uncertainty relation a certain 'Edge Diffraction Strength' (EDS) causes the particle to be diffracted according to the 'Diffraction Angle Probability Density Function' (DAPDF= $D(\varepsilon)$) The lower figure shows some angle window ('Zählfenster') used to count the diffracted particles and to add up their energies to the transmission degrees.

The idea of that DAPDF (with non-split-up particles) emerges from the UR. But it is more efficient (and physically equivalent) to split up the rays into new ones with partial energies according to the DAPD (because the effort to trace many rays from the source to the diffracting edges vanishes.)

After a 'detour into wave theory', i.e. the split-up according the DAPDF, again rays are traced and superposed energetically (fig 2).

2.1. The DAPDF

The DAPDF (see fig.3.) is derived from the Fraunhofer diffraction at a slit $\propto \sin^2 v / v^2$, where $v = \pi \cdot b \cdot \varepsilon$, valid for parallel incident and diffracted rays. The DAPDF,(averaged over a wide frequency band, similar as over an octave-band averaged as for 'white light'), is roughly approximated

$$D(v) = D_0 / \left(1 + 2v^2\right) \text{ with } v = 2 \cdot b \cdot \varepsilon, \qquad (1)$$

where *b* is the apparent slit width in wavelengths, ε is the deflection angle and D_0 *is a* normalization factor such that the integral over all deflection angles is 1. The D_0-factor must be computed for each edge by-pass since its value depends on b and the angle limits of the wedge. In the following all distances are expressed in wavelengths.

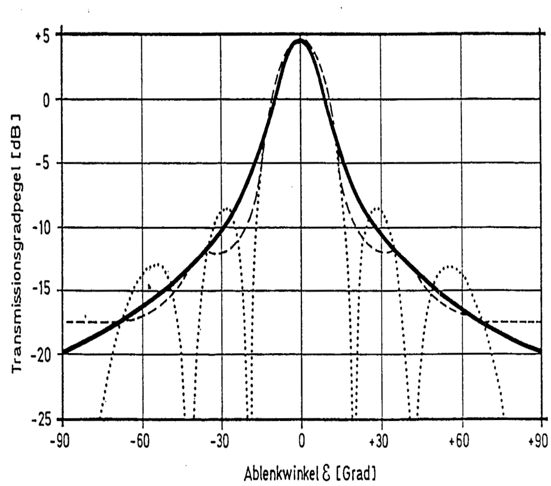

Fig. 3: Left: the derivation of the DAPDF (axes are the deflection angle ('Ablenkwinkel') and the transmission degree in dB) showing the function $\propto \sin^2 v / v^2$ (dashed curve) and the function $D(v)$ as fat curve

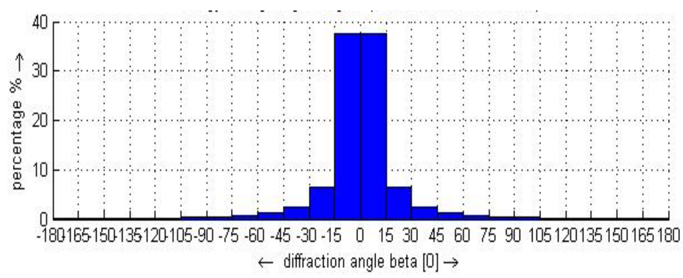

Fig.4: An energy histogram for a bypass distance of 1/2 and a slit width of 3. 75% of the incident energy is deflected into the angle range of -15…15°, only 2% into backward directions (<-90°, >+90°).

2.2. The EDS

To develop a modular model which is applicable also to several edges that are passed near-by simultaneously, the 'Edge Diffraction Strength' (EDS(a)) is introduced such that the EDS of several edges may be added up to a total TEDS ,

$$TEDS = \sum EDS_i \qquad (2)$$

To be used as input for the DAPDF, an 'effective slit width' is

$$b_{eff} = 1 / TEDS \ . \qquad (3)$$

By self-consistency-considerations (a slit should re-produce the energy distribution of itself) it turns out that

$$EDS(a) = 1/(6 \cdot a) \qquad (4)$$

So, with only one edge, a by-passing particle would 'see' a relative slit-width of b_{eff}=6a.
(A somewhat improved approach for the DAPDF was used in [11], now described in [12]. In [10], in stead of equ. 1 it is proposed a gaussian distribution and an evaluation only of the distance to the nearest edge.)

2.3. Method of evaluation

Many – typically 10…100 – particles are shot over the edge. In the first approach, their energies are counted in 'angle windows' on the other side to compute from that an angle dependant transmission degree of the semi-infinite screen and to compare it with reference functions.

The transmission degree is defined as T = intensity with the diffraction of an obstacle relative to the intensity in free field where 'intensity' in 2D is 'sound power/width' instead of 'power/surface' but the proportion of T is the same in 3D. In order to simulate also finite receiver distances, the particle diffraction model has been combined with a full 2D sound particle tracing algorithm utilizing a grid of quadratic particle detectors [2].instead of the angle windows (fig.2).

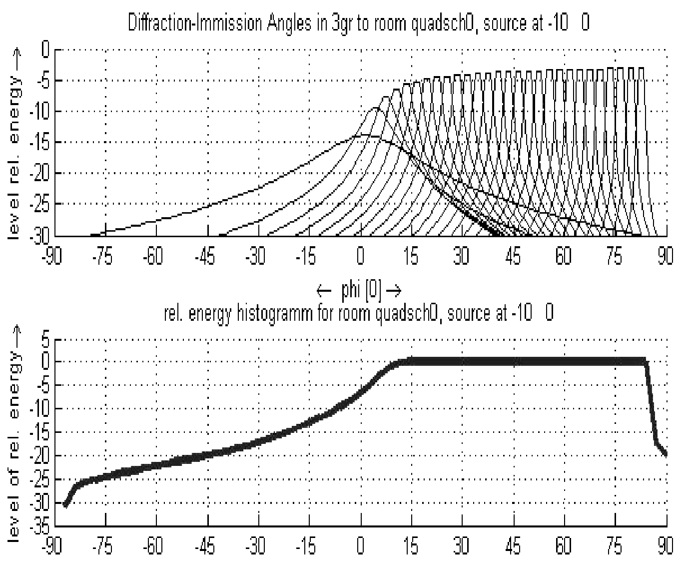

Fig. 5: Above: The superposition of angle-dependant DAPDFs in dB of single particles from a source at -10 λ passing at different distances (see fig. 2) summing up to the screen transmission function (bottom, as in fig. 7).

3. Results of ray diffraction experiments

For a systematic analysis, the 2D ray tracing was evaluated for sources and receivers (detectors of convenient sizes) at finite distances of 1,3,10,30,100 (wavelengths) and 15 angles -84…+84° (in steps of 12°, seen from the edge), applied to the semi-infinite screen, in total 375 combinations. This was first compared with the known angle function of the screen [3].
At the first go (without any parameter fitting), the agreements with the reference function (Maekawa) were very good for almost all cases, also for finite distances (standard deviation of <0.5dB, curves similar as in fig.7). It turned now out also that the reciprocity principle is fulfilled (same levels with a permutation of source and receiver). This is not self-understanding, hence, this is an important indication of the correctness of the model. It turned further out that, numerically, a decisive quantity is the number of incident particles within a close by-pass distance, a_{min} (which should be about 0.1 λ), and a maximum by-pass distance of a_{max}= 7 λ (see fig.2). Beyond that, direct transmission may be performed. The orientation of the 'diffracting surface' 'above' the screen (dashed lines in fig. 1) has only a weak influence (at +-45° less than 1dB). This is important for the practical implementation of the model in sub-divided rooms.

4. From ray to beam diffraction

The re-unification capability of QPBT requires beam tracing rather than particle tracing to be combined with diffraction. The number of secondary diffracted beams can be reduced considerably with beams. A thorough analysis showed that, in order to reach a certain numerical accuracy, particles require a much (at leat 10 times) higher number of crossings of each detector than beams do, since for mirror image sources, there is no stochastic variation and the 1/r² -distance law may be applied to compute the immitted intensities at the receiver points (in 2D a 1/r-law). Any ray tracing algorithm can be equivalently be replaced by a beam tracing algorithm which is much more effective. For one receiver, only one loop over all incident beams is necessary, each only with one diffracted beam, not a secondary loop over each time an additional number of secondary particles. (fig.6.)

A criterion for the valid by-pass distance of a beam is the middle ray's distance within the beam.

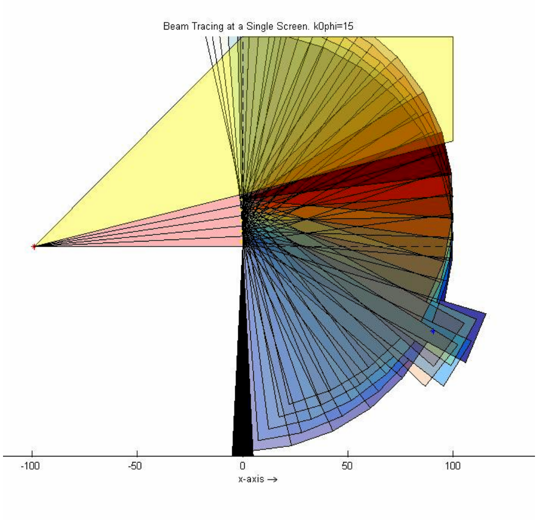

Fig. 6: 2D beam diffraction, specialized for the screen (black wedge in the middle): Typically 10…100 beams ('fans' in 2D) (left, pink) arrive within the decisive by-pass distance range of 0…7 λ (here exaggerated). The direct sound passes above (yellow). To reach <u>all</u> receivers beams are split up into each typically 10..100 secondary beams. To the right the diffracted beams: the darker the colour the higher the intensity –and this mainly in straight forward directions (see fig. 2, resp. the DAPDFs in fig. 3); bottom right the beams relevant for one specific receiver are drawn elongated.

5. Results

For the 5*5*15 source-receiver position combinations, comparisons were carried out between the beam formulation and the former ray diffraction.
- The agreements were very good (standard deviation of only 0.67dB), but beam tracing is in the order of more than 10 times faster than ray tracing with same accuracy.

- To exclude any numerical error due to the finite number of beams, a comparison with an 'infinite number' of beams i.e. a (numerical) beam integration was also carried out. The difference between those results was on average only 0.38dB standard deviation (figures look similar as fig.7)

- The direct comparison between beam tracing and the Maekawa screen transmission functions yielded a standard deviation of 0.74dB,
- the comparison with Svenssons's exact coherent secondary edge source model as analytical reference model [13] yielded only 0.39dB! (see fig.7).

(Svensson succeeded in deriving analytically directivity functions based on an exact time-domain solution for an infinite and finite rigid wedge. In contrast to the UTD, the model is valid also for lower frequencies, but only for hard wedges. Letting edge-sources re-radiate following edges, the method can be recursively applied for higher orders. The impulse responses of this reference model were Fourier transformed and the transfer functions octave band averaged.)

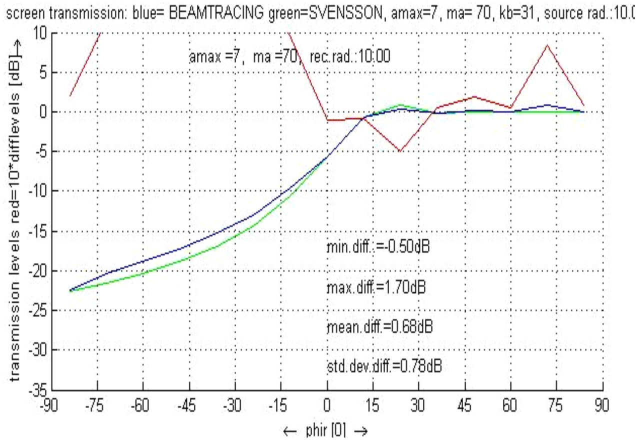

Fig. 7: Example of a comparison between beam tracing (green) and Svensson's reference method (blue, falling to the left) The transmission degree in dB is given as function of the receiver angle, to the left the 'shadow' region:. (red curve, rising to the left: deviation* 10) (1000 incident * 1000 diffracted particles, vs. 70 incident * 31 diffracted beams within a_{max}=7 λ, source and receiver distance: 10 λ, source at y=0).

Also, the influence of the inner wedge angle was investigated: For smaller inner angles their influence is low, but for the case of 90°, compared with 0°, the differences in the transmission levels are up to 4dB (mean difference are typically 0.4dB). However, in Svensson's reference model, hard flanking walls are assumed whereas in the interaction model based on the UR only the position of the edge is relevant, not any flanking walls.

Finally, the diffraction at two edges in parallel forming a slit was investigated (as a self-consistency-test). So, now the EDS of the two edges were added (Eqs. 2-4).

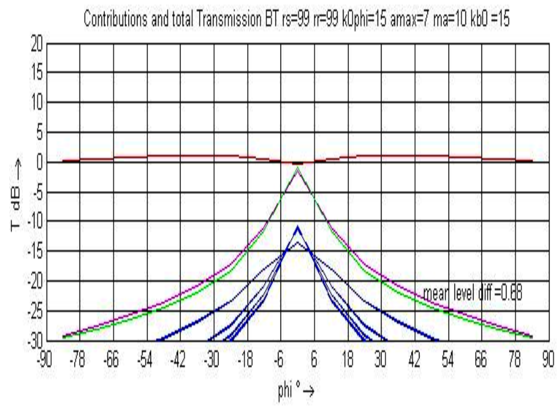

Fig. 8: Addition of the DAPDFs of beams crossing a slit of two edges (below); (Source and receiver distance = 99 λ); in the middle (green and violet) the sum i.e. the total transmission as the function of the receiver angle (compared with the free field transmission for the same sound power as incident on the slit); above the deviation curve).

The reference function was the DAPDF of the respective slit width itself. The result was again a very good agreement – at least for far sources and receivers (fig.8.). (For nearer distances, i.e. non-parallel incidence, the agreement can not be good, as the classical slit diffraction function is not valid).

6. Conclusions and Outlook

The agreements were in all cases very good.. Consequently, it seems like Heisenberg's UR may be applied also to acoustics and sound may be handled as particles even with diffraction. In principle, it should not be a problem to extend the presented model to 3D and to multiple diffractions (if the edge = the *z*-axis is infinite, then $\Delta z \rightarrow \infty$ and there is no reason for any diffraction in the *z*-direction, we do correctly get $\Delta k_z = 0$). Edge diffraction happens only in the area perpendicular to the edge, it is basically a 2D effect.

The strong frequency dependence of diffraction (influencing the question what are 'near' edges) remains problem. This concerns also the question of the limiting distance of edges for 'independent' subsequent diffractions. Also the assumption of incoherent superposition will cause difficulties in critical cases.

Results of a generalization to higher order diffraction (for example the 'thick wedge') will probably have been presented orally at the congress in Paris.

A combination of beam diffraction procedures with QPBT seems now possible without explosion of computation time. The application to room and city acoustics comes closer.

Acknowledgment

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