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Some algebra and statistics on isolated noise events

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Acoustical indices related to isolated acoustical noise events are calculated on variable finite durations T_{ev}, as opposed to instantaneous levels or index on constant periods T, (1h, 24h, …). They are the TEL (transit exposure level) or Leq_{tev} on every t_{ev}, and there are formulas to pass from many TEL_i to Leq_T on a rather long T including n events. In Algebra one may develop some relations and conditions between total duration T, the total contribution of $t_{ev,i}$ and their number n. Then one obtains the result that "events kill events", say that too many events render not emerging any new one. This is also the case with Statistics and other methods. Coming on with probabilistic tools, the Beta distribution renders possible some characterization of the time evolution of emerging noise events.

1 Introduction

Of course the notion of "time duration" is included into the definitions of acoustic indices, and one has to make a distinction between several sorts of "time". The time devoted to isolated acoustic events is one of them, and about these indices there are some interesting algebraic and/ or statistical processings which yield specific results for isolated events.

2 The principal "times" in environmental acoustics

2.1 The different times

First there are the indices corresponding to a null duration, this is the time for instantaneous levels (or supposed instantaneous, [7]), and besides this there are the indices corresponding to a finite positive time interval Δt. In this second category we have fixed conventional and standard durations for different reasons (regulations and/or habits and conveniency such as second, minut, hour, 24 h, from 6 to 22h, …), and the case of variable durations related to isolated events over the ground noise level, [8, 9]. Last we may add very special (and scarce) cases such as the very very long standard option "*over a 40 year period*", [10].

2.2 Relations between the different times

- The simplest transfer relation is the general passage between instantaneous levels and the very classical time integrated levels, $Leq_T = 10 \log\{1/T \int_T 10^{L_t/10} dt\}$, corresponding to the definition of the equivalent level Leq ;

- reciprocally the L_{max} on any given positive duration is a way to pass from a Δt index to an instantaneous one ;

- and concerning a relation between instantaneity and the time for events, the transit exposure level index, TEL, [2], is the Leq formula's adaptation to any variable Δt.

2.3 From the time for events to a longer standard duration

Usually an event duration is rather short and the conventional ones are longer. So the last interesting relation is how to pass from the indexes related to some factual events to an integrated Leq corresponding to a Δt including the different isolated events. In a general way, as a part of the logic of time integration, one has the simple relation

$Leq_{1+2} = 10 \log\{1/(T_1+T_2) \int_{T1+2} 10^{L_t/10} dt\}$

$= 10 \log\{1/(T_1+T_2) (\int_{T1} 10^{L_t/10} dt + \int_{T2} 10^{L_t/10} dt)\}$

$= 10 \log\{1/(T_1+T_2) (T_1 10^{Leq_{T1}/10} + T_2 10^{Leq_{T2}/10})\}$

which is also $(T_1 + T_2) 10^{Leq_{1+2}/10} = T_1 10^{Leq_{T1}/10} + T_2 10^{Leq_{T2}/10}$, for respective Leq on two separate time intervals and the resulting global Leq.

The relation may be extended to several separate durations T_k. Then, whenever we have different time durations Δt_i related to respective events TEL_i and a ground level L_f on the remaining time $T_f = T - T_{ev}$ with $T_{ev} = \sum_i \Delta t_i$, (L_f for the French "bruit de fond"), we get the relation

$Leq_T = 10 \log\{1/T (T_f 10^{L_f/10} + \sum_i \Delta t_i 10^{TEL_i/10})\}$.

3 The isolated events number intervention

3.1 An associated calculation about the acoustical dominance

Let $L_{x,y} = 10 \log (x+y) = 10 \log x + 10 \log (1+y/x)$, in these conditions the part of y in the final global level is negligeable as soon as the second term 10 log (1+y/x) is for instance lower than 1 decibel ; and this occurs under the trite condition $1+y/x \leq 10^{0,1}$, that is to say $y/x \leq 0,258$ or $x/y > 3,87$, (or something like that).

3.2 Application for isolated events on a T duration

Following this the contribution of the emerging noises during a T period is practically alone in the global Leq whenever the condition $(\sum_i \Delta t_i)/T_f\ 10^{(TEL_i-L_f)/10} > 3,87$ is checked, and one has the common approximation $Leq_T = 10 \log\{1/T \sum_i \Delta t_i 10^{TEL_i/10}\}$.

When the isolated acoustical emergences are more or less equal, with common data TEL and Δt, one obtains $Leq_T = 10 \log\{T_f /T\ 10^{L_f/10} + n \Delta t/T\ 10^{TEL/10}\}$. Then the previous dominance condition yields the simpler formula $Leq_T =$

$10 \log \{n \Delta t/T\ 10^{TEL/10}\} = TEL + 10 \log n + K$, with the special term 10 log n, in which the events number n on the

T period appears.

When the emergences are slightly different, TEL_i and Δt_i, one has again $\sum_i \Delta t_i\ 10^{TEL_i/10} = (\sum_i \Delta t_i)\ 10^{TEL_{eq}/10}$ with an equivalent TEL_{eq} and a mean event duration $\Delta_{moy}t = (\sum_i \Delta t_i)/n$ and an analogous relation $Leq_T = TEL_{eq} + 10 \log n + K'$, (moy for the French "moyenne").

Then, by quasi continuity of the formula's algebraic expressions, one obtains a relation in which there is the number of repetition of emerging events included in a longer duration. This is of course the classical and simplified transfer relation between related acoustical times, for instance one has this expression in a French psophic index $R = L_{moy} + 10 \log n - 34$, and the composite noise rating, $CNR = L_{moy} + 10 \log n - 12$, [4, 6].

There is also the case of categorical events (into K classes) with their own respective data n_k, $\Delta_{moy}t_k$ and TEL_{eqk}, in which case $\sum_i \Delta t_i\ 10^{TEL_i/10}$ is $\sum_k n_k\ \Delta_{moy}t_k\ 10^{TEL_{eqk}/10}$. It happens for instance for the noise exposure forecast, $NEF = L_{moy} + 10 \log (n_d + 16{,}7\ n_n) - 88$; or again for the $CENEL = SENEL_{moy} + 10 \log(n_d + 3\ n_s + 10\ n_n) - 49{,}4$ with the single event noise exposure level, SENEL, [6], when the different classes of noise isolated events are discriminated with fixed regulations penalties in relation to the hour in the day (evening + 5 dB, night + 10 dB).

4 Some Algebra with emergences

4.1 Isolated noises dominance

We come back to the condition $T_{ev}/T_f\ 10^{(TEL_{eq}-L_f)/10} > 3{,}87$ with the level emergence $\delta = TEL_{eq} - L_f$ and the percentage of emergence's duration $q_{ev} = T_{ev}/T = 1 - T_f/T$, $\delta > 0$ and $0 \le q_{ev} \le 1$. This condition is equivalent to $\delta \ge 10 \log (3{,}87\ (1-q_{ev})/q_{ev})$; in the $\{\delta, q_{ev}\}$ plane, the checking area is at the right of the curve of equation $\delta = 10 \log (3{,}87\ (1-q_{ev})/q_{ev})$, a decreasing curve versus q_{ev} and equal to 0 for $q_{ev} = 0{,}795$, (figure 1).

4.2 A condition for loneliness

However for important q_{ev} , close to 1, and then for a large number of emerging isolated noises, the condition is fitted for a weak δ and even for negative emergences (above $q_{ev} = 0{,}8$). Then one has to introduce another emergence coefficient (all the TEL_i are supposed equal in order to be more simple), $TEL - Leq_T = 10 \log 10^{TEL/10} - 10 \log(q_{ev}\ 10^{TEL/10} + q_f\ 10^{L_f/10}) = - 10 \log(q_{ev} + q_f\ 10^{-(TEL-L_f)/10}) = - 10 \log(q_{ev} + (1-q_{ev})\ 10^{-\delta/10})$, with the argument $q_{ev} + (1-q_{ev})\ 10^{-\delta/10} = 10^{-\delta/10} + q_{ev}\ (1-10^{-\delta/10})$, decreasing versus δ and increasing versus q_{ev}.

Now we pose the new condition $TEL - Leq_T \ge s$ in order that the global equivalent levels Leq_T does not drown or mask the isolated noises. This leads to the inequality $q_{ev} + (1-q_{ev})\ 10^{-\delta/10} \le 10^{-s/10}$, and to the curves family of equations $q_{ev} + (1-q_{ev})\ 10^{-\delta/10} = 10^{-s/10}$, $0 \le q_{ev} < 10^{-s/10}$, increasing versus q_{ev}, equal to $\delta = s$ at the origin, and tending to $+ \infty$ when q_{ev} tends to $10^{-s/10}$, figure 1.

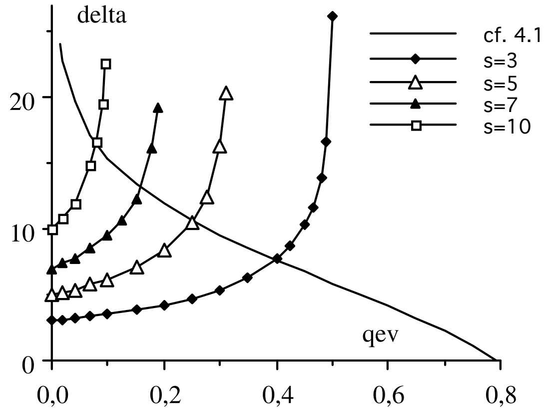

figure 1, emergence and loneliness conditions

Then the events which are at the same time isolated and emerging have to check a minima these two conditions. And coming back to the number of events $n = q_{ev}\ T/\Delta t$ this show that n cannot be too large. Following an image or an analogy one may say that "events kill events", understood as an identified isolated acoustical phenomenon.

5 Some Statistics with emergences and L_X indices

5.1 The L_X calculation

This same idea may be accounted for with the L_x statistical indices and another technical formalism.

Indeed, the noise levels are not time constant and they show a temporal evolution ; then they scan a level interval $[L_{min}\ L_{max}]$ for every time interval T, and their related statistical distribution is summarized by a cumulative distribution function, cdf, F_L. For every $0 \le x \le 1$, the L_x index corresponds to the level overpassed during x% of time and then obeys to the relation $F_L(L_x) = 1-x$, (note that it is the opposite convention for usual statistical fractiles $F_U(q_x) = x$).

We suppose that the ground level $\mathbf{L_f}$ follows a distribution with $F_f(L_f)$ on $[L_{f,min}, L_{f,max}]$, and that the emerging noises $\mathbf{L_{ev}}$ follow another distribution with $F_{ev}(L_{ev})$ on $[L_{ev,min}, L_{ev,max}]$, under condition $L_{f,max} \le L_{ev,min}$. We also pose a model into which the resulting noise level $\mathbf{L}$ follows a linear mix of the two previous distribution with $F(L) = (1 - q_{ev})\ F_f(L) + q_{ev}\ F_{ev}(L)$. Whenever there is $x \le q_{ev}$, one obtains $L_x \ge L_{ev,min}$, and then one may calculate L_x with the cdf F_{ev} alone because the general relations $(1 - q_{ev})\ F_f(L_x) + q_{ev}\ F_{ev}(L_x) = 1 - x$ becomes $(1 - q_{ev}) + q_{ev}\ F_{ev}(L_x) = 1 - x$ for $L_x > L_{ev,min}$ with the solution

$L_x = F_{ev}^{-1}[(q_{ev}-x)/q_{ev}]$.

5.2 The L_X evolution or tendency

When the part of the emerging noises tends to 1, that is to say when the emerging noises become more numerous, the more the indices $L_x = F_{ev}^{-1}[(q_{ev}-x)/q_{ev}]$ are definite, L_{80}, L_{90}, L_{95}, …, and that they respectively tend to $F_{ev}^{-1}[(1-x)/1]$ that is to say $L_{x,ev}$. This accounts for the fact that the more emerging noises there are above the ambiant noise, the more the statistical distribution of resulting noise levels **L** tends to the distribution of emerging noises, following a convergence in distribution, [11].

This very simple model is sufficient to show again that "events kill events" with this time a statistical reasoning. But unfortunately for silence and environment, the killing is not in the "good" direction because the highest noise levels do not disappear. It is better to say that "events absorb events" into the growing population of emerging high noise levels, and that what is killed is rather the identification of isolated events, and the related suitability of the 10 log n algebraic term into the formulation of Leq_T .

6 A probabilistic development with Beta laws (or distributions)

6.1 The distribution of isolated events

When one introduces a probabilistic (or statistical) distribution for noise levels as in § 5, one gives up the more or less implicit simple rectangular diagram for the emergences as we did in § 3 and 4. But this offers the new possibility to envisage the design of the event's temporal evolution, because here we have two ways to describe this isolated event. First the classical acoustical temporal signature $L(t_j)$ on the ΔT_{ev} interval varying from $L_{ev,min}$ to $L_{ev,max}$ or more simply from L_{min} to L_{max}, and second with the statistical distribution coming from the histogram of the $L(t_j)$ values and represented by the cdf $F_{ev}(L)$ and the different fractiles L_x, a distribution on $[L_{min}, L_{max}]$,

6.2 The Beta laws

One may be more detailed when using some known probabilistic laws and F_{ev}, here for instance the Beta laws Be(p,q) with the two parameters p and q, [5, 11] ; note the q notation is classical and has no relation with the previous q_{ev} percentage of time for emerging events.

The probability density of the Be(p,q) law is $f_{p,q}(u) = u^{p-1} (1-u)^{q-1}/B(p,q)$ on [0, 1], with B(p,q) and Γ(p) the very classical eulerian functions $B(p,q) = \Gamma(p)\,\Gamma(q)/\Gamma(p+q)$. Let $F_{p,q}$ be the related cdf, then the cdf for the noises distribution is $F_{ev}(L) = F_{p,q}[(L-L_{min})/(L_{max}-L_{min})]$ fot $L_{min} \leq L \leq L_{max}$, and one obtains $L_x = L_{min} + \Delta L_{ev}\, F_{p,q}^{-1}(1-x/q_{ev})$ with $\Delta L_{ev} = L_{max} - L_{min}$. notation.

The associated calculations are simple when one of the parameters is equal to 1, for instance

- Be(1,2) law : $F_{1,2}(u) = 1 - (1 - u)^2$ and $L_x = L_{min} + \Delta L_{ev} [1 - (x/q_{ev})^{1/2}]$,
- Be(p,1) law : $F_{p,1}(u) = u^p$ and $L_x = L_{min} + \Delta L_{ev} (1 - x/q_{ev})^{1/p}$,
- Be(1,q) law : $F_{1,q}(u) = 1 - (1-u)^q$ and $L_x = L_{min} + \Delta L_{ev} [1 - (x/q_{ev})^{1/q}]$.

6.3 A mix of the two technical approaches, and return to TEL

One obtains $10^{TEL/10} = \int_{\Delta Lev} 10^{L/10} f_{ev}(L)\, dL = 10^{Lmin/10} \int_0^1 10^{u\,\Delta Lev/10} f_{p,q}(u)\, du = 10^{Lmin/10} \int_0^1 e^{au} f_{p,q}(u)\, du$ with $a = \ln 10\, \Delta L_{ev}/10$ # $0{,}23\, \Delta L_{ev}$ and $TEL = L_{min} + 10 \log (\int_0^1 e^{au} f_{p,q}(u)\, du)$. Calculations are easy in some cases, for instance with Be(1,2) law one has $I_{1,2} = \int_0^1 (1-u)\, e^{au}\, du = (e^a - 1 - a)/a^2$; $10^{TEL/10} = 10^{Lmin/10}\, 2\, I_{1,2} = 10^{Lmin/10}\, 2\, (e^a - 1 - a)/a^2$, $TEL = L_{min} + 10 \log(2/a^2 (e^a - 1 - a))$, and then $TEL - L_x = 10 \log(2/a^2 (e^a - 1 - a)) - \Delta L_{ev} [1 - (x/q_{ev})^{1/2}]$, for $q_{ev} \geq x$.

This shows that the difference TEL - L_x is decreasing as the part of the emerging noise levels increases (figures 2), and also for the most simple Be(n,m) laws, [9].

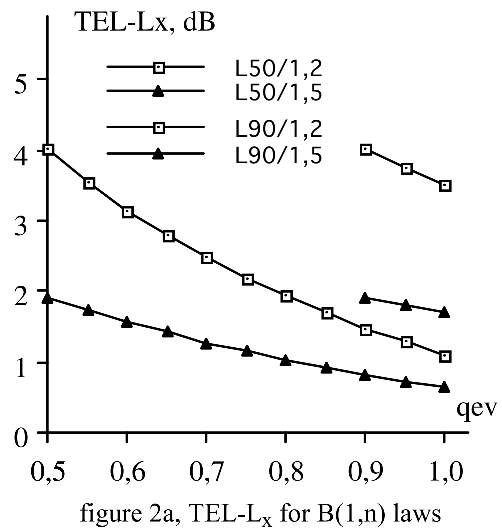

figure 2a, TEL-L_X for B(1,n) laws

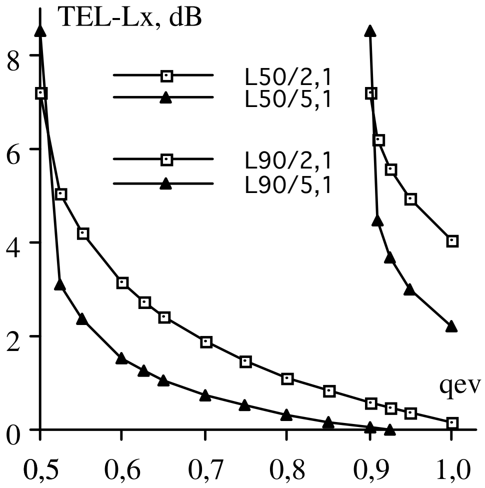

figure 2b, TEL-L_x for B(m,1) laws

In the general Be(p,q) law one remarks that the integral $\int_0^1 e^{au} f_{p,q}(u)\,du$ is the Kummer confluent hypergeometric function M(p, p+q, a), [1], and then TEL = L_{min} + 10 log {M(p, p+q, a)}. There are some numerical tables for the Kummer function and notably

- for p = 1 , TEL - L_x = 10 log {M(1, q+1, a)} - ΔL_{ev} $[1 - (x/q_{ev})^{1/q}]$,
- for q = 1 , TEL - L_x = 10 log {M(p, p+1, a)} - ΔL_{ev} $(1 - x/q_{ev})^{1/p}$.

6.4 The p and q parameters, the design of the temporal signature

Lastly Beta laws render possible a characterization for the design of temporal evolution. Whenever the probability density is high around the origin there are few levels closed to L_{max} and the design is as a peak with a progressive growth in time ; this is the case for Be(p,1) laws with p < 1 and Be(1,q) laws with q > 1. Inversely when the density is high close to 1, the noise levels are more concentrated aroud the L_{max} and then the design is flatter with an impulsive growth ; now this is the case for Be(p,1) laws with p > 1 and Be(1,q) laws with q < 1.

Then the parameters estimation of Beta law may provide some indication about the design of the L(t) signal. The fitting of moments Pearson method is easy to use [5], from the mean and variance formulae $m = \frac{p}{p+q}$ and $\sigma^2 = \frac{p\,q}{(p+q)^2\,(p+q+1)}$. One then obtains the estimates $p^* = M\,[\frac{M\,(1-M)}{\Sigma^2} - 1]$ and $q^* = (1-M)\,[\frac{M\,(1-M)}{\Sigma^2} - 1]$. Besides there are two possible acoustical options for calculation of the corresponding statistics M and Σ^2, with levels L or with power $10^{L/10}$, and we may apply them to many examples of emerging isolated noise events, [3].

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