Some algebra and statistics on isolated noise events

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Acoustical indices related to isolated acoustical noise events are calculated on variable finite durations $T_{ev}$, as opposed to instantaneous levels or index on constant periods $T$, (1h, 24h, ...). They are the TEL (transit exposure level) or $Leq_{Tev}$ on every $t_{ev}$, and there are formulas to pass from many TELi to $Leq_T$ on a rather long $T$ including n events. In Algebra one may develop some relations and conditions between total duration $T$, the total contribution of $t_{ev,i}$ and their number $n$. Then one obtains the result that “events kill events”, say that too many events render not emerging any new one. This is also the case with Statistics and other methods. Coming on with probabilistic tools, the Beta distribution renders possible some characterization of the time evolution of emerging noise events.

1 Introduction

Of course the notion of “time duration” is included into the definitions of acoustic indices, and one has to make a distinction between several sorts of “time”. The time devoted to isolated acoustic events is one of them, and about these indices there are some interesting algebraic and/or statistical processings which yield specific results for isolated events.

2 The principal “times” in environmental acoustics

2.1 The different times

First there are the indices corresponding to a null duration, this is the time for instantaneous levels (or supposed instantaneous, [7]), and besides this there are the indices corresponding to a finite positive time interval $\Delta t$. In this second category we have fixed conventional and standard durations for different reasons (regulations and/or habits and conveniency such as second, minut, hour, 24 h, from 6 to 22h, ...), and the case of variable durations related to isolated events over the ground noise level, [8, 9]. Last we may add very special (and scarce) cases such as the very long standard option “over a 40 year period”, [10].

2.2 Relations between the different times

- The simplest transfer relation is the general passage between instantaneous levels and the very classical time integrated levels, $Leq_T = 10 \log \{1/T \int_T 10^{L_t/10} dt\}$, corresponding to the definition of the equivalent level $Leq$ ;
- reciprocally the $L_{max}$ on any given positive duration is a way to pass from a $\Delta t$ index to an instantaneous one ;
- and concerning a relation between instantaneousity and the time for events, the transit exposure level index, TEL, [2], is the $Leq$ formula’s adaptation to any variable $\Delta t$.

2.3 From the time for events to a longer standard duration

Usually an event duration is rather short and the conventional ones are longer. So the last interesting relation is how to pass from the indexes related to some factual events to an integrated $Leq$ corresponding to a $\Delta t$ including the different isolated events. In a general way, as a part of the logic of time integration, one has the simple relation

$$Leq_{T_1+2} = 10 \log \{1/(T_1+T_2) \int_{T_1+2} 10^{L_{t}/10} dt\}$$

$$= 10 \log \{1/(T_1+T_2) (\int_{T_1} 10^{L_{t}/10} dt + \int_{T_2} 10^{L_{t}/10} dt)\}$$

$$= 10 \log \{1/(T_1+T_2) (T_1 10^{Leq_{T_1}/10} + T_2 10^{Leq_{T_2}/10})\}$$

which is also $(T_1 + T_2) 10^{Leq_{T_1+2}/10} = T_1 10^{Leq_{T_1}/10} + T_2 10^{Leq_{T_2}/10}$ for respective $Leq$ on two separate time intervals and the resulting global $Leq$.

The relation may be extended to several separate durations $T_k$. Then, whenever we have different time durations $\Delta t_i$ related to respective events TELi and a ground level $L_f$ on the remaining time $T_f = T - T_{ev}$ with $T_{ev} = \sum \Delta t_i$, ($L_f$ for the French “bruit de fond”), we get the relation

$$Leq_T = 10 \log \{1/T (T_f 10^{L_f/10} + \sum \Delta t_i 10^{TEL_i/10})\}.$$

3 The isolated events number intervention

3.1 An associated calculation about the acoustical dominance

Let $L_{x,y} = 10 \log (x+y) = 10 \log x + 10 \log (1+y/x)$, in these conditions the part of $y$ in the final global level is negligible as soon as the second term $10 \log (1+y/x)$ is for instance lower than 1 decibel ; and this occurs under the trite condition $1+y/x \leq 10^{1/10}$, that is to say $y/x \leq 0.258$ or $x/y > 3.87$, (or something like that).

3.2 Application for isolated events on a T duration

Following this the contribution of the emerging noises during a $T$ period is practically alone in the global $Leq$ whenever the condition $(\sum \Delta t_i)/T_f 10^{TEL_i-L^f/10} > 3.87$ is checked, and one has the common approximation

$$Leq_T = 10 \log \{1/T \sum \Delta t_i 10^{TEL_i/10}\}.$$

When the isolated acoustical emergences are more or less equal, with common data $TEL$ and $\Delta t$, one obtains

$$Leq_T = 10 \log \{T_f/T 10^{TEL/10} + n \Delta t/T 10^{TEL/10}\}.$$

Then the previous dominance condition yields the simpler formula

$$Leq_T = 10 \log \{n \Delta t/T 10^{TEL/10}\} = TEL + 10 \log n + K,$$

with the special term $10 \log n$, in which the events number $n$ on the
T period appears.

When the emergences are slightly different, TEL and ∆ti, one has again ∑i ∆ti 10TELi/10 = (∑i ... (1 - q ev) Ff(Lx) + qev Fev(Lx) = 1 - x  becomes (1 - qev) + qev Fev(Lx) = 1 - x for L x > L ev,min with the solution

Then, by quasi continuity of the formula’s algebraic expressions, one obtains a relation in which there is the number of repetition of emerging events included in a longer duration. This is of course the classical and simplified transfer relation between related acoustical times, for instance one has this expression in a French psophic index R = Lmoy + 10 log n - 34, and the composite noise rating, CNNR = Lmoy + 10 log n - 12. [4, 6].

There is also the case of categorical events (into K classes) with their own respective data n k, Δmoy k, and TEL eqk in which case ∑i ∆ti 10TELi/10 is ∑k nk Δmoy k 10TELeqk/10. It happens for instance for the noise exposure forecast, NEF = Lmoy + 10 log (n a + 16,7 n b ) - 88 ; or again for the CENEL = SENELmoy + 10 log(na + 3 nb + 10 nh) - 49,4 with the single event noise exposure level, SENEL, [6], when the different classes of noise isolated events are discriminated with fixed regulations penalties in relation to the hour in the day (evening + 5 dB, night + 10 dB).

4 Some Algebra with emergences

4.1 Isolated noises dominance

We come back to the condition T ev/T f 10TEL eq-Lp/10 > 3,87 with the level emergence δ = TEL eq - L f and the percentage of emergence’s duration qev = T ev/T f - 1 - T f/T, δ > 0 and 0 ≤ qev ≤ 1. This condition is equivalent to δ ≥ 10 log (3,87 (1-qev)/qev) ; in the {δ, qev} plane, the checking area is at the right of the curve of equation δ = 10 log (3,87 (1-qev)/qev), a decreasing curve versus q ev and equal to 0 for qev = 0,795, (figure 1).

4.2 A condition for loneliness

However for important q ev , close to 1, and then for a large number of emerging isolated noises, the condition is fitted for a weak δ and even for negative emergences (above q ev = 0,8). Then one has to introduce another emergence coefficient (all the TELi are supposed equal in order to be more simple), TEL - LeqT = 10 log (TEL eq + qf 10TEf/10 + qf 10TEf/10 - 10 log(qev + qf 10TEL-Lf/10) = - 10 log(qev + (1-qev) 10-δ/10), with the argument qev + (1-qev) 10-δ/10 = 10-δ/10 + qev (1-10-δ/10), decreasing versus δ and increasing versus q ev.

Now we pose the new condition TEL - LeqT ≥ s in order that the global equivalent levels LeqT does not drown or mask the isolated noises. This leads to the inequality qev + (1-qev) 10-δ/10 ≤ 10-s/10 , and to the curves family of equations qev + (1-qev) 10-δ/10 = 10-s/10 , 0 ≤ qev < 10-s/10 , increasing versus q ev, equal to δ = s at the origin, and tending to +∞ when qev tends to 10-s/10, figure 1.

Then the events which are at the same time isolated and emerging have to check a minima these two conditions. And coming back to the number of events n = q ev T/Δt this show that n cannot be too large. Following an image or an analogy one may say that “events kill events”, understood as an identified isolated acoustical phenomenon.

5 Some Statistics with emergences and Lx indices

5.1 The Lx calculation

This same idea may be accounted for with the Lx statistical indices and another technical formalism.

Indeed, the noise levels are not time constant and they show a temporal evolution ; then they scan a level interval [Lmin, L max] for every time interval T, and their related statistical distribution is summarized by a cumulative distribution function, cdf, F(L). For every 0 ≤ x ≤ 1, the Lx index corresponds to the level overpassed during x% of time and then obeys to the relation F(Lx) = x, (note that it is the opposite convention for usual statistical fractiles F(x(qx)) = x).

We suppose that the ground level Lx follows a distribution with F(Lx) on [Lx,min, Lx,max], and that the emerging noises L ev follow another distribution with F ev(L ev) on [L ev,min, L ev,max], under condition L f,min ≤ Lx,min, and then one may calculate L x with the cdf F ev alone because the general relations (1 - q ev) F(Lx) + q ev F ev(Lx) = 1 - x becomes (1 - q ev) + q ev F ev(Lx) = 1 - x for L x > L ev,min with the solution...
5.2 The Lx evolution or tendency

When the part of the emerging noises tends to 1, that is to say when the emerging noises become more numerous, the more the indices Lx = \( F_{ev}^{-1}\left[\frac{(q_{ev}-x)}{q_{ev}}\right] \) are definite, L80, L90, L95, ..., and that they respectively tend to \( F_{ev}^{-1}\left[\frac{1-x}{1}\right] \) that is to say Lx,ev. This accounts for the fact that the more emerging noises there are above the ambient noise, the more the statistical distribution of resulting noise levels L tends to the distribution of emerging noises, following a convergence in distribution, [11].

This very simple model is sufficient to show again that “events kill events” with this time a statistical reasoning. But unfortunately for silence and environment, the killing is not in the “good” direction because the highest noise levels do not disappear. It is better to say that “events absorb events” into the growing population of emerging high noise levels, and that what is killed is rather the identification of isolated events, and the related suitability of the 10 log n algebraic term into the formulation of \( L_{eqT} \).

6 A probabilistic development with Beta laws (or distributions)

6.1 The distribution of isolated events

When one introduces a probabilistic (or statistical) distribution for noise levels as in § 5, one gives up the more or less implicit simple rectangular diagram for the emergences as we did in § 3 and 4. But this offers the new possibility to envisage the design of the event’s temporal evolution, because here we have two ways to describe this isolated event. First the classical acoustical temporal signature L(tj) on the \( \Delta T_{ev} \) interval varying from \( L_{ev,min} \) to \( L_{ev,max} \) or more simply from \( L_{min} \) to \( L_{max} \) and second with the statistical distribution coming from the histogram of the \( L(tj) \) values and represented by the cdf \( F_{ev}(L) \) and the different fractiles \( L_x \), a distribution on \([L_{min}, L_{max}]\).

6.2 The Beta laws

One may be more detailed when using some known probabilistic laws and \( F_{ev} \), here for instance the Beta laws Be(p,q) with the two parameters p and q, [5, 11] ; note the q notation is classical and has no relation with the previous \( q_{ev} \) percentage of time for emerging events.

The probability density of the Be(p,q) law is \( f_{p,q}(u) = u^{p-1}(1-u)^{q-1}/B(p,q) \) on \([0, 1]\), with \( B(p,q) \) and \( \Gamma(p) \) the very classical eulerian functions \( B(p,q) = \Gamma(p) \Gamma(q)/\Gamma(p+q) \). Let \( F_{p,q} \) be the related cdf, then the cdf for the noises distribution is \( F_{ev}(L) = F_{p,q}(L-L_{min})/(L_{max}-L_{min}) \) fot \( L_{min} \leq L \leq L_{max} \), and one obtains \( L_x = L_{min} + \Delta L_{ev} \frac{1}{1-x/q_{ev}} \) with \( \Delta L_{ev} = L_{max} - L_{min} \).

The associated calculations are simple when one of the parameters is equal to 1, for instance

- Be(1,2) law : \( F_{1,2}(u) = 1 - (1 - u)^2 \) and \( L_x = L_{min} + \Delta L_{ev} [1 - (x/q_{ev})^{1/2}] \),
- Be(p,1) law : \( F_{p,1}(u) = u^p \) and \( L_x = L_{min} + \Delta L_{ev} [1 - (x/q_{ev})^{1/p}] \),
- Be(1,q) law : \( F_{1,q}(u) = 1 - (1-u)^q \) and \( L_x = L_{min} + \Delta L_{ev} [1 - (x/q_{ev})^{1/q}] \).

6.3 A mix of the two technical approaches, and return to TEL

One obtains \( 10^{TEL/10} = \int_{L_{min}}^{L_{max}} f_{p,q}(L) \frac{dL}{10} = 10^{L_{min/10}} \int_0^1 10^\Delta L_{ev/10} f_{p,q}(u) \frac{du}{10} \int_0^1 10^\Delta L_{ev/10} f_{p,q}(u) \frac{du}{10} \) with \( a = \ln 10 \Delta L_{ev}/10 \neq 0,23 \Delta L_{ev} \) and TEL = \( L_{min} + 10 \log \int_0^1 e^a f_{p,q}(u) \frac{du}{10} \). Calculations are easy in some cases, for instance with Be(1,2) law one has

\[ I_{1,2} = \int_0^1 (1-u)^e u^a du = \left( e^a - 1 - a\right)/a^2 ; \quad 10^{TEL/10} = 10^{L_{min/10}} 2 I_{1,2} = 10^{L_{min/10}} 2 \left( e^a - 1 - a\right)/a^2 , \quad TEL = L_{min} + 10 \log(2/a^2 (e^a - 1 - a)) , \quad \text{and TEL} - L_x = 10 \log(2/a^2 (e^a - 1 - a)) - \Delta L_{ev} \left[1 - (x/q_{ev})^{1/2}\right], \quad \text{for} \ q_{ev} \geq x. \]

This shows that the difference TEL - Lx is decreasing as the part of the emerging noise levels increases (figures 2), and also for the most simple Be(n,m) laws, [9].
In the general Be(p,q) law one remarks that the integral
\[
\int_0^1 e^{au} f_{p,q}(u) \, du
\]
is the Kummer confluent hypergeometric function \( M(p, p+q, a) \), \([1]\), and then \( \text{TEL} = L_{\text{min}} + 10 \log \{M(p, p+q, a)\} \). There are some numerical tables for the Kummer function and notably
- for \( p = 1 \), \( \text{TEL} - L_x = 10 \log \{M(1, q+1, a)\} - \Delta L_{\text{ev}} (1 - (x/qev)^{1/q}) \),
- for \( q = 1 \), \( \text{TEL} - L_x = 10 \log \{M(p, p+1, a)\} - \Delta L_{\text{ev}} (1 - x/qev)^{1/p} \).

6.4 The p and q parameters, the design of the temporal signature

Lastly Beta laws render possible a characterization for the design of temporal evolution. Whenever the probability density is high around the origin there are few levels closed to \( L_{\text{max}} \) and the design is as a peak with a progressive growth in time ; this is the case for Be(p,1) laws with \( p < 1 \) and Be(1,q) laws with \( q > 1 \). Inversely when the density is high close to 1, the noise levels are more concentrated around the \( L_{\text{max}} \) and then the design is flatter with an impulsive growth ; now this is the case for Be(p,1) laws with \( p > 1 \) and Be(1,q) laws with \( q < 1 \).

Then the parameters estimation of Beta law may provide some indication about the design of the \( L(t) \) signal. The fitting of moments Pearson method is easy to use \([5]\), from the mean and variance formulae \( m = \frac{p}{p+q} \) and \( \sigma^2 = \frac{pq}{(p+q)^2(p+q+1)} \). One then obtains the estimates \( p^* = M \frac{[M(1-M)^2 - 1]}{\sum^2} \) and \( q^* = (1-M) \frac{[M(1-M)^2 - 1]}{\sum^2} \). Besides there are two possible acoustical options for calculation of the corresponding statistics \( M \) and \( \sum^2 \), with levels L or with power \( 10^{\frac{L}{10}} \), and we may apply them to many examples of emerging isolated noise events, \([3]\).