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Determination of Condition for Fastest Negative Group Velocities of Lamb-Type Waves under each Density Ratio of Solid and Liquid Layers

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Lamb-type waves are coupling modes of leaky Lamb waves on a layer structure. The Lamb-type waves have complicated propagation characteristics more than ordinary Lamb waves on a uniform elastic plate. In the characteristics, we examine the negative group velocities. Generally, the negative group velocities of Lamb waves are slower than positive group velocities under the same condition. If the negative group velocities are applied to fabricating some new device, it is desired that the speeds of negative group velocities are comparable to those of positive group velocities. Consequently, we aim to obtain the faster negative group velocities. Lamb-type waves show more discriminative characteristics in negative group velocities than ordinary Lamb waves. In this research, we consider the Lamb-type waves in a solid/liquid/solid structure. It is described the conditions for obtaining the fastest negative group velocities of Lamb-type waves. The conditions, which are the acoustic impedance ratio, are expressed as the function of the density ratio of solid and liquid layers. These results are verified by numerical analysis.

1 Introduction

In various elastic materials, there exist various elastic waves. The elastic waves are used in a wide range of fields [1-3]. Moreover, Lamb waves propagating in an elastic plate have long been studied by many researchers and their propagations in the elastic plates are applied to many fields [4-7]. The Lamb waves have several unique properties such as the existence of velocity dispersion and negative group velocity [8]. Negative group velocity means that the group propagation has an opposite direction of the phase propagation. This situation typically occurs on Lamb waves. Here, we consider the concrete example for applied fields of Lamb waves. For example, Lamb waves are used in the nondestructive test (NDT) [9]. In this situation, negative group velocity is an annoyance in NDT. Accordingly, it is important to determine the characteristics of negative group velocity. On the other hand, it is considered that negative group velocity is applied to acoustical flat lens among others [10]. This is an example showing that negative group velocity is actively utilized. In such a case, controlling negative group velocity is important. However, the characteristics of negative group velocities of Lamb waves are only determined by material constants, which cannot be changed dynamically. Generally, the negative group velocities of Lamb waves are slower than positive group velocities under the same condition. If the negative group velocities are applied to fabricating some new device, it is desired that the magnitudes of negative group velocities are comparable to those of positive group velocities. Consequently, we aim to obtain the faster negative group velocities. The negative group velocities appear near cut-off frequencies of particular propagation modes [11]. The range of the existing negative group velocities was found to depend on Poisson's ratio. Indeed, the propagation characteristics of Lamb waves depend on the parameters of each material, therefore, the controllable range of the negative group velocities of the Lamb waves is quite limited. In contrast, we found out that the negative group velocity of Lamb-type waves in a glass/water/glass structure depends not only on physical parameters but also on the thickness of the water layer by numerical calculation [12]. This means that the phase velocity and the group velocity of Lamb-type waves can be changed without changing the frequency of Lamb-type waves. However, the materials are fixed in this research. In this paper, we consider the Lamb-type waves in a solid/liquid/solid structure. It is described the conditions for obtaining the fastest negative group velocities of Lamb-type waves under some condition, qualitatively.

2 Theory of Lamb Waves and Lamb-Type Waves

We examine a thin water layer embedded between two identical elastic plates. This structure and mode shapes are shown in Fig. 1, where d and d_W are the thickness of elastic plates and the thickness of water layer, respectively. Lamb-type waves propagate in the direction of the x-axis. z-axis is the direction of the thickness of the structure. The origin is on the central surface of the structure.

First, we consider the Lamb waves propagating in one elastic plate. We show the dispersion relation using the following equations [13,14]. Here, the plane strain model is assumed. There exist symmetrical (S) and antisymmetrical (A) modes in Lamb waves.

$$D_S(\omega,k)D_A(\omega,k)=0, \tag{1}$$

with the following:

$$D_S(\omega,k)=4k^2k_{zL}k_{zT}\cot(k_{zT}d/2)+\left(2k^2-k_T{}^2\right)^2\cot(k_{zL}d/2), \tag{2}$$

$$D_A(\omega,k)=4k^2k_{zL}k_{zT}\tan(k_{zT}d/2)+\left(2k^2-k_T{}^2\right)^2\tan(k_{zL}d/2). \tag{3}$$

Here, k is the wave number of the Lamb waves and ω is the angular frequency. Using the longitudinal wave velocity c_L, its wave number k_L, transversal wave velocity c_T, and its wave number k_T, the following equations are given,

$$k_{zL}=\sqrt{k_L^2-k^2}, \tag{4}$$

$$k_{zT}=\sqrt{k_T^2-k^2}. \tag{5}$$

Next, we consider the Lamb-type waves propagating in a solid/liquid/solid structure. In Fig. 1(a), when the boundaries $z=\pm d_W/2$ contact the water layer, Lamb waves partly leak longitudinal waves into the water layer with their propagations. The longitudinal waves are partly converted back into Lamb waves on the boundaries. By this means, the

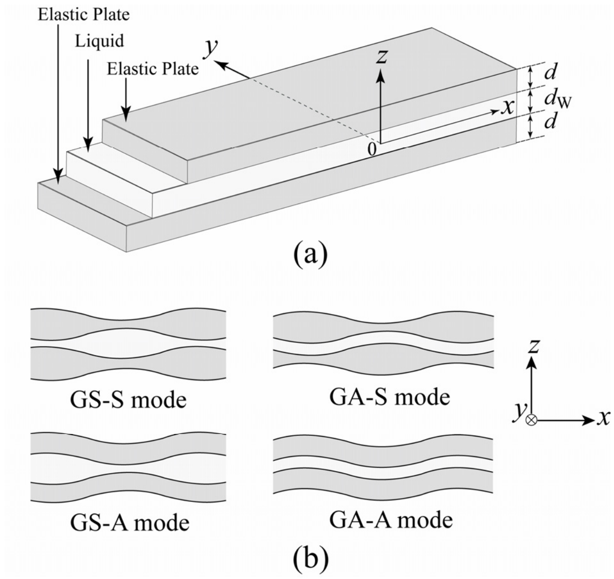

Fig. 1 Geometry and mode shapes in a solid/liquid/solid structure. d: thickness of elastic plates, and d_W: thickness of liquid layer.

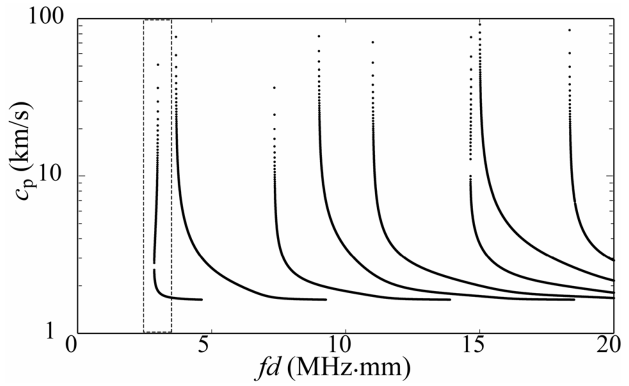

Fig. 2 Dispersion curves of Lamb wave of phase velocity with respect to product fd.

Lamb-type waves propagate as a coupling mode of the leaky Lamb waves in the solid/liquid/solid structure. The dispersion relation of the Lamb-type waves in the solid/liquid/solid structure [13,15] is written as

$$D_{GS}(\omega,k)D_{GA}(\omega,k)=0, \tag{6}$$

with the following:

$$D_{GS}(\omega,k)=2D_S D_A + \tau(D_A - D_S)\cot(k_{zW}d_W/2), \tag{7}$$

$$D_{GA}(\omega,k)=2D_S D_A - \tau(D_A - D_S)\tan(k_{zW}d_W/2), \tag{8}$$

$$\tau = \frac{\rho_W}{\rho}\frac{k_{zL}}{k_{zW}}k_T^4, \tag{9}$$

$$k_{zW} = \sqrt{k_W^2 - k^2}, \tag{10}$$

where ρ_W, c_W, k_W (=ω/c_W), and ρ are the water density, wave velocity in water, wave number in water, and elastic plate density, respectively. We define the modes whose deformation on the surface $z=\pm(d_W/2+d)$ is symmetric with respect to the central plane as global symmetrical(GS) modes.

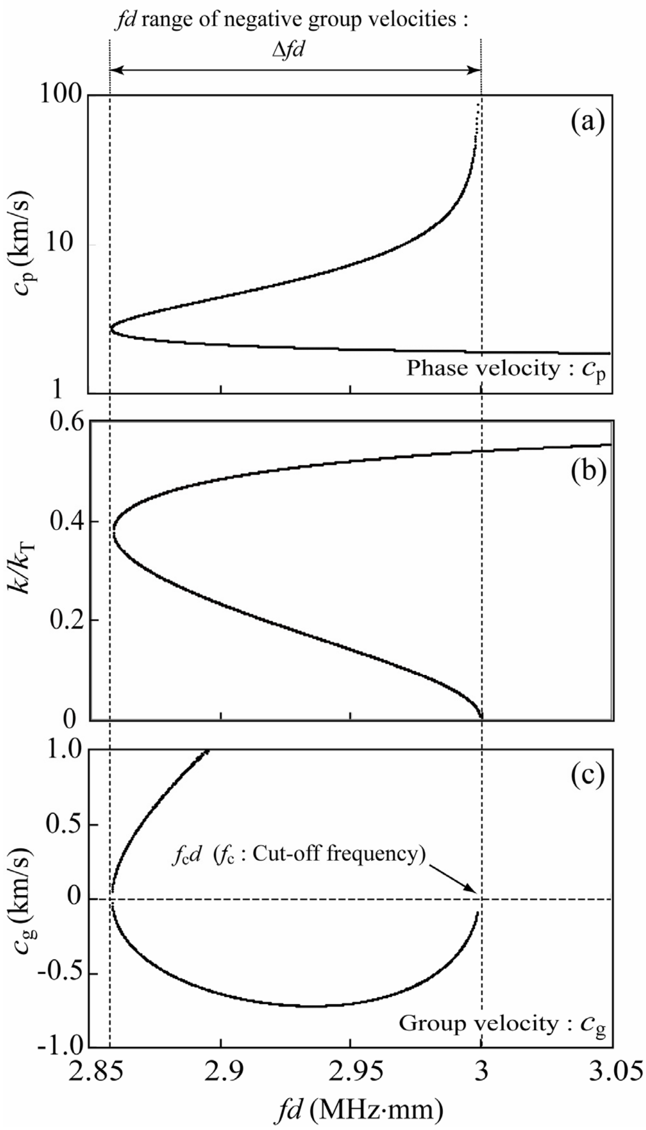

Fig. 3 Existence range of negative group velocities.

Global antisymmetrical (GA) modes are also defined similarly. Additionally, each mode can be divided into two modes with respect to the mode shapes on two identical elastic plates. When S mode is on each elastic plate and the entire structure is in the GS mode, the mode is referred to as the GS-S mode. On the other hand, when S mode is on each elastic plates and the entire structure is in the GA mode, the mode is referred to as the GA-S mode. The GS-A mode and the GA-A mode are also defined, similarly. The deformations of these modes are shown in Fig. 1(b). Here, the Lamb-type waves we consider are composed of only the coupling modes, which propagate with the same wave number.

The wave number of each mode is obtained by solving eqs. (7) and (8), and group velocity is given by $c_g=\partial\omega/\partial k$. Lamb waves show velocity dispersion, that is the phase velocity depends on the product fd (f: frequency, d: the thickness of the elastic plate). Here, the dispersion curve of Lamb waves is shown in Fig. 2. The negative group velocity appears near cut-off frequencies of certain propagation modes. Lamb-type waves have such characteristics, as well. The cut-off frequency means the frequency at which phase velocity is divergent. In Fig. 2, the negative group velocities are in the dotted rectangle. This region is magnified in Fig. 3(a). Figure 3(b) shows the wave number of Lamb waves normalized by the transversal wave number of elastic plates with respect to the same *fd*. In this figure, it is confirmed that the wave number of negative group velocity were lower than transversal wave number of elastic plates generally. Figure 3(c) shows the group velocity c_g with respect to the same *fd*. In these figures, the relationships between the existence range of negative group velocity Δf and the cut-off frequency f_c is shown. The negative group velocity exists in the range from the point that the gradient sign of phase velocity changes to the cut-off frequency. Here, in Fig. 3, the curve is broken in the vicinity of 0 m/s of group velocity. The reason is that the curve is almost normal to the *fd* axis because the gradient of group velocity is divergent to infinity in the vicinity of 0 m/s of group velocity, consequently, the solutions here are not found during numerical search.

3 Negative group velocities of Lamb-Type Waves

In this research, we consider the case d_W/d=0.016 (d=5 mm and d_W=0.08 mm) and the GS-S_1 mode. In this shape, we can obtain the biggest existence range of negative group velocities and fastest negative group velocities in the glass/water/glass structure from the results of our previous research. The parameters of a glass (BK7) are used for sold layers. In fact, the materials are fixed in this research. Consequently, we research the quantitative relationships between the existence of negative group velocity and the parameters of each layer material under this shape by numerical calculation.

We consider that the parameters of two elastic plates as the solid layers are variable. The density and Poisson's ratio of the elastic plates are changed. Here, the water is dealt as a liquid layer. The density and velocity of the water are 1×10^3 kg/m^3 and 1500 m/s, respectively. It is shown that the change of the negative group velocities dependent on Poisson's ratio σ when the density of the elastic plates are fixed at 3×10^3 kg/m^3 in Fig. 4. As Poisson's ratio σ increase from 0.01 to 0.188, the negative group velocities become fast and as Poisson's ratio σ increase from 0.188 to 0.35, the negative group velocities become slow. Also, about the negative group velocities under the same density, the bigger Poisson's ratio σ gives the lower dispersion curves of the negative group velocities about the values of *fd*. In addition, the existence range of the negative group velocity Δf becomes wider as the negative group velocity becomes faster.

We also regard about the characteristics of negative group velocities when the densities of elastic plates are changed. The results are shown in Fig. 5. Figures 5(a) and (b) show $\Delta f/f_c$ as an existence range of negative group velocities with respect to Poisson's ratio σ. Here, Fig. 5(a) shows the $\Delta f/f_c$ of S_1-mode Lamb wave consisting of one elastic plate, and Fig. 5(b) shows the $\Delta f/f_c$ of GS-S_1 mode Lamb-type wave. Figure 5(c) shows max ($-c_g$) with respect to Poisson's ratio σ. In fact, the line composed of density $\rho=3\times10^3$ kg/m^3 is obtained by connecting the minimum value of each dispersion curve in Fig. 4. Here, in order to change Poisson's ratio σ, the transversal wave velocity is changed whereas the longitudinal wave velocity is fixed at 5000 m/s. In Lamb-type waves, it was observed that the existence range of negative group velocities changed widely depending on the density ρ. In Figs. 5(b) and 5(c), the peaks of lines composed of the same density ρ are high and move in the higher direction of Poisson's ratio σ as the density ρ increases, to some degree. However, for the lines of $\Delta f/f_c$, the peaks move lower in the case that ρ is larger than about 15×10^3 kg/m^3. In the lines of max ($-c_g$), the peaks move lower in the case that ρ is larger than about 9×10^3 kg/m^3.

It is described the conditions for obtaining the fastest negative group velocities of the Lamb-type waves. The conditions, which are the acoustic impedance ratio, are expressed as the function of the density ratio of the solid and liquid layers. The result is shown in Fig. 6. The positions that they were circled in Fig. 6 are where the fastest negative group velocities under the same density were obtained. In fact, the values of the positions are the same as the values of peak of lines composed of the same density in Fig. 5(c). In addition, the result is shown when the longitudinal velocity c_L is fixed at 7000 m/s. In Fig. 6, the fitted curves are obtained by connecting the circles that they were obtained by numerical analysis quantitatively. The fitted curves are expressed as following equations,

c_L=7000 m/s:

$$Z_T/Z_W = -7.83\times10^{-5}(\rho/\rho_W)^2+2.35\rho/\rho_W+0.94, \qquad (11)$$

c_L=5000 m/s:

$$Z_T/Z_W = -2.71\times10^{-4}(\rho/\rho_W)^2+1.70\rho/\rho_W+1.12. \qquad (12)$$

Here, Z shows acoustic impedance. Subscripts T and W indicate the transversal wave velocity of elastic plates and the wave velocity of water, respectively, as

$$Z_T=\rho\times c_T, \qquad (13)$$

$$Z_W=\rho_W\times c_W. \qquad (14)$$

The fastest negative group velocity is obtained by using these equations and by choosing the appropriate assortment of elastic plate and liquid material, to a limited extent, that is, from 1 to 100 about ρ/ρ_W. This result can be applied to acoustical flat lenses and the nondestructive test for layer structure among others.

4 Conclusions

In this research, we considered that the negative group velocities of the Lamb-type waves in a solid/liquid/solid layer. While the negative group velocities are annoying in some cases, they are very useful in certain case. Consequently, it is important to understand and control them. If the negative group velocities are applied to fabricating some new device, it is desired that the magnitudes of negative group velocities are comparable to those of positive group velocities.

In this paper, we aimed to obtain the faster negative group velocities. By using characteristics of negative group velocities of Lamb-type waves obtained by numerical analysis quantitatively, it was described the conditions for obtaining the fastest negative group velocities of Lamb-type waves qualitatively. The conditions, which are the acoustic impedance ratio, are expressed as the function of the

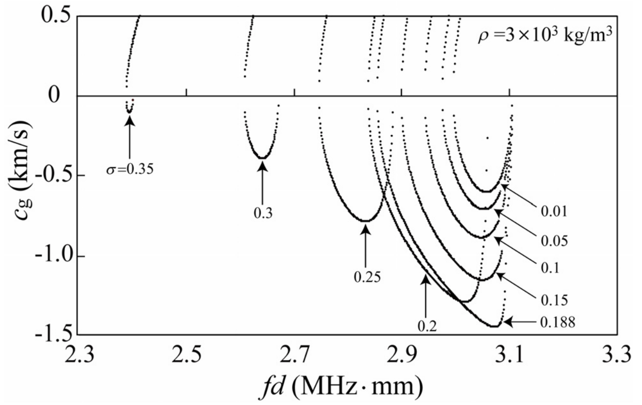

Fig. 4 Changes of negative group velocities dependent on Poisson's ratio.

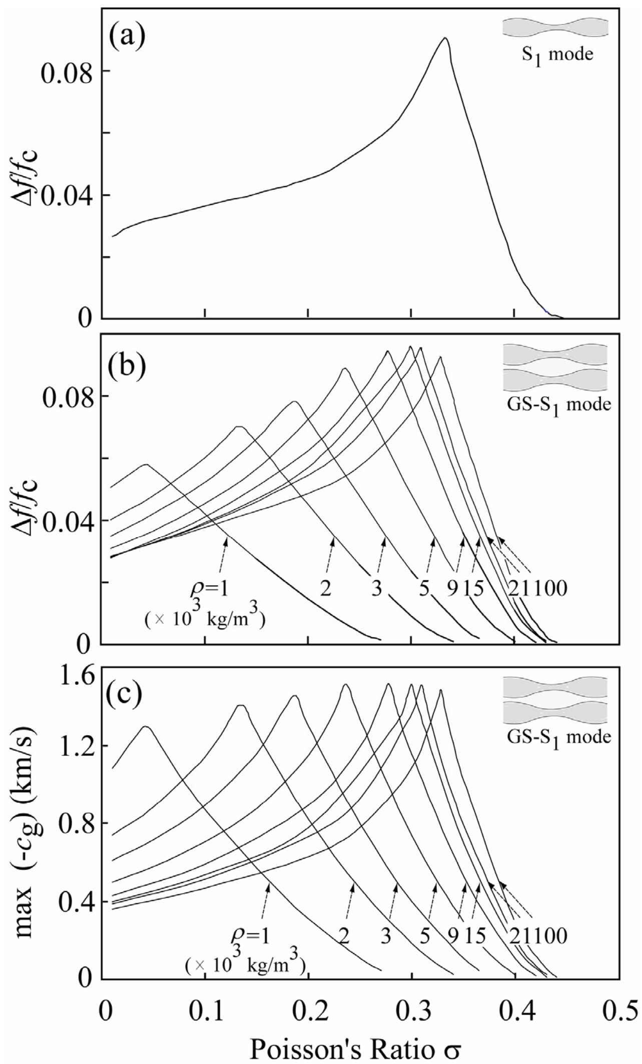

Fig. 5 Characteristics of existence of negative group velocity of S_1 mode Lamb waves and GS-S_1 mode Lamb-type waves when the parameters of solid layers are variable and the liquid layer is considered as water.

density ratio of solid and liquid layers. We expect that this result can be applied to acoustical flat lenses and the nondestructive test for layer structure among others. Especially, because the phase velocity and the group velocity of Lamb-type waves in a solid/liquid/solid layer can be changed without changing the frequency, it is expected to avail to the acoustical flat lenses of tunable focal spot.

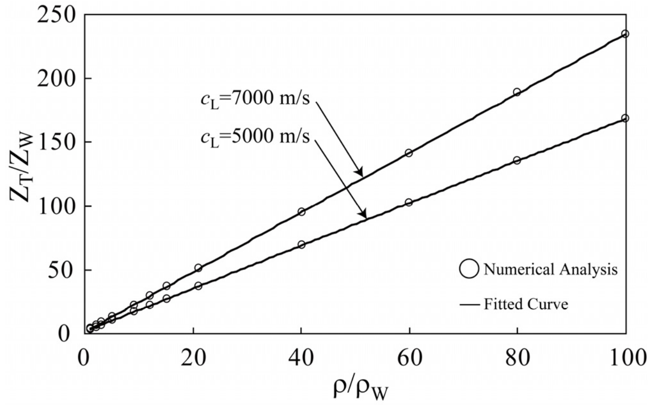

Fig. 6 Condition of fastest negative group velocities respect to density ratio of solid and liquid layers.

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