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Laboratory sound speed measurements on high water content sediment samples

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Laboratory measurements of sound speed in fluid viscous materials are known to be difficult, especially for frequencies of a few kHz. An experimental set up which allows such measurements is developed. Tests are run on sediment samples of various lengths (5cm - 20cm), all water-saturated but with different concentrations. When sound speed only depends on the concentration, over a narrow frequency bandwidth, its estimation originates from time-of-flight measurements on samples of different lengths. It appears that the concentration does play a significant role on the speed of sound. When the sound speed also depends on the frequency (dispersive waves) due to the sediment viscoelastic behaviour, that dependency can be taken into account. An inverse analysis according to the sample length will be given to characterize the sound dispersion for different concentrations. It will be shown that both the above studies on the experimental campaign enable sound speed estimations under various hypotheses. In these conditions, the estimation leads to information about saturated sediment behaviour.

1 Introduction

The difficulty of the laboratory measurement of the acoustic properties of sediments is well known. Still, there is some considerable work which concerns the acoustic properties of silts and sands, mostly at a few tens of kHz [1]. And there has also been experimental work on clays using ultrasound waves, the problem being the very high attenuation of the material. Measurements which are presented here were implemented on an experimental set-up which was developed especially to measure the acoustic properties of saturated sediments at a few kHz. Plane waves propagate through a sediment sample which acts like a waveguide, and multiple reflections occur in this waveguide, resulting in standing waves. The particular subject which is dealt with here is the dependency of the sound speed in clays with either the concentration, or with both the concentration and the frequency, when the viscoelasticity is not negligible anymore.

The first study deals with time-of-flight estimations, and provides some pieces of information about the dependency of the sound speed with the concentration for low-concentrated sediments. The limitation of this analysis is visible for higher concentrated sediments. The second study deals with an inversion method [2] which is made of an inverse analysis (least-square fitting between measurements and simulation), followed by a perturbation method. That method results in information in the frequency domain. Therefore, the dispersion, if there is any, is observable. In the end, the limitations of both these methods are given.

2 Laboratory set-up, and measurements

Sediment samples are cylinders of 5 cm diameter, and of various lengths (5, 10, 15 or 20 cm). They are placed in an envelope which was designed so that no coupling could occur between the sediment and the envelope. A mini-shaker emits a pulse (4 milliseconds) at one end of the sample. The pulse propagates along the sample, reflects itself at the other end of the sample, comes back at the bottom of the sample, and so on, until the pulse is so attenuated that no wave propagates in the sediment anymore. Two miniature accelerometers (Brüel & Kjaer 4374) are used to measure signals. Accelerometers are placed at both ends of the sample. They are connected to an oscilloscope, which has an average function over 64 pulses. Therefore, in the signals which are sent to the computer as a function of time, the noise is already diminished.

The reason for choosing accelerometers instead of hydrophones, while testing fluid samples, lies in the non intrusive character of the accelerometers, as they are located outside the sample. The different response times of the two accelerometers have been observed, in order to take them into account if necessary. The time delay between both accelerometers is about 1 microsecond, and this value is small compared to the measured time of flights. For each material concentration, four tests are run on different sample lengths.

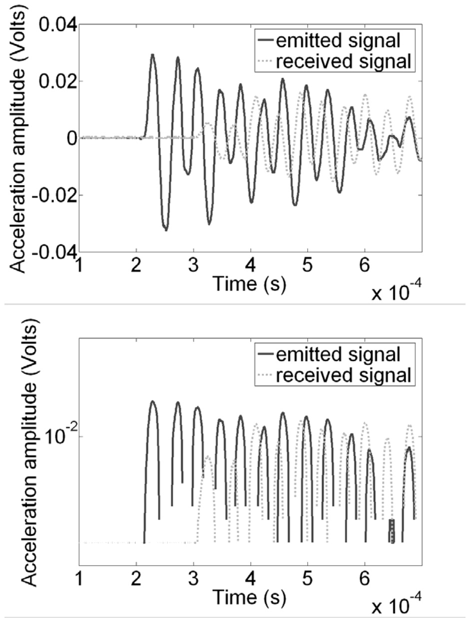
Concentrations are estimated by weight loss on drying. Three small samples are weighted and dried, for each material concentration, and the average uncertainty in the concentration is 0.3 % and it is at most 1.6 %, which is acceptable for our purpose. The concentration range is between 150 and 450 g/l (porosity being between around 85 and 95 %). When the concentration is below 150 g/l, the material is not stable, as far as the sedimentation is concerned. When the concentration is over 450 g/l, the sediment is extremely attenuating, and acceleration signals cannot be analysed. The natural clay which was tested throughout the measurement campaign was always the same, it only had different concentrations according to the quantity of water that was added in it.

3 First study : time-of-flight measurements

3.1 Water measurements : validation

In each measurement, the time of flight is determined by the duration separating the front edge of the acceleration in $x = 0$, and the one of the acceleration in $x = L$. In order to determine the very beginning of the signal, acceleration amplitude time records are plotted in a logarithmic scale, as can be seen on Fig. 1.

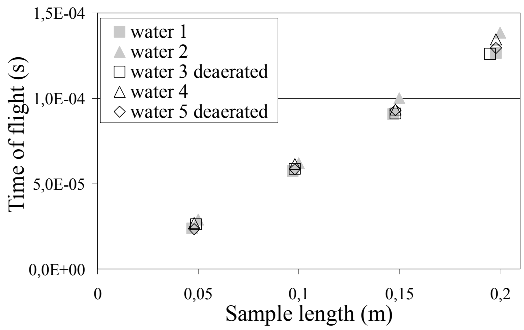
For each material sample, the time of flight can be represented as a function of the sample length. Sound speed can first be estimated with the ratio of the sample length to the time of flight. In that case, the sound speed is given by the signal velocity [3, chapter 9]. Also, if points stand almost in line, then this sound speed can be calculated as the inverse of the slope of the best linear fit.



.Fig.1 - Acceleration amplitude time recordsseries. Decimal scale (top) and semi-logarithmic scale (bottom).

The different times of flight obtained for five different tests on water are reported as a function of the sample length on Fig.2. They do stand in line, and the tests seem relatively repeatable.

The possible regression lines seem to have a negative origin. This could be perhaps related to the fact that sound speeds estimated as signal velocities are not the same for all sample lengths. In particular, the signal velocity is about 1900 m/s for 5 cm-samples, and about 1510 m/s for 20 cm-samples. Two possible experimental criticisms are considered.



.Fig.2 – Time-of-flight versus sample length. Water samples.

The first one concerns the frequency bandwidth of the pulse which could include cut-off frequencies of the first modes. In the waveguide, if the water sound speed is supposed to be 1480 m/s at 20°C [4], then the first cut-off frequency is 17.3 kHz, and the second is 28.8 kHz (the third is 36 kHz). The pulse contains frequency components from 1 to 30 kHz. Therefore, non plane waves of the first and second mode could propagate in the sample, with an infinite sound speed at their cut-off frequencies. The first non plan mode has a radial node line, while the second has a circumferential nodal line [3, chapter 5]. The source is a piston type source, and this is a precaution for letting the plane wave dominate. Therefore, non plan modes are not likely to develop in the duct, especially the first one.

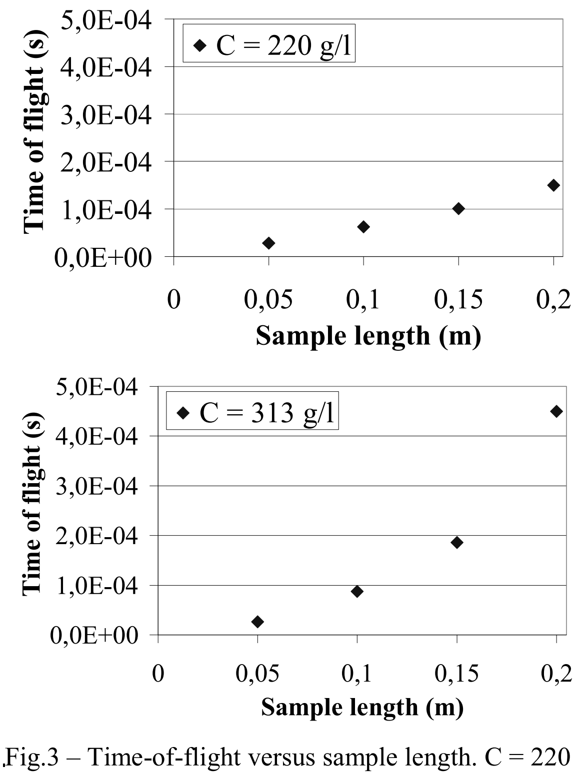
The second doubt concerns the ratio of the sample length to the wavelength. Indeed, in water, the wavelength at 1 kHz is 1.5 m. The condition of the waveguide being longer than a quarter of a wavelength is not observed and this could introduce wrong estimations of the sound speed. When the waveguide is very short, the sample behaves as a nearly incompressible medium, and the apparent wave speed is higher than the one characterizing the material itself. When the waveguide length is long enough, the apparent wave speed is the one characterizing the material. This overestimation of the sound speed according to the sample length could be an explanation of the high sound speed calculated for the 5 cm-samples.

As points stand in line, the sound speed can also be calculated as the inverse of the gradient of the best linear fit. These first measurements on water yield a sound speed of 1427 m/s, while we expect 1480 m/s at 20 °C [4]. The uncertainty is 3.3 %, and this is the best temporal estimation of the sound speed obtained until now.

These measurements lead to estimations of the signal velocity, which might be different from the phase velocity, in the case of a dispersive medium. In water, the dispersion is supposed to be negligible. But this approximation might not be justified in the case of concentrated clays, as shown in the next paragraph.

3.2 Clay measurements

The time method which deals with the slope of the regression line described in the previous paragraph is chosen for the estimation of sound speed in clays.



.Fig.3 – Time-of-flight versus sample length. C = 220 g/l (top) and C = 313 g/l (bottom).

Measurements on low-concentrated sediment prove to be reliable, as points of time of flight do stand in line as function of the sample length. However, there seems to be a limit in concentration, over which the sound speed cannot be estimated in the previous manner anymore. According to our tests, the limit, for our sediment, should stand between 220 g/l and 313 g/l, as can be seen on Fig.3.

Six sediment samples of a concentration below 220 g/l are then tested. Results are reported on Fig.4, where the ratio of the tested sediment sound speed over the reference water sound speed is plotted as a function of the sediment concentration for the water and the six sediment samples. The graph shows a tendency for the sound speed to decrease when the sediment concentration increases. This is consistent with the behaviour expected if the diluted sediment is first seen as a suspension of thin particles. In that particular case, the sound speed c can be expressed as a function of the material density ρ and its compressibility β [5] :

$$c = \sqrt{\frac{1}{\beta\rho}} \quad (1)$$

Assuming these two properties depend on the porosity n the following way :

$$\rho = n\rho_{water} + (1-n)\rho_{sediment} \quad (2)$$

$$\beta = n\beta_{water} + (1-n)\beta_{sediment} \quad (3)$$

Then, when the porosity decreases from 1 to 0, the sound speed is expected to decrease, to reach a minimum, and to increase afterwards. But, as far as suspensions of clays are concerned, this theory is only valid for very low concentrated clays (or very high porosities), in which the cohesion due to physical and chemical bonds is not too important, and the rigidity of the material is negligible.

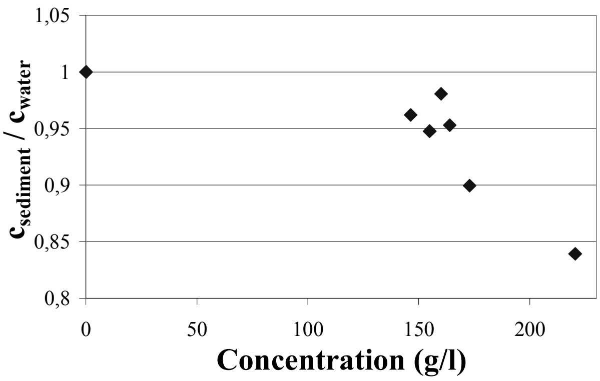


Fig.4 – Ratio of the tested sediment sound speed to the reference water sound speed, versus sediment concentration.

The fact that points do not stand in line for high concentrated sediments could partially be explained by the propagation conditions, but also by the viscoelastic behaviour of the sediment which might not be negligible when the concentration is important. The possibility of the presence of gas bubbles has been put aside, since some measurements have been realized on two clay samples with the same concentration, except one had been filled with demineralised water, and had been degassed for 72 hours before being tested. The two samples showed precisely the same acoustic behaviour.

If the tested material is viscoelastic, then the high components of its frequency content are attenuated. The pulse is made of all frequency components between 1 kHz and 30 kHz. For one sample length, let us say that frequencies over f_0 are attenuated. For a longer sample length, more frequency components will be attenuated, and frequencies over f_0-df are attenuated. As mentioned, the material is supposed to be viscoelastic. Therefore, the high frequency components propagate more rapidly than the low frequency components, as the sound speed is supposed to be dispersive. Then, the apparent sound speed for the first length we considered is going to correspond to the f_0 component. The apparent sound speed for the second length is going to correspond to the f_0-df component, and it therefore going to be lower. In the end, for a viscoelastic attenuating material, the apparent sound speed is going to be higher for short sample lengths than for long sample lengths. And this could explain, if the sound speed dispersion is important enough, why points of time of flight versus sample length do not stand in line. In the end, it seems that it reflects the fact that the group velocity can no longer be an approximation for the phase velocity, which is the sound speed we are looking for.

This consideration naturally introduces the second study, in which the sound speed is studied as a function of both the concentration and the frequency.

4 Second study : inversion method

4.1 Identification problem

An inversion method is developed for this purpose. In the previous study, time records were used for the estimation of the sound speed. Here, the work is being carried out in the frequency domain, which implies that the recording duration is long enough to justify a Fourier transform and obtain a frequency decomposition of signals which has a sufficient resolution. All along this study, signals are supposed to have undergone a Fourier transform, and the frequency domain is implicit.

First, the direct problem is modelled. The sediment is supposed to have a viscoelastic behaviour. Therefore, its acoustic properties (sound speed and attenuation), are very likely to depend on the frequency, and can both be expressed by a complex wavenumber noted k:

$$k = \frac{\omega}{c} - i\alpha \quad (4)$$

ω	Angular frequency [rd/s]
c	Sound speed [m/s]
α	Attenuation per length [Np/m]

Results in terms of attenuation are still under consideration and will not be presented here, as the work here only deals with the sound speed. Nevertheless, the solution of the identification problem is given in terms of wavenumber, and thus the method could also apply for the determination of the attenuation per metre α.

Boundary conditions were established as a function of the laboratory set-up, and, together with the wave propagation equation, they lead to the following displacement at the top of the sample u_L as a function of the displacement at the bottom of the sample u_0:

$$u_L = u_0 \frac{1+R}{e^{ikL} + Re^{-ikL}} \quad (5)$$

R Reflection coefficient in displacement at the top of the sample taking into account the receiver [2]

L Sample length [m]

Now that the displacement can be expressed as a function of the acoustic properties of the material and the displacement at the bottom of it, one shall deal with the identification problem. Eq.(2) shows that the wave number cannot be expressed easily as a function of the displacements at the bottom and at the top of the sample, which are now measured.

An inversion allows the estimation of the acoustic properties for each measurement, that is, for each sample length. Considering Eq. (2), let F(k) be the following function:

$$F(k) = R + \frac{1-(u_L/u_0)e^{ikL}}{1-(u_L/u_0)e^{-ikL}} \quad (6)$$

To deal with the identification of k, the method consists in seeking the wavenumber which minimizes $|F(k)|$, starting from an initial value k_{ini}. Complex wavenumbers are obtained with this method, but, as it is a non-convex problem, it may often lead to erroneous results, first because it depends on k_{ini}, second in case of error in the data (u_L/u_0). Obviously, the closer the initial value k_{ini} to the solution, the better the chance of obtaining a good estimation of the wavenumber.

To go further in terms of precision, the perturbation method makes it possible to linearize the function with regard not to the parameter itself but to its variation. Let k be the wavenumber which is the solution to the problem. A close value to that wavenumber, k_0, is supposed to be known. The solution can now be written $k = k_0 + \varepsilon$, and it becomes possible to linearize Eq.(2) with regard to the perturbation ε, yielding the following expression:

$$(u_L/u_0) \approx f + \varepsilon g \quad (7)$$

where f and g depend on parameters which appear inside the reflection coefficient, that is characteristics of the system which contributed to the establishment of the boundary conditions and can be expressed under certain hypotheses which are reliable in our case [2]. Eq. (7) is established for each sample length. However, four measurements for (u_L/u_0) are available, and the perturbation ε ought not to depend on the sample length. For each length L_j (j=1,2,3,4), Eq.(7) can be written $g_j\varepsilon \approx (u_L/u_0)_j - f_j$. Given these considerations, the simple matrix equation can be written:

$$\overline{\overline{G}}\,\overline{\varepsilon} = \overline{F} \quad (8)$$

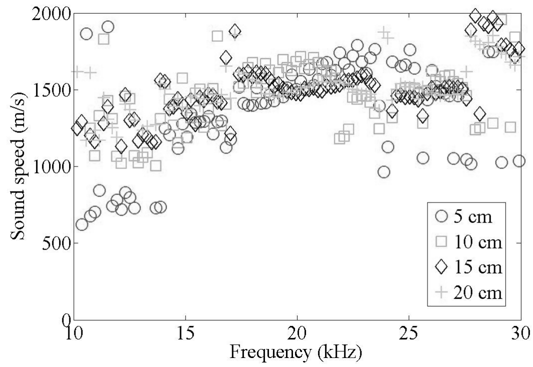
The solution of which is given by the following expression:

$$\overline{\varepsilon} = \left(\overline{\overline{G}}^T . \overline{\overline{G}}\right)^{-1} . \overline{\overline{G}}^T . \overline{F} \quad (9)$$

The combination of these two methods has been compared numerically to the inverse analysis alone, in terms of efficiency and robustness [2]. In this paper, we discuss this same comparison on laboratory measurements on clays.

4.2 Results

In this section, sound speed obtained from the identification of the complex wavenumber is presented as a function on frequency, for different concentrations. Pulses frequency content is between 1 kHz and 30 kHz, slightly depending on the mini-shaker loading. The amplitude is low for frequency components between 1 and 10 kHz, and results are presented for 10 to 30 kHz frequencies only.



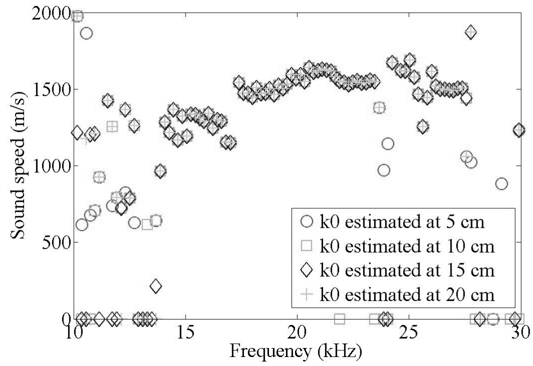


Fig.5 – Sound speed versus frequency for a water sample. Non-convex analysis (top) and non-convex followed by convex analysis (bottom).

First, sound speed computed from water measurements is presented on Fig.5. At the top, one can observe all sound speeds that are obtained with the non-convex problem resolution. They are roughly around 1500 m/s and this is the order of magnitude we expect, as it is quite close to the water sound speed. Computations are made for each frequency component, and are therefore independent one from the other. The fact that sound speeds of close frequencies are close themselves helps in gaining some confidence in the results. The sound speed computed from the perturbation method can be observed on the bottom of Fig.5. There are still four sets of results, because the perturbation method was computed with the four different wavenumbers which originated from the inverse analysis as initial wavenumbers. But the method seems to have had these wavenumbers converge to the same value as, for each frequency, the four sound speeds are identical in most cases. Indeed, from Eq.(8), it appears that the method helps finding a wavenumber which strongly depends on the ratio (u_L/u_0). It gives more weight to wavenumbers which are obtained at resonance frequencies, while the previous method did not take that parameter into account.

Then, sound speed computed from low-concentrated sample measurements is presented on Fig.6. At the top, sound speeds which are obtained with the non-convex problem resolution seem rather consistent with each other. The sound speed computed from the perturbation method can be observed on the bottom of Fig.6. Here also, the method seems to have had these wavenumbers converge to the same value.

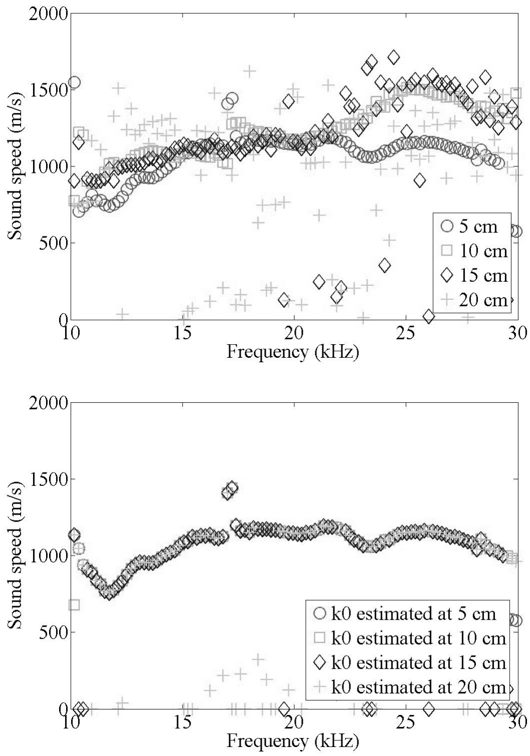


Fig.6 – Sound speed versus frequency for a low concentrated sediment (C = 173 g/l). Non-convex analysis (top) and non-convex followed by convex analysis (bottom).

This perturbation method was also tested alone on these measurements, independently from the non-convex problem, and with an initial wavenumber similar to that of the water. Results were perfectly consistent with the results that are presented here.

The limitations of this method shall be pointed out. The exact same computations are done on measurements which had been implemented on higher concentrated samples. In that case, it seems that the sound speed can neither be obtained with the inverse analysis nor with the inverse analysis followed by the perturbation method. Both analyses lead to scattered plots, and the perturbation does not improve the identification of the sound speed. This could have been anticipated, as high-concentrated sediments do strongly attenuate waves, and the ratio (u_L/u_0) in this case is always low.

Finally, the perturbation method helps the identification of the wavenumber with respect to the importance of the ratio (u_L/u_0), and thus with respect to the reliability of the wavenumber obtained with the inverse analysis. The use of this method is a real improvement in the identification problem, which is usually dealt with by an inverse analysis alone. Still, just like the time-of-flight method, it shows limitations in relation with the material attenuation. Laboratory measurements which were made on clays till now were either with ultrasound, or the attempts to measure the acoustic properties at a few kHz failed. Sound speed in clay is expected to be lower than that of water, but the importance of the ratio of sound speed in the sediment to the sound speed in water has never been measured at these concentrations for clays. And at these concentrations (150 g/l – 450 g/l), we suspect it is too much of a hypothesis to see sediments as suspensions.

5 Conclusion

Two methods were developed to estimate the sound speed from acceleration measurements in a sediment sample, and each has shown its interesting points and its limitations. The first consists in time-of-flight measurements in the time domain. The sound speed (signal velocity) appears to decrease while the concentration increases, and this is consistent with the expected behaviour if the diluted sediment is first seen as a suspension of particles. The second consists in solving, in the frequency domain, an identification problem which was set-up after the experiment was modeled. Computations occur on Fourier transforms of acceleration signals, and sound speed (phase velocity) is estimated as a function of frequency. Both methods have been tested on saturated clays. They are consistent with one another, but they both show limitations with the material attenuation. A technical improvement would be to emit more powerful signals to the sediment.

One further development would be to be able to include the set-up characteristics, such as time delay, in the identification method, in order to improve the confidence in the sound speed. Measurements are delicate, and so is the post-measurement analysis, but these recent developments could allow measurements of the acoustic properties of fluids at a few kHz.

Acknowledgments

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References

[1] Buckingham M.J., Richardson M.D., "On tone-burst measurements of sound speed and attenuation in sandy marine sediments", *IEEE J. of Oceanic Engineering,* 27, 3, 429-453 (2002)

[2] Martin V., Martin V., Poullain P., "Stability of sound speed and attenuation identified in material samples", ICSV14, 9-12th July 2007, Cairns, Australia (2007)

[3] Morse, P.M., Ingard K.U., Theoretical Acoustics, ed. Princeton University Press, 927 p. (1986)

[4] Del Grosso V.A., Mader C.W., "Speed of Sound in Pure Water", *J. Acoust. Soc. Am.* 52, 5B, 1442-1446 (1972)

[5] Wood, A.B., A textbook of sound, ed. MacMillan, 519p. (1930)