A formulation based on modal optimization for predicting sound radiation from fluid-loaded aircraft structures

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This study is led in the context of understanding sound transmission through fluid-loaded aircraft structures with non-uniform damping. The fluid/structure coupling is here highlighted. Classic approaches using modal radiation impedances to formulate the fluid/structure coupling do lead to complexity and are computationally time-consuming. A method defining rigorously this coupling and avoiding the calculation of the modal radiation impedances is proposed. This method aims at developing a simple formulation taking advantage of current technical progress in optimization algorithms.
Sound radiation from a simply supported, baffled, fluid-loaded plate is here solved in optimizing the modal amplitudes so that they fit the governing equation with fluid loading. To perform this optimization, a sampling of the plate into observation points is first done, and then a modal decomposition into *in vacuo* modes is led. Comparison with results from the literature over [10–1000 Hz] for a reference case of plate immersed in air (critical frequencies equal to f_C=1.2 kHz) show excellent agreement within 1dB. The simplicity and computation time allow an extension to non-uniform damped aircraft structures and a prediction over a large frequency band. As perspectives, results from plates with local damping patch are presented.

1 Introduction

The turbulent boundary layer excitation is one of the main sources of aircraft interior noise over a large frequency range. An aircraft structure is strongly heterogeneous: it is made of an assembly of stiffened curved panels with non-uniform skin thickness. It includes beams to support the floor as well as large size windows in the flight deck. In the context of better understanding and predicting, a formulation has been developed with purpose of assessing quickly sound radiation from these complex aircraft structures.

This paper focuses on presenting the basics of an approach based on modal optimization and then its advantages in the case of fluid loaded plates. Only mechanical excitation is here considered. The calculation of radiated pressure leads to complexity and is time consuming even if the structure is a simple plate. Some models predicting sound radiation from plates with fluid loading have been proposed in the literature, for instance [1, 2, 3, 4, 5]. They all emphasize the difficulties to calculate the modal radiation impedances. This problematic has been also highlighted by ref. 6. The present method allows to avoid their calculations. Moreover the simplicity of the formulation offers the possibility to treat heterogeneous plates.

2 Methodology and modeling

The structure of interest is a simply supported thin rectangular plate inserted in a baffle (Fig. 1). The baffle is assumed to be infinite, plane and rigid. The plate-baffle system separates vacuum (z<0) from a fluid (z>0).

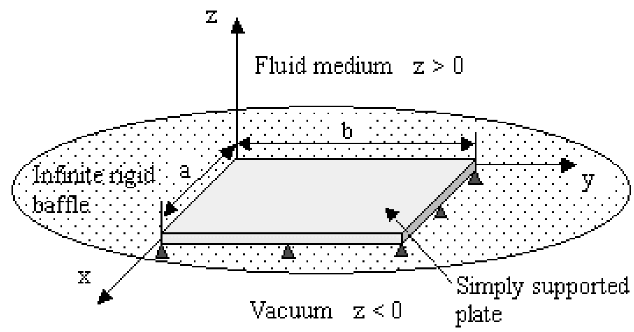

Fig. 1. Geometry of the problem.

Such indicators as Mean Square Velocity, Radiated Sound Power and Radiation factor are assessed.

The transverse motion $W(x,y)$ of the homogeneous plate is given by the following Love-Kirchhoff equation:

$$D\Delta^2 W(x,y) - \omega^2 \rho h W(x,y) = F(x,y) - p_0(x,y) \qquad (1)$$

where D is the bending stiffness, ρ is the mass density, h is the thickness, $F(x,y)$ is the driving force per unit area and $p_0(x,y)$ is the fluid loading pressure. $p_0(x,y)$ is defined by the well-known Rayleigh integral using Eq. (2), where S_p is the surface of the plate, $k_0 = \omega/c_0$ is the wave number, ρ_0 and c_0 are the density and the sound speed in the fluid, and $R^2 = (x-x')^2 + (y-y')^2$.

$$p_0(x,y) = \int_{S_p} j\rho_0\omega \, V(x',y') \, \frac{e^{-jk_0R}}{2\pi R} \, dx'dy' \qquad (2)$$

To compute the fluid loading pressure, the collocation method is used, which allows a relative easy calculation. To apply this approach, the plate is discretized into N patches. One gets the radiation impedances defined by

$$Z_{st} = \frac{p_0(x_s,y_s)}{V(x_t,y_t)} = \frac{1}{2\pi}\rho_0 j\omega \frac{\exp^{-jk_0 d_{st}}}{d_{st}} S_t \qquad (3)$$

$$Z_{ss} = \frac{p_0(x_s,y_s)}{V(x_s,y_s)} = \rho_0 c\left(1-\exp^{-jk_0 r}\right) \qquad (4)$$

where s denotes the patch index, (x_s,y_s) are the coordinates of the center of the patch l, d_{st} is the distance between patches s and t, S_t is the surface of the patch t, and r is the radius of a circular patch of surface S_s.

Using these radiation impedances, a new formulation of the fluid loading pressure is derived:

$$p_0(x_s,y_s) = \sum_{t=1}^{N} Z_{st} V(x_t,y_t) = j\omega \sum_{t=1}^{N} Z_{st} W(x_t,y_t) \qquad (5)$$

To model the plate behavior, a sample of observation points verifying Eq. (1) is used:

$$\begin{aligned} &D\Delta^2 W(x_i,y_i) - \omega^2 \rho h W(x_i,y_i) \\ &= F(x_i,y_i) - j\omega \sum_{t=1}^{N} Z_{it} W(x_t,y_t) \, , \forall i \in [1,N] \end{aligned} \qquad (6)$$

where i denotes the observation point index, N is the number of samples equal to the number of patches used in radiation impedances calculation, and (x_i,y_i) are the coordinates of the observation point i corresponding here to the patches centers.

To solve this radiation problem thanks to modal optimization, modal decomposition of plate displacement is introduced. The flexural motion of the plate is expanded into series of *in* vacuo modes. The following expression is used:

$$W(x_i, y_i) = \sum_{m,n} a_{mn} \Phi_{mn}(x_i, y_i) \quad (7)$$

where a_{mn} and Φ_{mn} are the modal amplitude and the modal shape, respectively, of the (m,n) mode. In the case of a simply supported plate, Φ_{mn} is defined by

$$\Phi_{mn}(x_i, y_i) = \sin(\frac{m\pi x_i}{a})\sin(\frac{n\pi y_i}{b}) \quad (8)$$

The set $\{a_{mn}\}$ are the unknown coefficients to be determined. Using truncation of the series, one gets the following plate transverse motion:

$$D\sum_{m=1}^{m_{max}}\sum_{n=1}^{n_{max}} a_{mn}\Delta^2\Phi_{mn}(x_i, y_i) - \omega^2\rho h\sum_{m=1}^{m_{max}}\sum_{n=1}^{n_{max}} a_{mn}\Phi_{mn}(x_i, y_i)$$
$$+ j\omega\sum_{t=1}^{N} Z_{it}\sum_{m=1}^{m_{max}}\sum_{n=1}^{n_{max}} a_{mn}\Phi_{mn}(x_t, y_t) = F(x_i, y_i),\ \forall i \in [\![1, N]\!] \quad (9)$$

where m_{max} and n_{max} are the maximal orders of the modes in the x and y directions, respectively. For the sight of simplicity, the equation of motion can be rewritten as

$$\sum_{m=1}^{m_{max}}\sum_{n=1}^{n_{max}}\begin{pmatrix} D\nabla^4\Phi_{mn}(x_i, y_i) - \omega^2\rho h\Phi_{mn}(x_i, y_i) \\ + j\omega\sum_{j=1}^{N} Z_{ij}\Phi_{mn}(x_j, y_j)\end{pmatrix} a_{mn} \quad (10)$$
$$= F(x_i, y_i),\ \forall i \in [\![1, N]\!]$$

3 Optimization

The present approach is a modal method based on the optimization of the modal amplitudes, which are here the unknowns of the governing equation of the plate. The modal amplitudes are consequently the only variables of the problem, i.e. the variables to optimize, while the number of samples represents the number of equations. Solving this problem in optimizing the modal amplitudes so that they fit Eq. (9) represents an important simplification compared to classical methods computing direct and cross-modal radiation impedances, that is to say $(m_{max} \times n_{max})^2$ calculations of oscillating integrals.

The first approach consists in taking the same number of equations and unknowns to get a square system, *i.e.* same number of observation points as modes considered. All algorithms studied deal with this case without any problem of convergence. Note that a simple Gauss elimination algorithm is sufficient for such a system. However, one can anticipate the case of more complex plates (heterogeneous, stiffened, …) where a fine mesh is required and, as a result, a large number of samples must be considered in the simulation. For practical reasons, the square system described before is avoided because increasing the number of modes in the model is naturally too inconvenient. Therefore a rectangular system is used, *i.e.* more samples than modes.

Contrary to square system, problems of convergence are observed when the system to optimize is over-determined. This undesirable effect is due to the well known phenomenon in sampling called aliasing. To correct this phenomenon, a filtering based on Daubechies wavelets is applied to the system, which is then optimized in the least square sense. The filter effect is illustrated in Fig. 2.

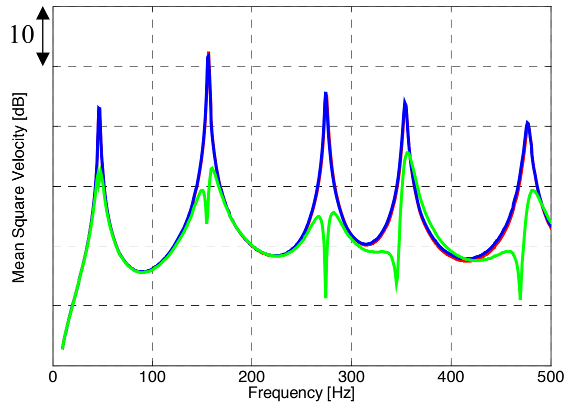

Fig. 2. Filter effect on Mean Square Velocity of a plate by the proposed method in case of square system (red), rectangular system without filtering (green) and with filtering (blue). Case of sound radiation into air from a simply supported thick plate (a/b=1, f_c=22.7 kHz) driven by a point force at (a/3,b/4).

4 Validation

4.1 Homogeneous plates

The modal optimization is applied to a simply supported plate 1000 mm x 1000 mm, immersed in air and driven by a point force F_p at (0.7,0.2) of 1 N ($F_p = \|F_p\|\ \delta(x\text{-}0.7)\ \delta(y\text{-}0.2)$ and $\|F_p\| = 1$). To take into account this point force, the plate is again discretized into N patches. Assuming a uniform value of the force inside the patch excited, one gets for all samples:

$$F(x_i, y_i) = \begin{cases} \|F_p\|/S_i & \text{if sample } i \in \text{patch excited} \\ 0 & \text{else} \end{cases} \quad (11)$$

where S_i is the surface of the patch i.

The following properties of air are chosen to simulate the fluid loading: ρ_0 = 1.2 kg.m^{-3} and c_0 = 340 m.s^{-1}. In all cases, the value of the structural loss factor is taken to be η = 0.01. This factor is introduced in the complex Young modulus E^* such as

$$E^* = E(1 + j\eta) \quad (12)$$

Fig. 3 presents the simulation results compared to reference results from C-Valor, a commission for vibroacoustic software validation led by the French Acoustical Society (S.F.A.) and the French Mechanical Society (S.F.M.). An excellent agreement within 1dB is observed over [10-1000Hz]. This simulation was realized using a regular sampling of the plate with N = 21 x 21 samples and 36 modes.

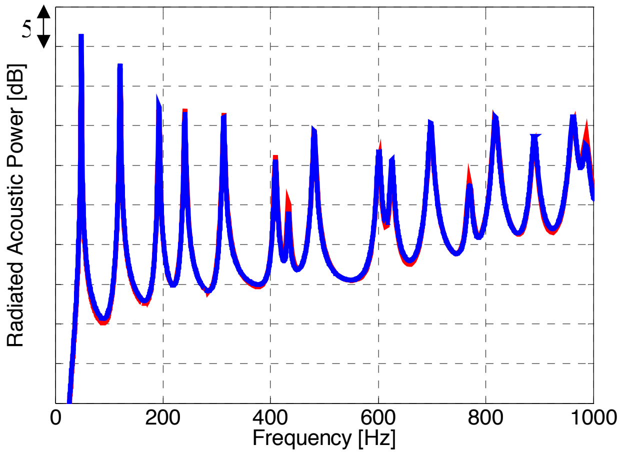

Fig. 3. Radiated Acoustic Power by the proposed method (blue) compared to reference results from C-Valor (red). Case of sound radiation into air from a simply supported thick plate (a/b=1, f_c=1.2 kHz) driven by a point force at (0.7,0.2).

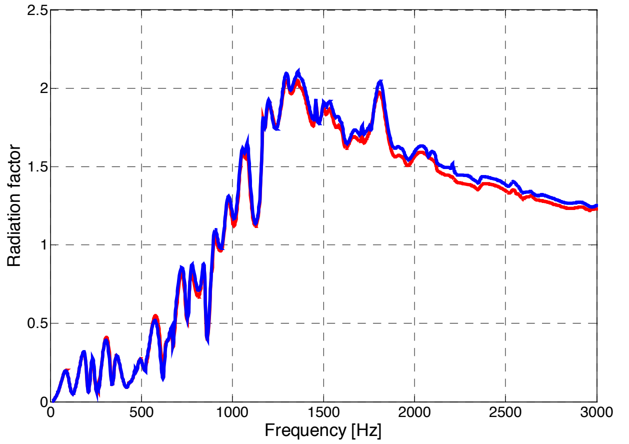

Fig. 5. Radiation factor (right side) into air by the proposed method (blue) compared to results from Miloudi *et al.* (red). Case of a simply supported thick plate (a/b=1, a/h=100, f_c=1.2 kHz) with varying thickness (α=1) driven by a point force at (0.33,0.38).

4.2 Non-homogeneous plates

Modal optimization is then applied to non-homogeneous plates. The present model can be easily generalized. The governing equation is rewritten in considering the non-homogeneity of the plate, it yields

$$D\Delta^2 W - \omega^2 \rho h W + 2\frac{\partial D}{\partial x}\frac{\partial(\nabla^2 W)}{\partial x} + 2\frac{\partial D}{\delta y}\frac{\partial(\nabla^2 W)}{\partial y} + (\nabla^2 D)(\nabla^2 W) - (1-\upsilon)\left(\frac{\partial^2 D}{\partial y^2}\frac{\partial^2 W}{\partial x^2} - 2\frac{\partial^2 D}{\partial x \partial y}\frac{\partial^2 W}{\partial x \partial y} + \frac{\partial^2 D}{\partial x^2}\frac{\partial^2 W}{\partial y^2}\right) = F - p_0 \quad (13)$$

where v is the Poisson ratio. Next, applying the sampling to Eq.(13), one gets the unique change in the present theory compared to the homogeneous case.

To validate modal optimization in the non-homogeneous case, the prior work of Miloudi *et al.* [7] is used as comparison. Miloudi *et al.* have put their focus on the influence of varying thickness on sound radiation from a baffled, simply supported plate into air. The studied thickness is only varying in the x direction (Fig. 4) as follows

$$h(x) = h_0(1 + \frac{\alpha}{a}x) \quad (14)$$

where h_0 is the thickness at x = 0 and α is a given constant.

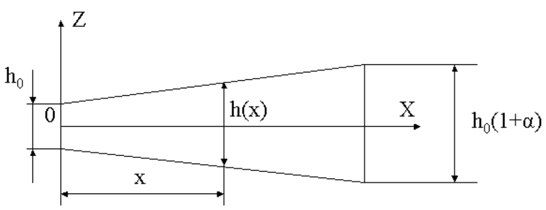

Fig. 4. Geometry of the non-homogeneous plate.

The radiation efficiency of a simply supported steel plate 1000 mm x 1000 mm with h_0 = 8 mm and α = 1, immersed in air and driven by a point force at (0.33,0.38) of 1 N, is here investigated. Fig. 5 shows an excellent agreement between optimized results and those ones from Miloudi's work. This observation gives credit to the generalized model, which seems to be also powerful with non-homogeneous plates. In addition, one can also notice here a real advantage of this approach compared to classical ones, that is the simplicity of changing plate structures in just modifying the differential equation of motion.

5 Perspectives

As perspectives, an extension of the proposed formulation to local damping is presented. The modal response of the simply supported square plate with damping patch described by Kung and Singh [9] is here investigated. The homogenized properties of the multilayer damping patch are computed using an approach developed by Guyader and Cacciolati [8].

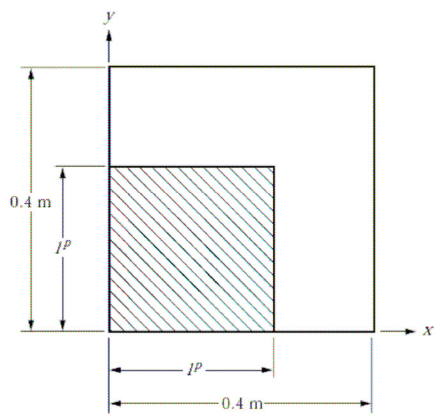

Fig. 6. Geometry of the non-homogeneous plate with varying square damping patch.

Results from a parametric study led on patch size variation, as described in Fig. 6, are compared to the prior work of Kung and Singh. Good agreement is observed (Fig. 7) between the predictions of the first natural frequency ω_{11}.

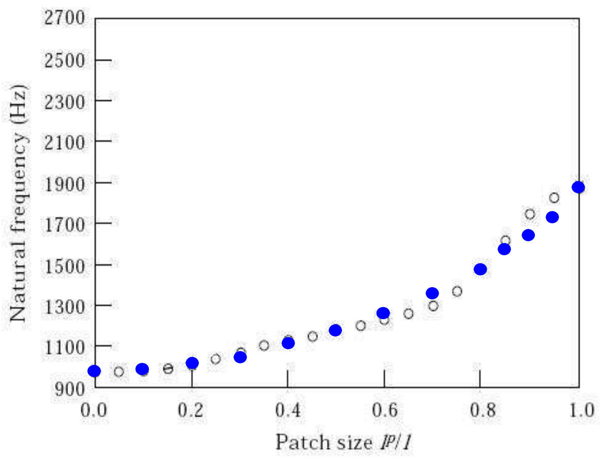

Fig. 7. Calculation of the first natural frequency ω_{11} into air by the proposed method, ●, compared to results from Kung and Singh, ○.

The authors studied only the vibration characteristics with energy based approach whereas the proposed method permits to predict also sound radiation as shown in Fig. 7. In fact, Fig. 8 presents sound radiation results from the Kungh and Singh' plate over [100–2000 Hz] with the present method. One can observe that radiated acoustic power decreases when damping patch size increases whereas radiation factor increases. As a result, damping patches influence on aircraft structures can be investigated thanks to this formulation.

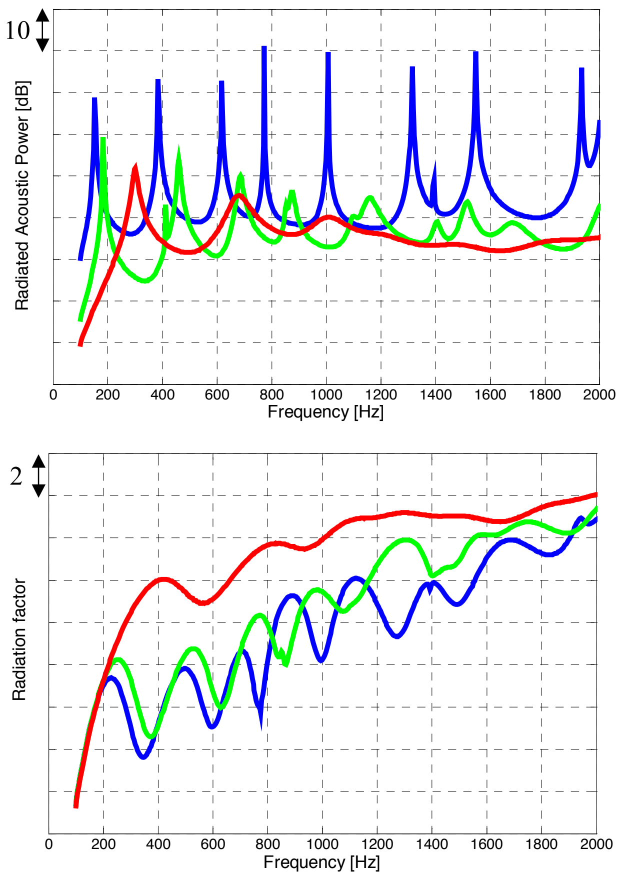

Fig. 8. Radiated Acoustic Power and Radiation factor into air by the proposed method. Case of a simply supported square plate 0.4 x 0.4 m driven by a point force at (a/3,b/4) with varying damping patch: base plate (blue), ¼-covered (green) and fully covered (red).

6 Conclusion

A formulation based on modal optimization has been presented. Its simplicity through the use of powerful existing algorithms was attractive, and comparison with reference results and literature has demonstrated the power of this approach. Modal optimization permits to predict accurately and above all very quickly sound radiation from baffled plates. The fluid/structure coupling is thus precisely included whereas the difficulties, both theoretical and numerical, of calculating the exact radiation impedances are avoided. Investigation of sound radiation over large frequency band becomes thus possible. The simplicity of this formulation offers also the possibility to treat different excitations and different boundary conditions.

Furthermore, the generalization to non-homogeneous plates allows us to investigate new applications as sandwich plates or damping patches. Finally, the present approach can be depicted as a powerful tool to assess sound radiation from complex aircraft structures.

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