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Sound propagation in a street canyon: A study by modal decomposition

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An urban, U-shaped, street canyon being considered as an open waveguide in which the sound may propagate, one is interested in a multimodal approach to describe the sound propagation within. The key point in such a multimodal formulation is the choice of the basis of local transversal modes on which the acoustic field is projected. For a classical waveguide, with a simple and bounded cross-section, a complete orthogonal basis can be analytically obtained. The case of an open waveguide is more difficult, since no such a basis can be exhibited. However, an open resonator, as displays for example the U-shaped cross-section of a street, presents resonant modes with complex eigen frequencies, owing to radiative losses. This work first presents how to numerically obtain these modes and, then, how they can be used as a basis for the modal decomposition of the sound field in a street canyon.

1 Introduction

The investigation of sound propagation in urban environments and streets has been the subject of extensive researches in the past three decades, as the response to a growing social demand (see, *e.g.* [1, 2, 3, 4, 5, 6, 7, 8, 9]).

An urban, U-shaped, street canyon being regarded as a waveguide in which sound may propagate (Fig. 1), one is interested in a multimodal description of the wave propagation within, as such a formulation is shown to be particularly adapted to take into account geometrical (cross-section [10], curvature [11]) or physical (condition at the walls [12]) non-uniformities along a waveguide, as well as the radiation or continuity condition at its extremities. In particular, the aim of the present communication is to show the suitability of a multimodal formulation to take into account the opening of the street canyon on the infinite space above the street.

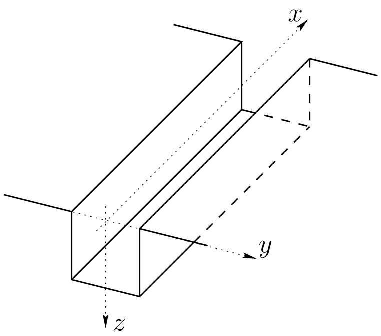

Figure 1: The street canyon considered as an open waveguide

The basic idea of the multimodal formalism is the following: infinite sets of coupled differential equations are built for the components of the wave field (pressure, velocity), projected on a basis of local transverse modes.

A key point in the multimodal formulation developed in above cited references is the choice of the basis of local transverse modes, on which the wave field is developed. In "classical" waveguides having a simple and bounded cross-section, a complete orthogonal basis of eigenmodes can be analytically obtained (or numerically for more complicated cross-sectional shapes). The case of an open waveguide is more difficult, since no such a basis can be exhibited. However, an open resonator, as displays the cross-section of an open waveguide, is known to also exhibit resonant modes, with complex eigenfrequencies, owing to the radiative losses [13, 14, 15, 16].

In this paper we propose to describe how the resonant modes of the open cross-section of an open waveguide can be used to give a multimodal formulation of the sound propagation in long open enclosures. A general method to compute the resonant frequencies and modes in the open cross-section of the duct is first descibed, then the multimodal propagation in a straight open waveguide is formulated and numerical examples are given and discussed.

2 Eigenmodes of the transverse problem

2.1 Theory and formulation

We are interested in the resolution of the transverse problem in the open waveguide shown in Fig. 1. Its cross section can be seen as a rectangular cavity open on the semi-infinite space (Fig. 2). A similar problem, with elastic waves, has been treated by Maradudin *et al.* [14]. Thus, part of the following equations are derived from this work.

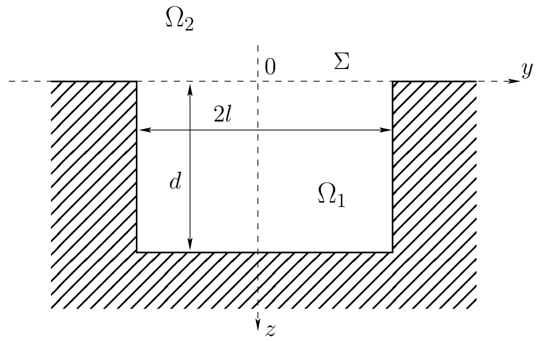

Figure 2: The cross section is defined as a rectangular cavity Ω_1 open on a semi-infinite domain Ω_2.

The transverse modes are the discrete solutions of the eigenproblem

$$(\nabla_\perp^2 + k^2)\phi = 0, \tag{1}$$

satisfying conditions of rigid and perfectly reflecting boundaries :

$$\frac{\partial \phi}{\partial n} = 0, \tag{2}$$

n the normal to domain boundaries.

Continuity conditions are written at $z = 0$:

$$\phi(y, z = 0^+) = \phi(y, z = 0^-), \tag{3}$$

and
$$\frac{\partial p(y,z=0^+)}{\partial z}=\frac{\partial p(y,z=0^-)}{\partial z}. \qquad (4)$$

The solution of Eq.(1) in Ω_1 satisfying the condition (2) is written as a discrete sum of functions,
$$\phi=\sum_{n\in\mathbb{N}} A_n\cos\left(k_{z_n}(d-z)\right)\psi_n(y) \qquad (5)$$
where
$$k_{z_n}^2=k^2-k_{y_n}^2 \quad , \quad k_{y_n}=\frac{n\pi}{2l} \qquad (6)$$
and
$$\psi_n(y)=\sqrt{2-\delta_{n0}}\cos\left(\frac{n\pi}{2l}(y-l)\right), \qquad (7)$$
where δ_{mn} is the Kronecker symbol.

In Ω_2, the solution is written as a continuous sum of functions,
$$\phi=\int dk_y\left(A^{(2)}e^{jk_z z}+B^{(2)}e^{-jk_z z}\right)e^{jk_y y}, \qquad (8)$$
where
$$k_z^2=k^2-k_y^2. \qquad (9)$$

Using solutions (5) and (8), continuity conditions (3) and (4) give
$$\sum_{n\in\mathbb{N}} A_n\cos\left(k_{z_n}d\right)\psi_n(y)=\int dk_y\left(A^{(2)}+B^{(2)}\right)e^{jk_y y}, \qquad (10)$$
and
$$\sum_{n\in\mathbb{N}} A_n k_{z_n}\sin\left(k_{z_n}d\right)\psi_n(y)=\int dk_y jk_z\left(A^{(2)}-B^{(2)}\right)e^{jk_y y}. \qquad (11)$$

The eigenmodes are the non trivial solutions of the system (10-11) with $A^{(2)}=0$. It follows that:
$$A_m\cos(k_{z_m}d)=-j\sum_{n\in\mathbb{N}} k_{z_n}\Pi_{mn}A_n\sin(k_{z_n}d) \qquad (12)$$
where
$$\Pi_{mn}=\int\frac{dk_y}{2\pi}\frac{S_m\bar{S_n}}{k_z}, \qquad (13)$$
with
$$S_m=\int_{-l}^{l}\psi_m(y)e^{-jk_y y}dy. \qquad (14)$$

Finally, Eq.(12) can be written in the matricial form
$$D\vec{C}=\vec{0}, \qquad (15)$$
where
$$D_{mn}=\cot(k_{z_m}d)\delta_{mn}-j\Pi_{mn}k_{z_n}, \qquad (16)$$
and
$$C_n=A_n\sin(k_{z_n}d). \qquad (17)$$

Eigenvalues of the problem are the values k_i of k for which $\det(D)=0$ and eigenfunctions are determined by the $\{A_m^{\{i\}}\}$, solutions of Eqs. (15) and (17). Owing to the radiation losses at the opening, eigenfrequencies are complex, with a negative imaginary part [14].

2.2 Numerical resolution and results

Eigenvalues of the transverse problem are computed by numerically locating the zeros of the determinant of the matrix D in the complex k-plane after truncation at a finite number of functions ψ_n (in the following the 20 first ψ_n will be used). Fig. 3 shows the spectrum for an open rectangular cavity with an aspect ratio $d/2l=1.4$ in the complex k-plane. Eigenvalues are complex owing to radiation losses. The spectrum displays families of eigenvalues corresponding to either symetric (blue circles) or antisymetric modes (red crosses).

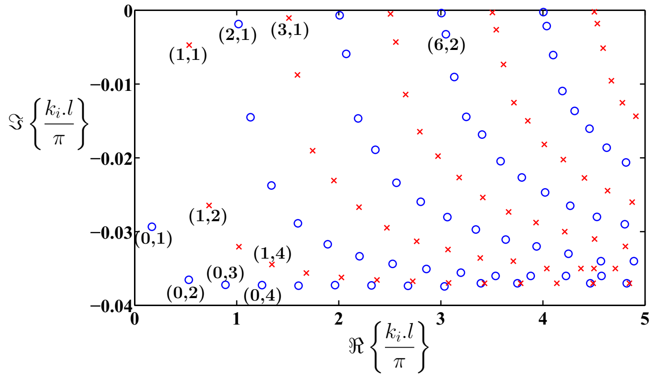

Figure 3: Complex eigenvalues for a rectangular open cavity with an aspect ratio $d/2l=1.4$. Blue circles (o) corresponds to symetric modes and red crosses (x) to antisymetric modes.

Examples of mode shapes are shown in Fig. 4. They are also complex with a non negligible imaginary part. However, they can be easily associated with the real modes $\phi_{(p,q)}$ of the more simple problem with a Dirichlet condition ($\phi=0$) at $z=0$ instead of the "exact" radiating condition. Thus, for convenience, the complex modes $\phi^{\{i\}}$ are labelled in the same way in Figs. 3 and 4.

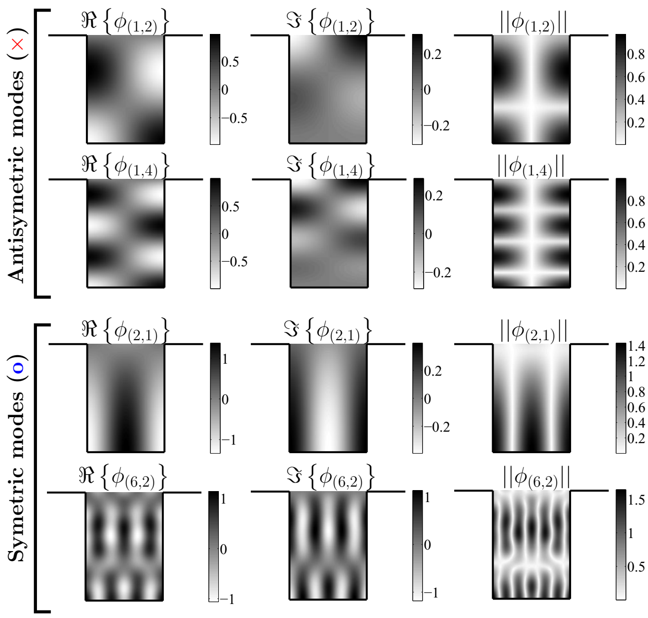

Figure 4: Mode shapes of several leaky modes.

Now that the transverse eigenmodes and their eigenvalues are determined, they can be used to establish a multimodal formulation of the sound propagation within the 3D open waveguide that displays a street canyon (Fig. 1). As we are interested in the sound field propagating inside the waveguide, the $\{\phi^{\{i\}}\}$ in the following are the restriction of the above described modes to the domain Ω_1. They are then

$$\phi^{\{i\}}(y,z) = \sum_{n\in\mathbb{N}} A_n^{\{i\}} \cos\left(k_{z_n}^{\{i\}}(d-z)\right)\psi_n^{(1)}(y) \quad \forall\ (y,z) \in [-l,l]\times[0,d], \qquad (18)$$

with $\left(k_{z_n}^{\{i\}}\right)^2 = k_i^2 - (n\pi/2l)^2$.

3 Propagation along the street

The general solution for the acoustic pressure field in the waveguide is decomposed on the modal basis:

$$p(x,y,z,\omega_0) = \sum_i (a_i e^{jk_x^{\{i\}}x} + b_i e^{-jk_x^{\{i\}}x})\phi^{\{i\}}(y,z), \qquad (19)$$

where $\phi^{\{i\}}$ are the eigenmodes defined in Eq.(18) and $\left(k_x^{\{i\}}\right)^2 = k_0^2 - k_i^2$ with k_0 the source frequency. Since the eigenvalues k_i are complex, each mode in the waveguide is exponentially decreasing. The two sets of modal coefficients $\{a_i\}$ and $\{b_i\}$ are determined as functions of end conditions at the waveguide extremities (source conditions, radiation conditions), taking care that modes $\phi^{\{i\}}$ are not orthogonal [17].

In the following, for simplicity, we will consider the wave field downstream from a source distribution in an infinite waveguide. Then, $b_i = 0$.

The source distribution shown in Fig.5(a) is described as a given pressure field at $x = 0$ with frequency $k_0 l/\pi = 1.2$, and chosen as

$$P_{(in)}(y,z,0) = \sum_{k=1}^{3} \frac{1}{\sigma_k\sqrt{2\pi}}\, e^{\frac{-(y-y_k)^2-(z-z_k)^2}{2\sigma_k^2}}, \qquad (20)$$

where $\sigma_k \in \mathbb{R}^+$ and $(y_k, z_k) \in [-l,l]\times[0,d]$. This input condition is chosen as a non trivial solution for the modal formulation.

Then, the $\{a_i\}$ are deduced from Eq. (20) by a least square method [17]. The modal reconstruction is shown in Fig.5(b). Using the 86 modes $\phi^{\{i\}}$ that have been numerically computed, the input pressure condition is well reproduced.

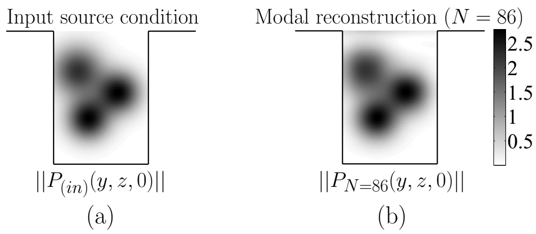

Figure 5: (a) Input pressure field condition at $x = 0$ and (b) modal reconstruction using 86 modes.

Fig. 6 shows an estimation error as a function of the number of modes taken into account defined as

$$\epsilon = \sqrt{\frac{\int_0^d \int_{-l}^{l} ||p_N - P_{(in)}||^2 dydz}{\int_0^d \int_{-l}^{l} ||P_{(in)}||^2 dydz}}, \qquad (21)$$

where p_N is the modal reconstruction with N modes taken into account.

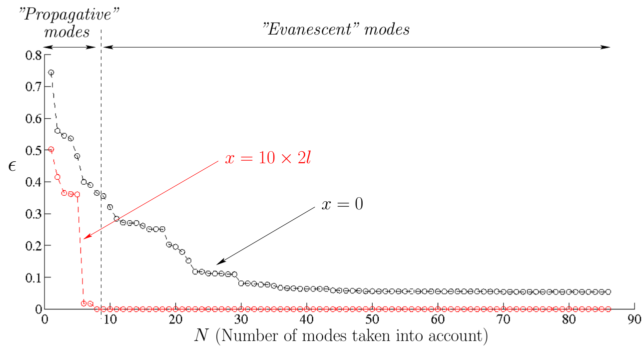

Figure 6: Convergence of the multimodal method at the waveguide entrance (black curve) and at $x = 10\times 2l$ (red curve) .

In Fig. 6, modes are sorted by increasing values of $\Re\{k_i\}$, as it allows to roughly discriminate "propagative" modes (weakly attenuated) and "evanescent" modes (strongly attenuated) at a given frequency (see below). However, there is no doubt that this choice is not optimal for the convergence, as the contribution of the modes can be more or less significant, depending on the source distribution. The error ϵ on the field in the input plane $x = 0$ reaches an asymptotic value ($\epsilon \simeq 0.06$) from $N \simeq 50$. This limitation in the convergence is probably due to the finite number (here, 20) of functions $\psi_n(y)$ considered in the computation of the modes $\phi^{\{i\}}(y,z)$ in Eq.(18).

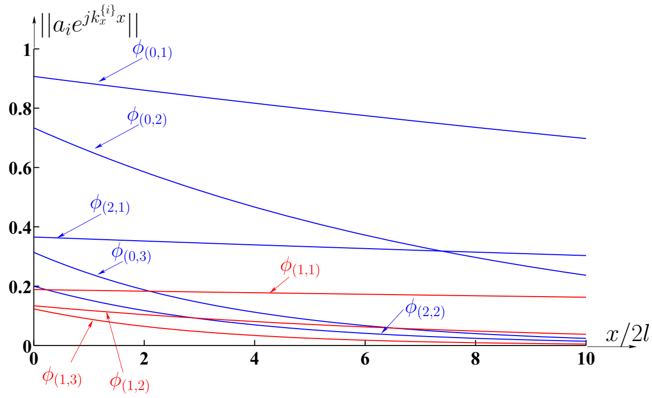

Figure 7: Amplitudes of the 8 "propagative" modes along the waveguide.

The wave field can then be propagated in the waveguide. Far from the input plane $x = 0$, all the "evanescent" modes, naturally, are attenuated and a limited number of modes contribute to the transport of energy. But more interestingly, as seen in Figs. 7 and 8, it appears that among the so-called "propagative" modes, only a few remain significant. These modes contribute strongly to the convergence of ϵ obtained at

$x = 10 \times 2l$ (Fig. 6; in this case, the reference field is the field computed at $x = 10 \times 2l$ with 86 modes). The density of modes in the open waveguide is lower than in a closed - non radiating - waveguide. In general, for an arbitrary source, one can expect the field far from the source to be "carried" by a limited number of the less attenuated modes, giving this modal approach a particular, physical and numerical, interest.

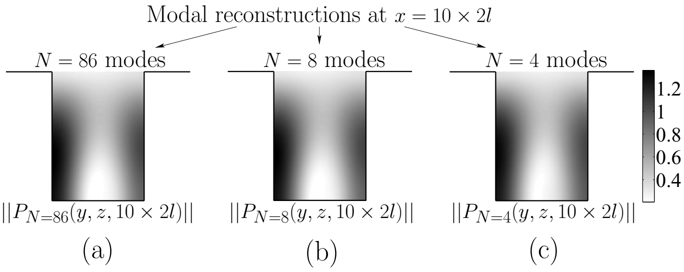

Figure 8: Multimodal fields at $x = 0$ using 86 modes(a), using the 8 "propagative" modes (b) and using the 4 less attenuated "propagative" modes (c).

4 Conclusions

The wave equation in an open uniform waveguide is solved using resonant modes, solutions of the transverse eigenproblem in the open cross-section. Thus, the computation of so called leaky modes is a key point of the method. Leaky modes are complex owing to radiation losses at the waveguide opening. This formulation particularly appears to be much less expensive than usual numerical methods. To obtain more realistic results in the urban street case, works are now in progress to take into account geometrical uniformities in the cross-section or more complex continuity conditions at the extremities as the coupling between two canyons, for example.

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