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Acoustic Computerized Tomography for Temperature Distribution Measurement in Rectangular Space

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In this paper, we propose new acoustic computerized tomography (A-CT) methods based on Radon transform. In these methods, temperature distribution in rectangular space is measured. Temperature distribution measurements are used in various fields, such as, air-conditioning control in offices, temperature control in greenhouses, and many of their measured spaces are rectangular. Ordinary A-CT method conducts pseudo linear scanning and rotational scanning by arranging acoustic transducers at equal intervals on a circle. However, it is difficult to take circular arrangements of the transducers in the rectangular spaces. Thus, we adopt to arrange the transducers on a rectangle. Therefore, a problem that projection data could not be obtained at equal angular intervals occurs. To solve this problem, we propose two methods. One is that the projection data of unequal angular intervals are superposed with weighting with respect to its angular intervals. The other is that the projection data of equal angular intervals are obtained from the projection data of unequal angular intervals by two-dimensional interpolation. We confirm the usefulness of proposed methods by numerical simulation.

1 Introduction

Temperature distribution measurement is used in various fields. Especially, this is important in greenhouses for temperature control [1, 2]. Thus, temperature distribution measurement methods are researched and proposed.

Among these, acoustic computerized tomography (A-CT) method [3, 4] is studied and applied to temperature distribution measurement [5-9]. In many cases, acoustic transducers are located at equal intervals along circumference of measurement space. Times of flight (TOFs) of sound waves between the transducers are measured. Switching the speaker and microphones, pseudo linear scanning and rotational scanning are conducted and projection data are obtained by interpolation to obtain more dense projection data. By use of filtered back projection (FBP) method [10], reciprocal of sound velocity is reconstructed by the projection data, and converted into temperature distribution. The A-CT method has several advantages, including noncontact measurement, the low space occupation, and the measurement capability of large-scale space.

However as mentioned before, many of the examples are used in rectangular spaces. It is difficult to take circular arrangements of the transducers in the rectangular spaces. Additionally, the advantage of the low space occupation is lost. In the rectangular space, it is efficient to arrange the transducers on a rectangle. However, arranging the transducers on a rectangle causes two problems. One is that projection data could not be obtained at equal angular intervals. Inverse transform of Radon transform [11] is essentially integral of rotational direction. However, the projection data of unequal angular intervals could not be directly addressed in order to perform inverse transform. The other is that the numbers of parallel sound paths are greatly different according to the projection angle. The interpolation by few parallel sound paths causes deterioration of reconstructed result.

To solve the former problem, the authors targeted nonuniform sampling theorem [12] and proposed weighted superposition method [9]. The latter problem is tried to solve to set threshold value of the sound paths. To set the threshold value, projection data interpolated by few parallel sound paths and its rotational angles are not used for reconstruction. Although these methods were useful, some sound paths could not be used for reconstruction. Moreover, it is difficult to determine the proper threshold value.

In this paper, we propose a new method. The projection data are ordinarily interpolated in one dimension which is r-axis. In the newly proposed method, projection data of equal angular intervals are obtained by interpolating the projection data of unequal angular intervals in two-dimensional surface which are r-axis and rotational angle θ. By the two-dimensional interpolation, the information of all sound paths are effectively used for reconstruction and no threshold values are necessary.

2 Principles

2.1 Weighted back-projection for unequal angular intervals

Fig.1 illustrates principle of A-CT in a rectangular space. The transducers are assumed to be point sources. Twenty four transducers are located at equal intervals along the circumference of the rectangular space. Aspect ratio of the rectangular space is two. $c(x,y)$ is sound velocity distribution in the rectangular space. Sound velocity outside of the rectangular space is assumed to be c_0. $c(x,y)$ and c_0 are sound velocity in air. An origin of x-y coordinate is located at the center of the rectangular space. In addition, r-s coordinate is assumed, which is rotated θ from the x-y coordi-

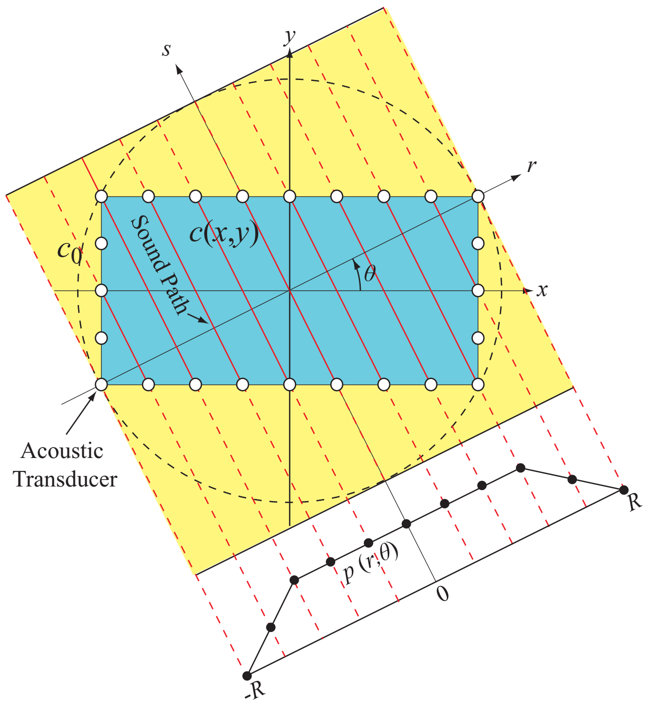

Fig.1 Principle of A-CT in a rectangular space.

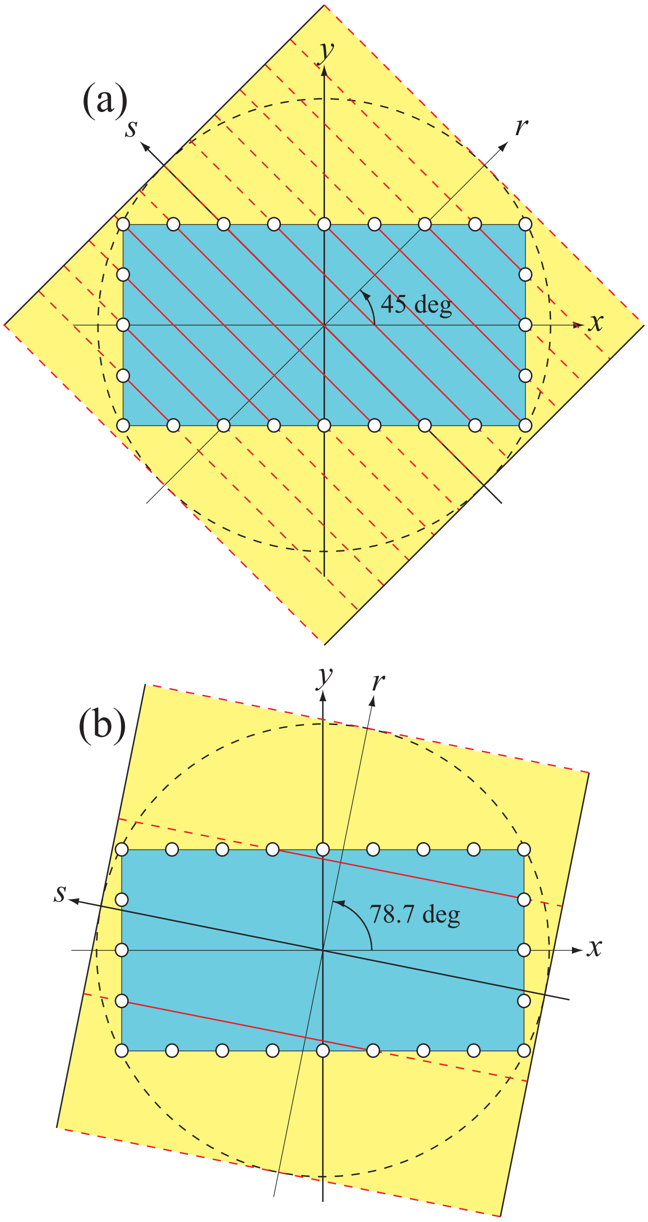

Fig.2 Difference of number of parallel sound paths according to the rotational angle of r-axis θ. (a) θ equals 45 deg. (b) θ equals about 78.7 deg.

nate.

Switching the speaker and microphones, pseudo linear scanning and rotational scanning of Radon transform are conducted and TOFs between the transducers are measured. Assuming virtual speaker array and microphone array which are parallel to the r axis, projection data $p(r,\theta)$ is obtained by converting the TOFs of sound paths parallel to s axis into TOFs between the virtual arrays. Interpolating the projection data, denser projection data are obtained.

However, two problems occur. One is that the numbers of parallel sound paths are greatly different according to θ, as shown in Fig.2. Fig.2(a) shows the example that θ is equal to 45 deg. There are eleven parallel sound paths. However, in the case that θ is equal to about 78.7 deg, there are only two parallel sound paths, as shown in Fig.2(b). Use of the projection data interpolated by few parallel sound paths causes deterioration of reconstructed result. To solve this problem, threshold value of parallel sound paths is set. Setting the threshold value, projection data interpolated by the number that is smaller than the threshold value and its an-

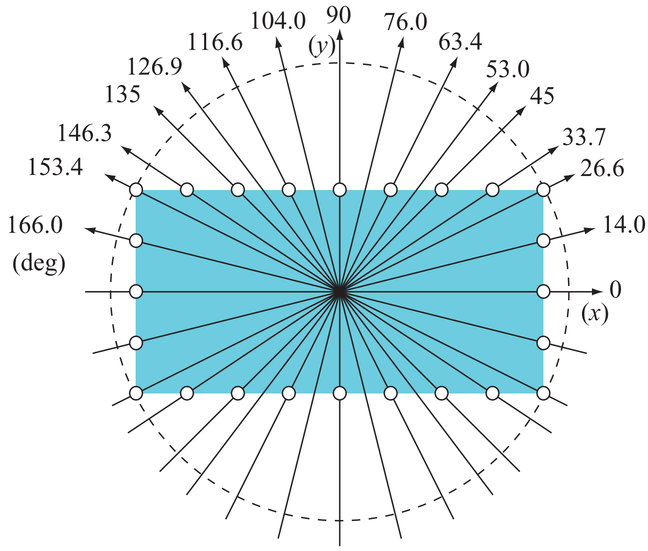

Fig.3 Rotational angles of r-axis when the threshold value is five.

gles of rotational scanning when the threshold value is five. The maximum angular interval is about 14.4 deg, and the minimum angular interval is about 7.1 deg. Thus, dense parts and sparse parts of the rotational direction, at which projection data are obtained, exist. Moreover, inverse transform of Radon transform is essentially integral of rotational direction. However, the projection data of unequal angular intervals could not be directly addressed in order to perform inverse transform in a summation form.

To solve this problem, we target nonuniform sampling theorem and propose weighted back-projection method. The weights are calculated depending on angular intervals. Rotational angles are set as $\theta_1, \theta_2, ..., \theta_n, ..., \theta_N$ (deg). θ_1 is greater than 0 deg, and θ_N is less than 180 deg. Each angular interval $\Delta\theta_n$ is calculated using

$$\Delta\theta_n = \theta_n - \theta_{n-1}. \qquad (1)$$

To calculate $\Delta\theta_1$, θ_0 is required. Because the transducers are located at equal intervals, the process of rotational angles becomes symmetry, as shown in Fig. 3. Thus, θ_0 is set as following equation.

$$\theta_0 = \theta_N - 180 \qquad (2)$$

In addition, the maximum angular interval is set as $\Delta\theta_{max}$. The weight w_n is calculated as following equation.

$$w_n = \frac{\Delta\theta_n - \Delta\theta_{n-1}}{\Delta\theta_{max}} \qquad (3)$$

Using the weights, the influence of the thick parts becomes light and that of the rough part becomes heavy. Reciprocal of sound velocity distribution $f(x,y)$ is calculated by the projection data of unequal angular intervals $p(r,\theta_n)$ with w_n by the following equations.

$$f(x,y) = \frac{\sum_{n=1}^{N} w_n \cdot Q(x\cos\theta_n + y\sin\theta_n, \theta_n)}{\sum_{n=1}^{N} w_n} \qquad (4)$$

$$Q(r,\theta) = \int_{-\infty}^{\infty} G(\rho)|\rho|\exp(j2\pi\rho r)d\rho \qquad (5)$$

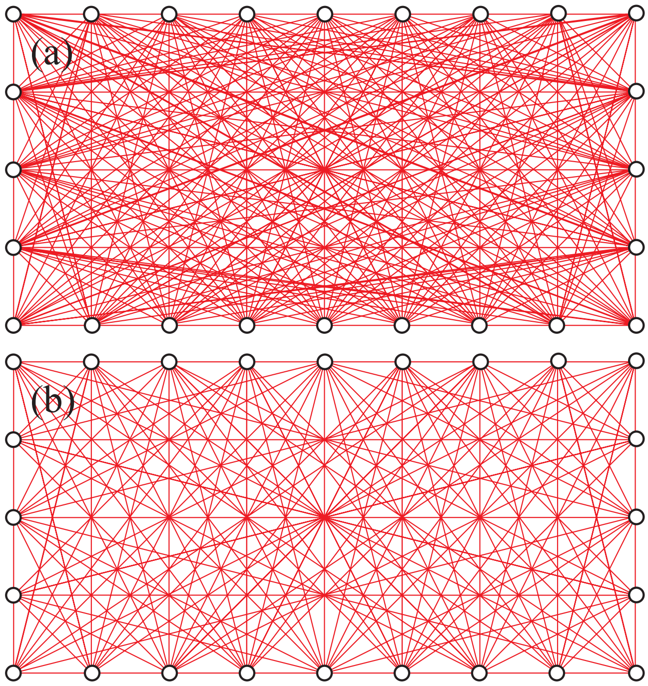

Fig.4 Sound paths in a rectangular space by twenty four acoustic transducers. (a) The threshold value is zero. (b) The threshold value is five.

$$G(\rho) = \int_{-\infty}^{\infty} p(r, \theta_n) \exp(-j2\pi\rho r) dr \quad (6)$$

These are based on filtered back projection (FBP) method.

Converting sound velocity into temperature, temperature distribution is reconstructed from $f(x, y)$ by following equation where c is sound velocity and T is temperature in air.

$$T = \frac{273.15}{331.32^2} c^2 - 273.15 \quad (7)$$

However, these methods have two problems. One is that it is difficult to decide the best effective threshold value for reconstruction. The other is that unused sound paths for reconstruction accrue. Fig.4(a) illustrates all sound paths. The number of the sound paths is 184. Fig.4(b) illustrates the sound paths for reconstruction when the threshold value is five. There are 116 sound paths. Thus, 68 sound paths are not used for reconstruction even though they are available.

2.2 Two-dimensional interpolation in *r*-*θ* surface

To solve the two problems mentioned above, we propose another method. The weighted back-projection interpolates the projection data at *r*-axis, as shown in Fig.1. This is one-dimensional interpolation. The unused sound paths problem is attributable to this interpolation. Thus, the new method interpolates the projection data in two-dimensional *r*-*θ* surface. Interpolating the projection data in *r*-*θ* surface, the threshold value is not required and all sound paths are used for reconstruction. Moreover, the projection data of unequal angular intervals are converted to equal angular intervals projection data.

Fig.5 illustrates principle of the two-dimensional interpola tion for A-CT method. The projection data are mapped to *r*-*θ* surface, as shown in Fig.5(a). Interpolating the *r*-*θ* surface, as shown in Fig.5(b), dense *r*-*θ* surface is obtained. In this paper, we use cubic spline interpolation. From the dense *r*-*θ* surface, projection data of equal angular are obtained.

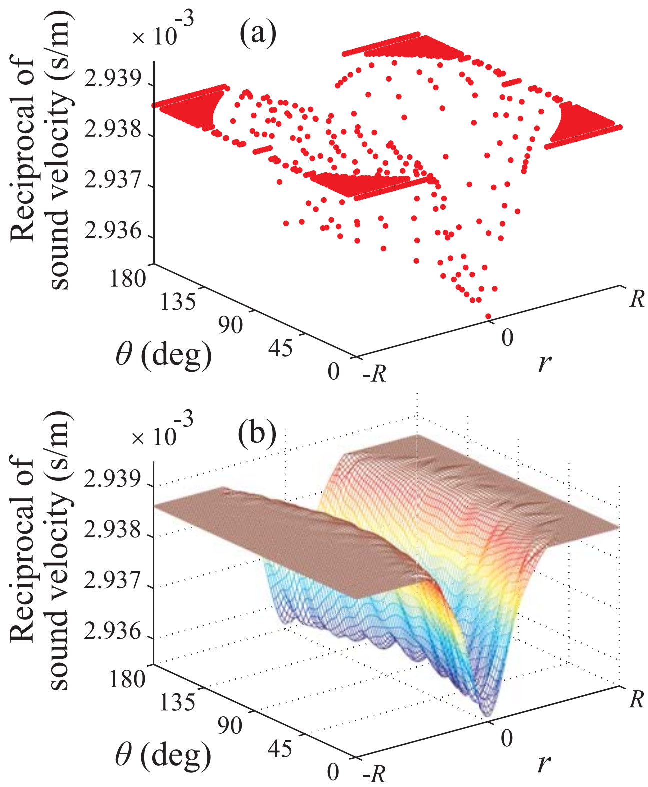

Fig.5 Two-dimensional interpolation of unequal angular intervals projection data at *r*-*θ* surface. (a) Sparse *r*-*θ* surface mapped from projection data. (b) Dense *r*-*θ* surface interpolated from the rough *r*-*θ* surface.

3 Numerical Simulations and Discussions

By numerical simulations, we confirm usefulness of proposed methods. Fig.6 presents original and reconstructed temperature distributions. Twenty four acoustic transducers are assumed to be located at equal intervals along the circumference of a rectangular space, as shown in Fig.6(a). The size of the space is 4.0 × 8.0 m^2, and the interval is 1.0 m. All calculations are performed on the 41 × 81 grid with a size of 0.1 × 0.1 m^2.

Fig.6(a) shows the original distribution. This distribution is given by Gaussian function whose 1/e width is 2.0 m. The base value of the distribution is 15.0 deg C, and the peak value of the distribution is 18.0 deg C and located at (1.0 m, 0.0 m). In the simulation, TOFs are calculated from this distribution integrating the reciprocal of sound velocity along its sound paths.

Fig.6(b) shows the distribution reconstructed from projection data of 45 deg intervals with ordinarily A-CT method. The peak value is about 18.20 deg C and located at (1.0 m, 0.0 m). The shape of the distribution is like octagon because the ordinary A-CT method uses only four projection data.

Fig.6(c) shows the distribution reconstructed by the weighted back-projection whose threshold value is five.

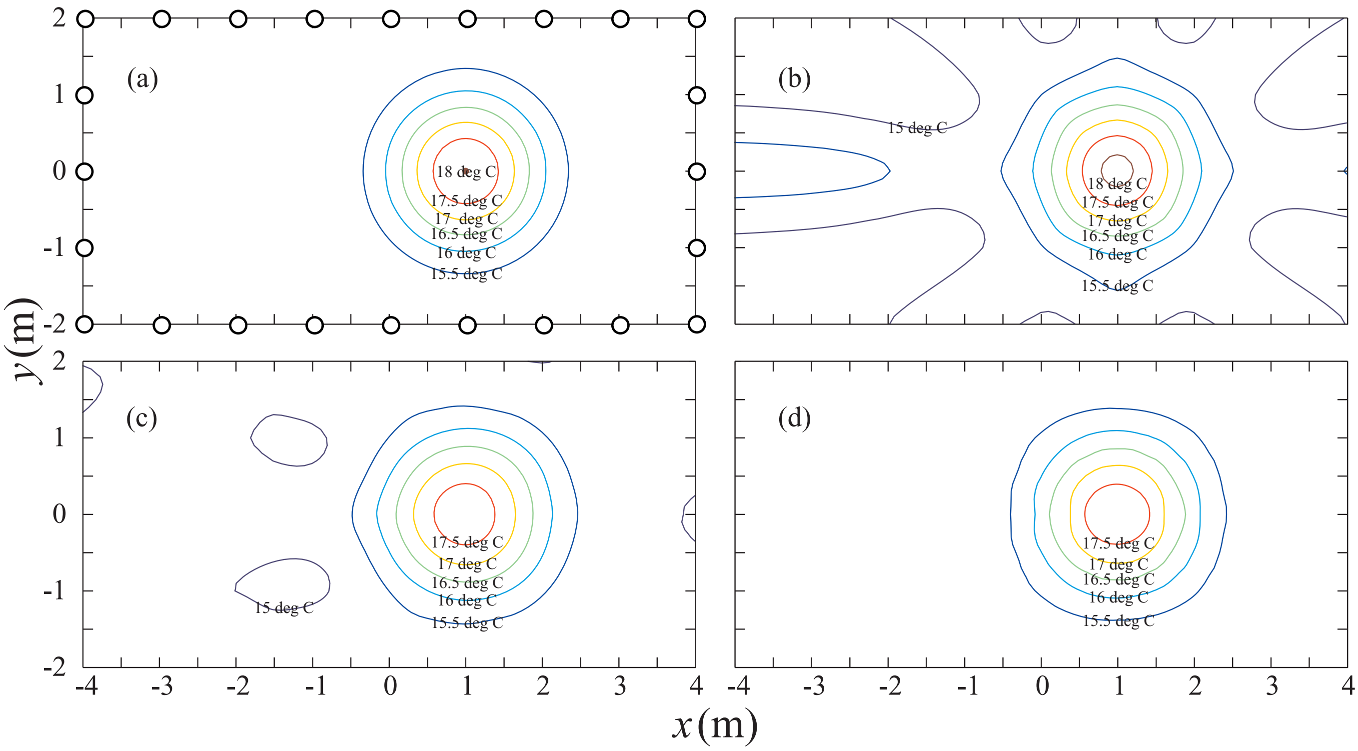

Fig.6 Original and reconstructed temperature distributions. (a) Original distribution. (b) Reconstructed by ordinary A-CT method using projection data of 45 deg intervals. (c) Reconstructed by the weighted superposition A-CT method whose threshold value is five. (d) Reconstructed by the two-dimensional interpolation A-CT method.

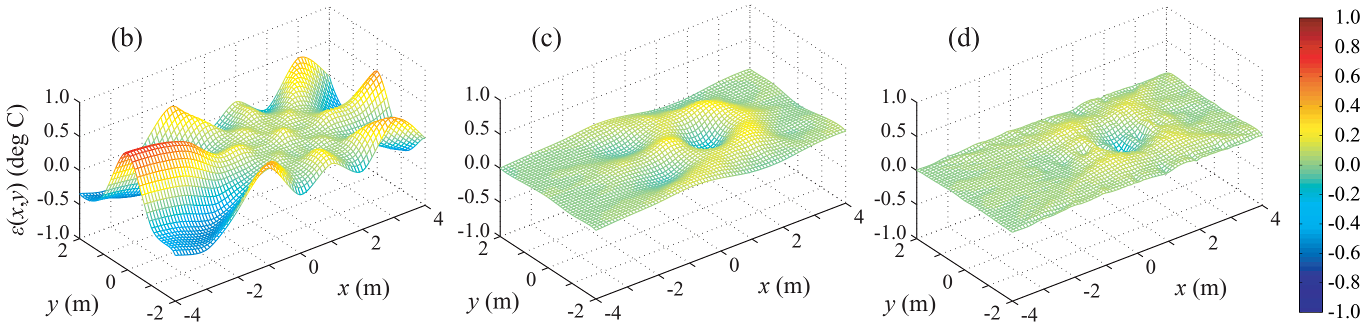

Fig.7 Error of reconstructed distribution to the original distribution $\varepsilon(x,y)$. (b), (c), and (d) correspond to those of Fig.6, respectively.

The peak value is about 17.86 deg C and located at (1.0 m, 0.0 m). Fig.6(d) shows the distribution reconstructed by the two-dimensional interpolation in A-CT method. 180 projetion data whose equal angular interval is 1.0 deg are obtained by the two-dimensional interpolation because twenty four acoustic transducers are used. The peak value is about 17.82 deg C and located at (1.1 m, 0.0 m).

Fig.7 indicates error of reconstructed temperature distribution to the original distribution $\varepsilon(x,y)$ which is calculated using

$$\varepsilon(x,y) = \hat{f}(x,y) - f(x,y). \tag{8}$$

$\hat{f}(x,y)$ is the reconstructed temperature distribution and $f(x,y)$ is the original temperature distribution. In Fig.7, (b), (c), and (d) correspond to those of Fig.6, respectively. RMS error of Fig.7(b) is about 0.236 deg C. The RMS error $\varepsilon_{\mathrm{RMS}}$ is calculated by the following equation.

$$\varepsilon_{\mathrm{RMS}} = \sqrt{\frac{1}{41 \times 81} \sum_x \sum_y \varepsilon(x,y)^2} \tag{9}$$

Fig.7(b) shows that $\varepsilon(x,y)$ in the vicinity of the circumference of the rectangular space are large. Moreover, radiated artifacts appear. These are attributable to that only four projection data are used for reconstruction.

The RMS error of Fig.7(c) is about 0.078 deg C, and that of Fig.7(d) is about 0.066 deg C. Comparing Fig.7(d) with Fig.7(c), the RMS error of Fig.7(d) is smaller than that of Fig.7(c). $\varepsilon(x,y)$ in the vicinity of the location of Gaussian function decrease. In addition, the shape of the Fig.6(d) is more similar to that of the original distribution than that of Fig.6(c). These advantages are owing to the number of projection data. Setting the threshold value as five, sixteen projection data are available for the weighted back-projection.

On the other hand, although 180 projection data are used for the two-dimensional interpolation, the error becomes greater about the peak value in Fig.6(d) than that in Fig.6(c). This is attributable to the two-dimensional interpolation, as shown in Fig.8. Fig.8(a) plots projection data mapped to r-θ surface at θ axis. Because the original distribution is given by Gaussian function, peak of the projection data is constant in principle. This constant peak is shown as the broken lines in Fig.8. Fig.8(b) shows interpolated projection data.

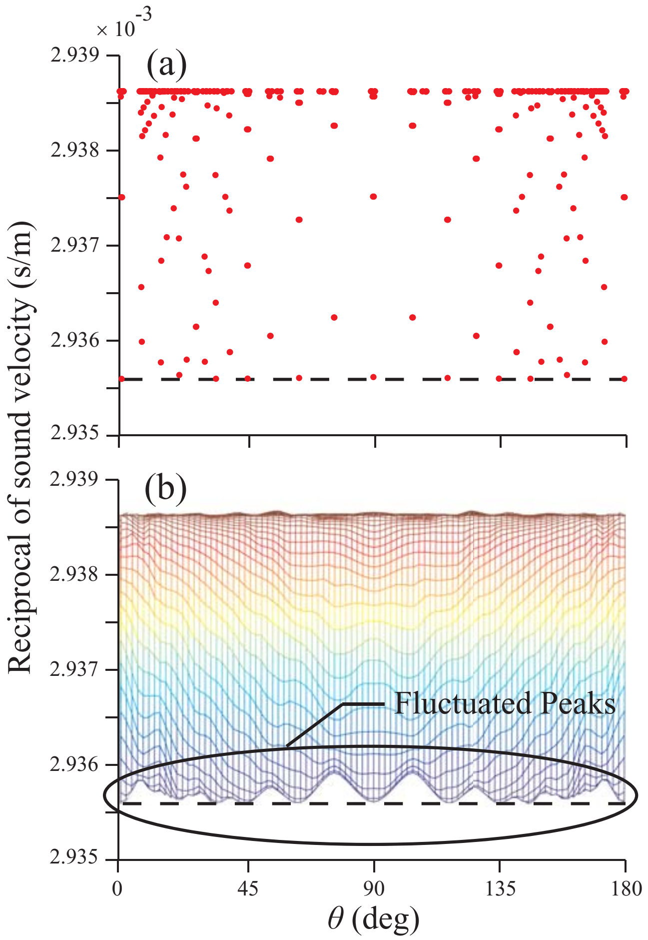

Fig.8 Two dimensional interpolation projection data at θ axis. (a) Measured data. (b) Interpolated data.

The peaks of the projection data are not constant. Thus, the peak value of Fig.8(d) becomes lower.

4 Conclusions

By the numerical simulation, we confirm the usefulness of the proposed methods, weighed back-projection method and two-dimensional interpolation in A-CT method. Comparing the distribution reconstructed by the latter method with that by the former method, the distribution approaches to the original shape and RMS error decreases. However, the peak value error of the distribution becomes large. It is ascertained that this problem is attributable to the two-dimensional interpolation.

As future works, other two-dimensional interpolation methods will be tried. Additionally, experimental verifications will be also scheduled.

Acknowledgments

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