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Variance in predicted structure borne sound power due to simplified characterisation of the source

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Abstract

Theoretically the structure borne sound power transmission from a vibration source to a receiving structure can be predicted using the mobility method. In order for all the power transmission mechanisms to be accounted for the source and receiver mobility matrices must contain the mobilities for all degrees of freedom (three translational and three rotational) for each contact point and the transfer mobilities between; each contact point, each degree of freedom and each degree of freedom at each contact point. In practice reliable measured data for some degrees of freedom is difficult to obtain, so simplified source characterisation methods are required, for example using only certain degrees of freedom. The novel contribution of this paper is to investigate the variance in the predicted structure borne sound power transmission calculated when some degrees of freedom are missing, and when the source is approximated as a single equivalent excitation. The investigation was carried out using numerical simulations of simplified source and receiver structures.

1 Introduction

In acoustics Sound Power can be used as an independent characterisation of a machines ability to deliver acoustical power to its surrounding environment. In structure borne sound the power delivered by a source into a receiver is a function of both the source and receiver, therefore the input power must be calculated from knowledge of the dynamic responses of the source and receiving structure and the free velocity or blocked force of the source. This is not straightforward as in order to account for all power transmission paths a full description of the source and receiver interaction is required, thus, the responses of the source and receiver need to be described for each degree of freedom (3 translational, 3 rotational) at each contact point.

In practice mobility and source activity can be difficult or impossible to measure reliably and efficiently [1]. For these reasons practitioners may often only have a coarse description of the real system, for example if a source and or receiver are described by only the translational mobilities at the contact points.

Calculating the structure borne sound power transmitted from source to receiver using this coarse data, results in an erroneous estimate as not all power transmission paths have been accounted for. This coarseness in the description is here on referred to as granularity. The more granular a data set is, the less information there is. The least granular system is one where data for all degrees of freedom at and between all contact points exists. A highly granular system is one where the system is described by a frequency dependent single equivalent figure.

There are numerous difficulties to consider in this investigation of granularity and its effect on the prediction of structure borne sound power. The severity of the error in the power due to a specific type of granularity is case specific, due to varying significance of different degrees of freedom in the source activity and mobilities. Also numerous cases are possible as a large number of source/receiver combinations can occur in practice.

A simulation tool based upon Monte Carlo methods has been developed in order to investigate this granularity problem. A Monte Carlo method is used to examine the validity of an analytical estimate of the variance experienced in the structure borne sound power prediction due to a specific type of granularity.

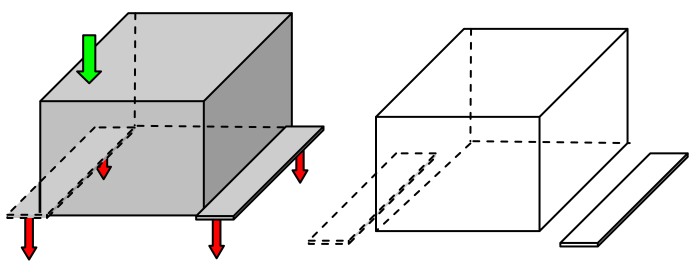
2 Investigation approach

2.1 Idealised structure borne sound source model

An approach based upon Monte Carlo methods has been developed, in order to investigate the errors in estimated structure borne sound power due to granular representations of the source-receiver data. The approach uses idealised analytical source and receiver models, which allow the power to be calculated using an exact, clean (no measurement noise etc) representation of the source and receiver for varying degrees of granularity.

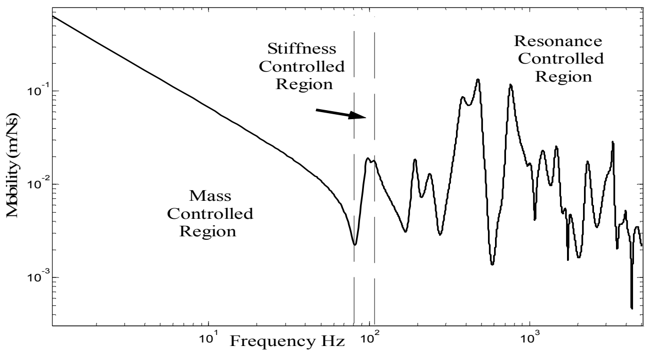
For the idealised receivers analytical models of plates of various boundary conditions [2] are used as plates represent a typical structure borne sound receiver.

Structure borne sound sources come in a variety of shapes and sizes and are more than often complex built up structures. Generally speaking a source can be seen to exhibit some characteristic mobility behaviour [3. This behaviour falls into 3 main frequency regions, mass controlled, stiffness controlled and resonance controlled. These three regions occur from low to high frequency respectively. Idealised source models that exhibit these characteristics have been created by sub structuring individual components to build up a complex system.



,Fig.1 Idealised Structure borne sound source, and sub structuring components, mass and two free plates, with applied force (green) and resultant blocked forces/free velocities (red)

Numerous source architectures have been modelled, in this paper a source based upon a mass with two flanges will be discussed. The flanges were created using analytical free plate models [2]. The source is assembled using the principles of continuity and equilibrium to connect each plate to the mass. The structural mobility for each degree of freedom, of the connected system is then calculated at two positions on each plate, these positions are considered the connection points of the source and represent the interface by which the source is connected to the receiver. The translational point mobility at a contact point of the assembled source is shown in Fig.2.



.Fig.2 Point translational mobility of idealised source, displaying characteristic mobility behaviour [3]

For each source architecture a set of source activities has been produced. This was done by applying a unit force to a point on the mass and calculating the resultant blocked forces and free velocities that occur at the source's contact points, see Fig.1. By varying the point of application of the force, numerous different sets of blocked forces and free velocities can be calculated. This creates an ensemble of sources which are structurally identical but differ in their activity, and which exhibit varying significance of degrees of freedom in terms of the excitation behaviour at the contact points.

Using an ensemble of sources allows us to investigate a type of granularity for numerous cases and allows validation of analytical estimates for the variance associated with a certain degree of granularity.

2.2 Analytical prediction for variance

An analytical prediction of the variance associated with a certain degree of granularity has been investigated. The investigation is limited to the simplified but nonetheless important case in which only translational forces and mobilities are considered and only four contact points exist. This simplification was adopted to allow the development of the methodology, however the method will enable other criterion to be taken into account at later stage.

For a force source case, whereby the source mobility is much higher than the receiver mobility, the real power transmitted from the source to receiver can be calculated using [4]:

$$P = F^{H}\operatorname{Re}(Y_R)F \qquad (1)$$

Where P is the complex transmitted power, F is a vector representing the blocked forces at the contact points of the source, and Y_R is a matrix of point and transfer mobilities for each contact point on the receiver, and the superscript H means conjugate transpose. Eq.(1) can be expanded in terms of the contribution to the power from the point and transfer mobilities:

$$P = \frac{1}{2}\sum_{i=i}^{N}|F_i|^2\operatorname{Re}(Y_{ii}) + \frac{1}{2}\sum_{i=i}^{N}\sum_{\substack{i=k\\k\neq i}}^{N}|F_i||F_k|\operatorname{Re}(Y_{ik})\cos\phi_{ik} \qquad (2)$$

Where Y_{ii} represents the receiver point mobility at connection point *i*, Y_{ik} represents the transfer mobility between point *i* and *k*. Φ_{ik} is the relative phase between point *i* and *k* and is given by:

$$\phi_{ik} = (\phi_{Fi} - \phi_{Fk}) \qquad (3)$$

Where Φ_{Fi} and Φ_{Fk} are the phases at point *i* and *k* respectively.

The investigated granularity case is one where the phases of the blocked forces are unknown. This can occur in practice where the forces are measured/calculated separately and an estimate for the phases is unattainable. Also, this may occur if one is expanding from a single equivalent source to a multipoint formulation, where the magnitude ratios of the forces is known but phase relationships are not. An equation for predicting the variance in the predicted power due to lack of knowledge of the force phase distribution is derived below.

An assumption about the statistical distribution of the relative phase of the forces, (Φ_{ik}) is required. In the high frequency or quasi infinite region it is reasonable to assume that the force phase difference can take any value between π and $-\pi$ with a uniform distribution [5]. Assuming all other variables in Eq.(2) are constant and averaging over the ensemble, the cross terms average out and the mean power can be written as:

$$E[P] = \frac{1}{2}\sum_{i=i}^{N}|F_i|^2\operatorname{Re}(Y_{ii}) \qquad (4)$$

The definition of variance is given as:

$$Var[P] = E\left[(P - E[P])^2\right] \qquad (5)$$

substituting Eq.(2) and Eq.(4) into Eq.(5) yields:

$$Var[P] = \frac{1}{4}E\left[\sum_{i=i}^{N}\sum_{\substack{i=k\\k\neq i}}^{N}|F_i|^2|F_k|^2\operatorname{Re}(Y_{ik})^2\int_{-\pi}^{\pi}\cos^2\phi_{ik}\,d\phi\right] \qquad (6)$$

evaluating the integral Eq.(6) is reduced to:

$$Var[P] = \frac{1}{4}E\left[\sum_{i=i}^{N}\sum_{\substack{k=i\\i\neq k}}^{N}(|F_i||F_k|\operatorname{Re}(Y_{ik}))^2\,\pi\right] \qquad (7)$$

Where the average E[..] denotes an average over N^2-N.

2.3 Comparison of calculated Powers and deviations

An investigation was carried out where the power injected into the receiver was calculated using Eq.(1) and was estimated using Eq.(4). The discrepancies between the two power formulations are hoped to fall within the deviation found from the variance predicted using Eq.(7). The four contact point source model described in Section 2.1 was used to calculate the power transmitted into a simply supported plate. The power transmitted calculated using

Eq.(1) is will be referred to as the true power P_{true}, and represents the power transmitted from source to receiver when all the (translational) degrees of freedom and force phases at each contact point are accounted for. An estimate of the power when the force phases are not known is calculated for the same source and receiver combination using Eq.(4); this power is referred to as E[P]. A predicted deviation δ_{pred} was obtained using Eq.(7) from:

$$\delta_{pred} = \sqrt{Var[P]} \qquad (8)$$

δ_{pred}, E[P] and P_{true} are calculated for each set of the ensemble such that the validity of Eq.(7) can be examined for a range of realistic force phase distributions. The results from this investigation are reported in the following section.

3 Results

The results of the investigation are presented in this section and a discussion of these results will ensue in the following section. In order for Eq.(1 ,4, 7) to be valid a force source scenario ($|Y_S|>>|Y_R|$) is required. Fig.3 shows that the mobility of the idealised source model used, is much higher than that of the simply supported plate receiver, at all frequencies.

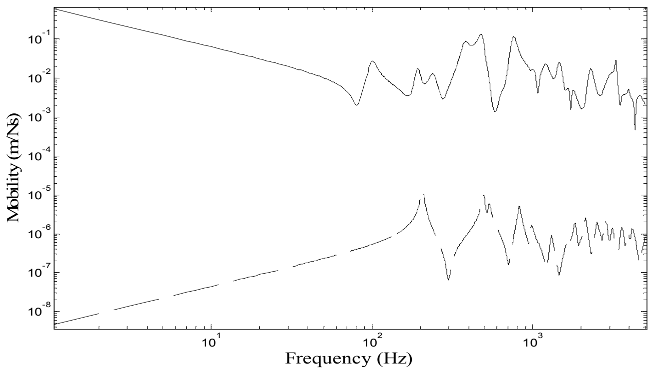


Fig.3 Point translational mobility of idealised source (solid line), point translational mobility of simply supported receiver plate (dashed line).

For a single case of the ensemble, δ_{pred}, E[P] and P_{true} are calculated. Fig.4 shows E[P], E[P]-δ_{pred}, E[P]+δ_{pred} and P_{true}.

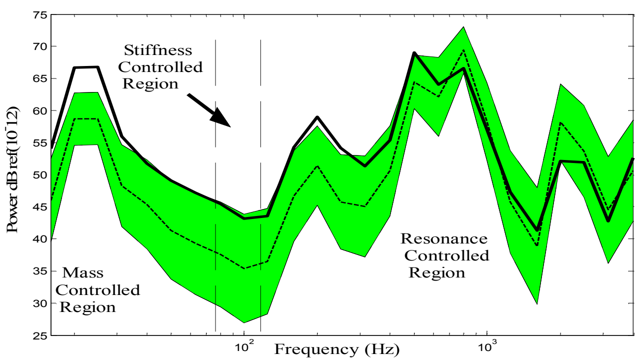


Fig.4 Single case P_{true} (thick line), E[P] (dashed line)], E[P]-δ_{pred}, E[P]+δ_{pred} (lines on either side of green band), all shown with frequency region indications.

Due to the different force magnitudes and phase ratios across the ensemble, E[P], E[P]-δ_{pred}, E[P]+δ_{pred} and P_{true} are different for each member of the ensemble. In order to investigate the effectiveness of the estimated power and the deviation prediction, the individual predictions have to be normalised to a property of the individual case of the ensemble. It was chosen to normalise all the parameters to the estimated power E[P], this was chosen as in practice the true power would not be known. Fig.5 shows the normalised true powers E[P], and the average (over ensemble) normalised E[P]-δ_{pred}, E[P]+δ_{pred}, for the 64 cases in the ensemble.

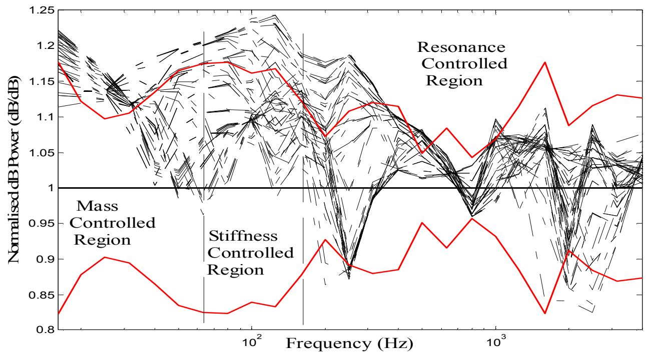


Fig.5 Normalised P_{true}s (dashed lines), normalised E[P] (solid line)], normalised E[P]-δ_{pred}, normalised E[P]+δ_{pred} (red lines), all shown with frequency region indications.

To aid in the discussion of this investigation Fig.6 has been produced. For each case in the ensemble the phase difference between the forces for each pair of feet was calculated, at each frequency a histogram was produced whereby the number of occurrences of a particular phase difference is logged. Fig.6 shows the histograms for all frequencies, the black areas represent more than 20 (out of a possible 384 phase differences) occurrences of that phase difference, and conversely white shows no occurrences of that particular phase difference.

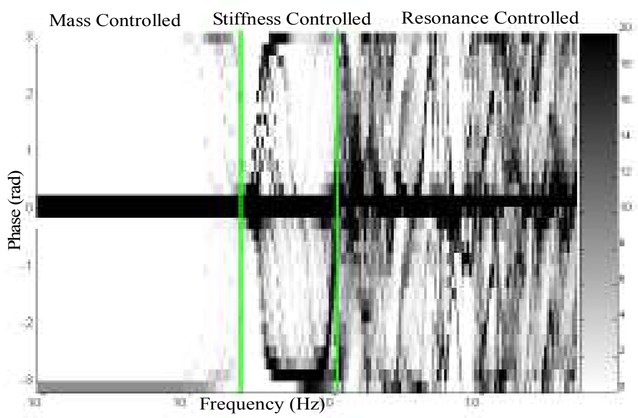


Fig.6 Occurrences of phase differences between all points, for all cases of ensemble, with frequency region indications.

The following section offers a discussion and critique of the results obtained here.

4 Discussion

The figure of most interest to this discussion is Fig.5 as it show how effective the variance and hence deviation prediction is. Its observed that for the a large part of the mass controlled region the deviation of the true power from the estimated power E[P] is large and is not predicted by the estimated deviation. By studying Fig.6 it can bee seen from the clustering of the dark regions in the mass controlled region that the assumption of a uniform phase distribution is invalid at lower frequencies. In the mass controlled region one would expect the phases of all the contacts to be equal [5] as the source behaves as a rigid body. It can be concluded that the mean power estimate and the variance estimate is invalid in the mass controlled region due to the invalidity of the uniform phase distribution assumption.

Similar discrepancies in Fig.5 are observed in the stiffness controlled region, and observing the uneven distribution of the dark patches in the stiffness controlled region of Fig.6, the discrepancies are again attributed to the invalidity of the uniform phase distribution assumption. In this frequency region one would be expect to observe large global resonances such as rocking modes [3].

Over the majority of the displayed resonance region in Fig.5 the calculated true powers fall within predicted deviation limits. Again referring to Fig.6, it can be seen in the resonance controlled region that the distribution of light to dark patches becomes more uniform and hence the phase distribution becomes more uniform. The discrepancies that occur at the start of the resonance controlled region can be attributed to the slightly uneven phase distributions observed at the corresponding region in Fig.6. This uneven distribution could be attributed to modes that occur within the flanges causing connection points on the same flange to vibrate in phase; however it is difficult to speculate as to the cause of this uneven phase distribution.

Improvements in the variance prediction could be achieved by using different phase assumptions in different frequency regimes. It is believed that an equal phase assumption will be more valid in the mass controlled region and work is being carried out by the authors to investigate this. An issue arises of how to define the frequency limits for each frequency regime, this is also an issue for further study.

5 Concluding remarks

The discussed investigation has shown that in the resonance controlled region there is a possibility to estimate the power and the deviation (from true power) in the power transmitted, for a force source scenario, where knowledge of the phase distributions of the forces is unknown.
The limitations of these predictions have been discussed and the causes of the discrepancies for other frequency regimes identified. It is believed that using different force phase distribution assumptions (such as equal phase in the mass region) could yield expressions that are valid for the other frequency regimes.

Acknowledgments

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References

[1] M.A. Sanderson, C.R. Fredo, "Direct Measurement of Moment Mobility",*Journal of Sound and Vibration.* 179(4), 669-684 (1995)

[2] P. Gardonio, M.J. Brennan, "Mobility and Impedance Methods in Structural Dynamics" *Advanced Applications in Acoustics, Noise and Vibration, e.d F.Fahy, J.Walke,* (Spon Press)

[3] R.A. Fulford, B.M. Gibbs, "Structure-Borne Sound Power and Source Characterization in Multi-Point-Connected Systems, Part 2: About Mobility functions and free velocities", *Journal of Sound and Vibration.* 220(2), 203-224 (1999)

[4] L. Cremer, M. Heckl, B.A.T Petersson, *Structure Borne Sound,* Springer (2004), Third Edition.

[5] R.A. Fulford, B.M. Gibbs, "Structure-Borne Sound Power and Source Characterization in Multi-Point-Connected Systems, Part 3: Force Ratio Estimates", *Journal of Sound and Vibration.* 225(2), 239-282 (1999)