Particular analysis of vibration energy transmission in connections of thin plates

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Analyses of energy balance in the mechanical systems are done by the quantitative and qualitative assessment. Qualitative assessment is possibly by the use of energy flow descriptors and observation of energy density distribution in the mechanical structure. The work presents the results of investigation of welded connections of rectangular steel plates with ribs. The aim of the analysis was the quantitative estimation of vibration energy transmitted though the welded connection of ribbed plates. The structural intensity was used as the parameter for the analysis. Method of structural intensity evaluation enabled the elaboration of intensity vector field. Unification of shape and size of finite elements made possible the summation of only the magnitude of intensity in the plate's cross sections far from the places of excitation and the damping. From obtained values the detailed information on vibration energy flow was achieved. The obtained results of calculation give the quantitative information on amount of energy transmitted, reflected and the damped in welded joints of plates. The analyzed case was intentional to show the utility of intensity method in diagnostics of joints in mechanical constructions. The calculation results were verified experimentally with application of thermograph measurements of stress and vibration velocity.

1 Introduction

The quantity of structural intensity was introduced with the purpose of extension of description of acoustic phenomena with the application of vector fields. It has found the exceptional application in investigations of vibrational energy flow in deformable elastic bodies. Structural intensity is an analogue quantity to the sound intensity and represents the same physical value – the averaged in time net mechanical energy flow through the unit area perpendicular to the direction of flow [9, 10]. Because of the measurement possibilities the analysis is conducted in the thin region close to the surface. The analysis in deeper parts of constructional element is out of practical meaning due mainly to their thin-wall nature. The analysis of spatial distribution of structural intensity vector fields enables determination and location of paths, sources and sinks of energy of vibrations in application to the mechanical systems. It gives the particular information on the streams of energy flow which is much advantageous than the other methods used earlier to such kind of analysis.

There were delivered the basic formulations and relationships [4, 5, 6] allowing to calculate the components of structural intensity vectors for simple constructional elements such as beams, plates, shells and pipes. Development of the intensity methods in application to the rigid bodies' analysis was started in the middle of seventies of last century. As the result there were elaborated the two main methods of measurements: simplified – using the definitional dependencies [1, 2, 5, 6] and exact based on the wave theory of acoustic field. The used measurement methods have still disadvantages in their application to the practical cases. They are the compromise between the measurement exactitude and simplicity of the measuring system meaning the number of measuring points. Structural intensity can not be measured directly. Methods for measurement use deflections (strains) instead of stress components.

The significant development of the structural intensity applications was noted in the second half of the eighties and beginning of nineties when have appeared works showing the possibility of numerical calculation of structural intensity [5, 6]. The basic dependencies were based on assumption of one or two dimensional structure as beam or plate. The numerical approach was based on the modal model. The possibility of computational analysis of structural intensity is very promising due to the prospects of use of already elaborated finite element models of mechanical structures. The practical examination on the computational way of vibrational energy flow in complex structures consisting of the beam, plates and shells haven't been solved satisfactory till now. The main problems are found in necessity of consideration of complex boundary conditions and complexity of the analyzed real structures.

2 Calculations of structural intensity components – modal approach

Instantaneous value of real part of structural intensity $i_k(t)$ is time dependent vector quantity equal to the change of energy density in the infinitively small volume [9]. Its *k-th* component is given by the equation:

$$i_k(t) = -\sigma_{kl}(t) v_l(t), \quad l = 1, 2, 3, \tag{1}$$

where $v_l(t)$ is the *l-th* component of velocity vector and $\sigma_{kl}(t)$ is *kl-th* component of stress tensor.

Averaged in time value of (1) represents the net energy flow in mechanical structure [9]

$$I_k = \langle i_k(t) \rangle \tag{2}$$

in the direction of *k-th* coordinate of rectangular frame of reference corresponding to analyzed constructional element. Values of structural intensity by the definition depend on particle velocity and stress. Components of structural intensity vector can be calculated for the beams, plates and shells as the functions of variables: bending and twisting moments, shear forces, linear and angular displacements. This approach significantly differs from the measurement methods.

Complex displacements and stresses needed for structural intensity evaluation can be obtained by the modal approach. [5, 6] The software used in computation with finite element method in most cases uses the real stiffness and mass matrices. As a result of calculation are obtained the real displacements and stresses. The procedure of structural intensity calculation is based on complex response of the structure with the modal representation of structure without the dissipation [8]. The damping is considered in two forms. The structural internal damping of the structure is taken into consideration as modal damping. The additional damping (energy absorber, sink), placed in known location is treated as external loading.

The complex response of the structure is obtained from the calculations done with the program operating on values obtained from classical FEM model in purpose to compute the complex modal response in modal coordinates. For the purpose of structural intensity calculations was elaborated the own program written in the MathLab environment [1, 2]. Program provides the calculations as well the presentation of the results in graphical form.

3 Rectangular ribbed plate

For the purpose of structural intensity usability verification in application to the identification of vibration energy flow in connections of thin plates the numerical experiment was performed. The model chosen for analysis was three dimensional structure of rectangular ribbed plate convenient for numerical modeling of structural intensity and experimental verification.

In the calculations there was applied elaborated program for calculations of complex modal model allowing the consideration of additional localised damping in the system.

3.1 Results of calculations

The first case was the simply supported plate with ribs. It has mechanical properties as structural steel and dimensions of 1.5m in width and 3m in length with thickness of 10^{-2}m. The FE model was prepared using the NASTRAN software. Plate was divided into 980 square elements of QUAD4 type. The harmonic excitation force attached to the plate in place indicated on figures by the star and damping force proportional to the velocity of vibration was attached to the plate in place indicated on figures by the triangle. The magnitude of damping force was set to 10^3 Ns/m. The direction of its action was chosen perpendicularly to the plane of plate. There was chosen the model of simply supported plate. The only feasible motion was the rotation around the edges of the plate. There were eliminated the translation motions in any direction edges of plate. Such model was in accordance with the most technical cases of plate element mounting in practice. The calculation has been done for 100 first mode shapes. In cases of low number of mode shapes there were observed significant changes in distribution of vectors for the same density of net of elements. The positions of excitation and damping forces attachments were clearly shown by the structural intensity vectors distribution. It was assumed that for the number greater than 60 mode shapes the numerical model of plate is exact and in a proper way represents the vibration energy flow in the system. The magnitude of structural intensity vectors get smaller with increase of number of finite elements. This inclined to the conclusion that in energy flow analysis it is not sufficient to observe only the intensity vectors. The better measure of energy flow seamed to be the total energy flow through the closed area around the places of excitation and damping or through the whole width of the plate. For that reason the method of summation of structural intensity vector magnitude along the path surrounding the source or through the whole cross section of the element should be applied. Significant changes were observed for the area around the excitation and sink. This is due to the changes of stress values in the region of point force attachment. For the other cross sections there were observed only slight change in total energy values. The obtained results of calculations done for ribbed rectangular plate are shown on Figures 1 and 2.

The main target of the analysis of the rectangular plate with ribs was the testing the distribution of structural intensity vectors especially in the regions of ribs attachment. For the comparison of results there is presented distribution of intensity vectors for plane rectangular plate of the same dimensions.

Obtained distribution of intensity vectors for ribbed plate is significantly different form result got for plate without ribs. Distributions of intensity vectors in places near by the places of excitation and damping are similar for both cases of plates. The positions of excitation and damping forces attachments were clearly shown. However in all cases the distribution of intensity vectors in the middle part has similar shape. The distribution of vectors has shown distinctly the direction of energy flow from the excitation to the damper. For the plates there were observed the vortices of vectors field in the region far from excitation and damping. Comparing obtained for the rectangular plates results of calculations presented here as the distribution of structural intensity vectors over the area of plate one can notice that there is no similarity with the distribution of displacements for each mode shape. This conclusion results from the fact that mode shapes are connected to the standing waves formed in the plate

In the system with small internal damping there is not observed the energy flow or at least it is very low. The effect of energy flow between the structure elements is observed only for systems with high internal damping caused e.g. by the local abrupt changes of properties or localized damping force.

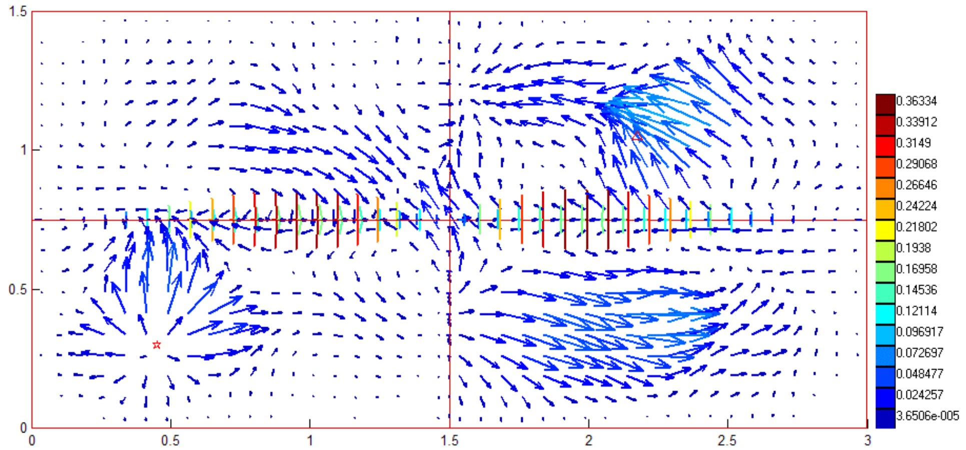

Fig.1 Distribution of structural intensity vectors for simply supported ribbed rectangular plate. Excitation frequency 52 Hz.

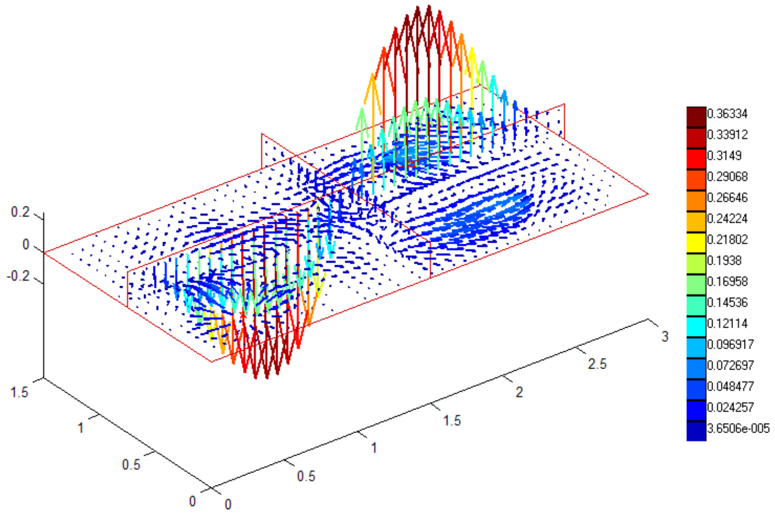

Fig.2 Distribution of structural intensity vectors for simply supported ribbed rectangular plate. Excitation frequency 52 Hz.

Different cases of damper location on plates were analyzed with different distance to the excitation on the horizontal plate. The distribution of structural intensity vectors on the set of plates strongly depends on mutual location of damper and excitation. The vibration energy flow is not straight but has shown the character of circulation. Passing through the edge connecting plates the structural intensity vectors focus in some places mostly the middle part of the connection. The similar behavior has been observed for ribs. The vibration energy flow described by the distribution of intensity vectors shown the flow also in the plate without the excitation and damper (the side way of energy transfer). The highest values of structural intensity are found in the middle part of plates due to their flexibility. There is very small vibration energy flow at the supported plate edges. Outside of the regions of excitation and damper location there is observed ordered and almost parallel, steady (regarding the vector orientation and magnitude) direction of intensity vectors.

4 Experimental verification

4.1 Non-contact measurements of stresses

Infrared thermography can be used for remote and non-contact detection of defects on the surface of materials and structures [1, 2].

When a solid material is rapidly stressed by external or internal load and adiabatically deformed, the variation in temperature occurs in the same way. When a solid material is under tensile loads, the temperature of the material is decreased in proportion to the tensile loads. When a solid material is under compressive loads, contrarily, the temperature of the material increases in proportion of the compressive loads. This is known as a thermoelastic effect.

Measurement system is only effective on the assumption that the adiabatic conditions for lock-in thermography are satisfied. The adiabatic conditions for a thermoelastic analysis are obtained by applying periodic loads (> 3 Hz) to a structure. In this case, the dynamic equilibrium (reversible state) is maintained between the mechanical and thermal forms of energy.

The study was conducted to measure the distribution of stress in a specimen using an infrared thermographic camera, which measures the fine variations in temperature, and applying the lock-in thermography method according to the principle of thermoelasticity.

Fig.3 Measurement system with infrared thermographic camera for lock-in-termography

Fig.4 Ribbed plate under investigation.

A camera system for non-contact imaging of stresses in materials and structures and for damage evaluation was used (Fig. 3). Based upon a high performance focal plane array camera and digital image processing software the Altair LI (Cedip) system produces high quality images of stress field in materials and structures under dynamic loading conditions. It provides full field stress images in real time by using the thermoelastic effect which states there is a linear relationship between the temperature changes induced by loading and the stress at the material surface. The required thermal resolution to achieve a resolution of 1MPa depends on the material properties. It is typically equal to 1 mK for steel.

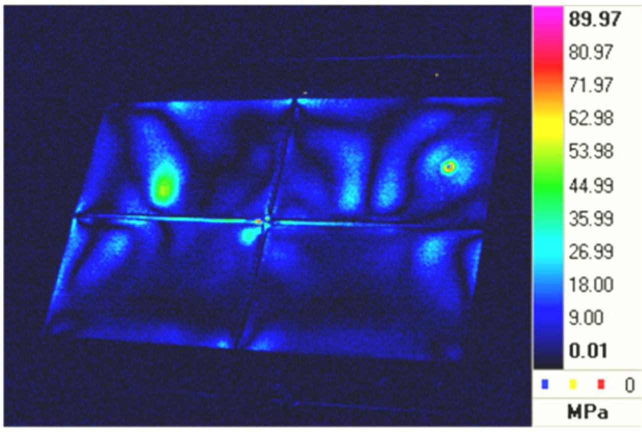

Fig.5 Results of non-contact stress distribution measurement with lock-in thermography. Excitation frequency 52,54 Hz

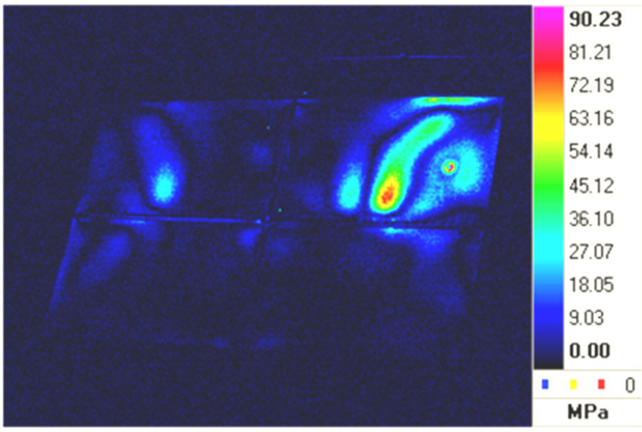

Fig.6 Results of non-contact stress distribution measurement with lock-in thermography. Excitation frequency 85,06 Hz

5 Conclusions

Obtained results of calculations of intensity vectors distribution for rectangular plate with two perpendicular ribs allows for formulating following conclusions:

- In region far from plate and ribs connections, external damping or excitation the directions of intensity vectors are ordered, parallel and steady regarding the vector direction and magnitude. It means steady regular flow of vibration energy in one direction defined by the sense of vectors.
- Ribs disturb the direction of energy flow forcing it to the directions parallel to the ribs. This was shown by the calculation results comparison with the plate of the same shape and dimensions without the ribs.
- In rectangular regions separated by the ribs with no excitation or damping the appearance of vortices of intensity vector field can be observed. This effect of vortices represented by the rotational distribution of intensity vectors means the circulation and conservation of vibration energy in the mechanical system.
- The energy flow can be observed also in the ribs. The presented case of analysis gave the significant observation for the assessment of energy flow in ribs.
- For some frequencies the energy flow in ribs was significantly greater than in the rest of the structure it means the plate.
- The experimental verification with the use of lock-in thermography has shown the high values of the stress in the ribs.
- The vibration energy flow in the perpendicular plates (ribs) is not straight but shows the circulation also in plate without the excitation or damper.

Modal analysis based on dense grid of finite elements enabled detailed analysis of vibration energy transportation. The results obtained from calculations are presented in graphical form on Fig. 1 and 2. The structural intensity vectors distributions over the surface of elements of plate structure with ribs are shown. There are clearly seen the very specific disturbances of distribution in places of attachment of damping element and application of exciting force. In region far from connections, places of attachment of external damping or excitation the directions of vectors are ordered, parallel and steady regarding the vector magnitude. It means steady regular flow of vibration energy in one direction defined by the sense of vectors. In some places especially those limited by the ribs connected to the plate one can observe the circulation of a structural intensity vectors. This effect of vortices represented by the rotational distribution of intensity vectors means the circulation and conservation of vibration energy in the mechanical system.

The distribution of vectors has shown there the rotational character of vector field due to the wave reflections and places of sink of mechanical energy. The particular analysis leads to conclusion that intensity vectors distribution by its nature enables localization of energy sink in large surface elements. This is the significant advantage compared to the other identification methods.

The distribution of structural intensity vectors gives the qualitative characteristic of vibration energy transportation in mechanical systems. Introduction of an additional measure in form of integral of magnitude structural intensity vector component perpendicular to the certain closed surface e. g. element cross section enables the quantitative assessment of energy transfer paths and its balance in the structure.

Presented method of structural intensity vector calculation enables its evaluation for chosen frequency range and mode shapes [6]. The frequency range of analysis can not be exactly defined in real numbers. It should be rather related to the frequency range covering approximately first hundred mode shapes of analysed constructional element or structure. This relates the frequency range of analysis with mechanical properties and dimensions of the structure.

Derived dependencies connect the structural intensity with linear and angular displacements, forces and moments exerted to the structures like beams, plates and shells [5]. The method of structural intensity estimation is based on calculation of displacements in nodes and stresses in inner points of finite elements. The calculations are done with the application of complex modal parameters counted by the use of the numerical modal analysis based on finite element method.

The main disadvantage of the method is its poor convergence. The number of modes which are used in calculation process should be properly chosen Usually for the calculations of displacements and their derivatives fields the number of modes is taken in the way that the highest eigenfrequency applied in the calculations is a few times higher than frequency of excitation..

The abrupt changes of stress which occur in points close to the excitation or discontinuities of structure can not be well described by the limited number of lower modes. This makes the main problem of calculation method accuracy.

The method of analysis of structural intensity distribution has been applied as a new diagnostic method for assessment of failure which could occur in structural elements under working loads. It enables the investigation in the regions of high concentration of vibration energy flow which consequently is exposed to the risk of damage or is propagating the vibration and sound waves to the environment. It can be also considered as the identification of the regions for application of additional damping in purpose of lowering of vibration level and resulting noise radiation.

Examples of examination of structural intensity distribution in elements of rectangular plate have confirmed the method usability to the investigation of vibration energy transportation paths and places of its concentration. The method enables the quantitative analysis relaying on structural intensity vectors distribution. Also there are possible the quantitative analysis based on calculated values of power of vibrations in particular elements of structure.

In respect of possibilities of chosen elements analysis of complex structures and limitation of large FEM models the structure intensity method seems to be very promising tool for energy vibration flow investigation in middle frequency range. However the convergence of the structural intensity calculation method is still poor and mode shapes of higher order should be taken into account.

Acknowledgments

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