Acoustics'08 Paris
June 29-July 4, 2008
www.acoustics08-paris.org

measurement of force and moment mobilities using a finite difference technique

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Measuring only force mobilities it is possible to derive a moment mobility without the need for an externally applied moment. The method uses finite differences of both forces and velocities about a central point. In this way responses due to pure forces and moments can be extracted. There are, however, errors associated with finite difference techniques. In this case the purity of the extracted quantities depends on the area over which they are measured and its significance in terms of wavelength. Meanwhile experimental errors such as noise dictate that the area is sufficiently large to provide differences which are not easily corrupted. The paper quantifies the finite difference error for beams in a general way free of situation specifics. Finally the method is validated with a moment mobility measurement of a beam compared to theory.

1 Introduction

Frequency response functions such as mobility, accelerance and receptance may be used to relate velocity, acceleration and displacement to a force respectively. Then, using mobility for example, we can calculate forces and velocities at points which we have characterised. FRFs have many uses but here we pay specific attention to their use in the characterisation of structure borne sound sources. The results presented are however general.

Since mechanical power is basically a product of force and velocity, the mobility Y is considered as a convenient quantity for structure borne sound source characterisation. A structure's mobility can be represented by a matrix N × D square, where N is the number of connection points and D the number of degrees of freedom. For a full description the number of degrees of freedom would be six to account for three orthogonal axis directions and a rotation about each of these axes. A full description must therefore relate exciting forces and moments to the resulting linear and angular velocities. They may also relate one point in space to another.

Translational degrees of freedom will for most cases be the dominant mechanism for power transmission. However, in some cases it may also be necessary to consider rotational degrees of freedom [1]. Also, when inverting mobility matrices, all significant degrees of freedom should be included for a representative impedance matrix to be obtained [2].

In general, rotational degrees of freedom are considerably more difficult to measure than their translational counterparts [3]. If rotational degrees of freedom are considered to be of importance, one will be required to measure exciting moments and the resulting linear/angular response velocities, often termed moment mobilities.

The requirement for moment mobility measurement techniques was highlighted some time ago [4,5]. Moment mobility measurement however still remains a current problem today for various reasons. A particular difficulty with moment mobility measurement is associated with applying and accurately measuring an exciting moment [6].

The aim of the paper is to demonstrate a useful method for measuring moment mobilities. The method is very fast to apply but has specific errors associated with it. Here we present a very brief history, a feasibility demonstration, an error analysis and a validation by measurement.

The technique is shown to yield good results for a frequency range well in excess of that usually considered for structure borne sound source characterisation. The results will also be of interest to other disciplines.

2 Background

Various methods for applying measured moments for the determination of moment mobilities have been proposed. Examples include moment exciters [7], magnetostrictive exciters [8], synchronized hammers [9], blocks [1,10]. A history and break down of methods is given in [11] and also conveniently tabulated in [12]. It is also noted here that, in general, the more costly and time consuming methods provide better results. Unfortunately, this is counter to the requirements of industry which requires fast, low cost, engineering approaches.

In this respect, one method of particular interest is the central difference technique [13] reported by Sattinger in 1980. In this paper a reasonable agreement between beam theory and measured beam moment mobilities was shown. It should be noted however that the equipment typically used to measure the required quantities has improved significantly since this time.

The main advantage of the central difference method is the ease with which it can be applied. Moments are excited by applying a force directly to the structure at a small distance from the point of interest; thus, effectively using the structure itself as a lever. The method requires a force hammer, two accelerometers and two excitations to determine a force mobility and moment mobility, together with their related cross mobilities simultaneously.

There are of course associated frequency limits imposed by the finite difference approximation. However, for the frequency ranges commonly of interest with respect to structure borne sound, this frequency limit may not be of concern. In which case, the problem is one of balancing the finite difference approximation with the unavoidable experimental uncertainties.

In order to optimise the central difference method for use in specific cases it is useful to have a general understanding of the errors associated with the method. Then, for any given case the finite difference error can be balanced with experimental errors for the specific frequency range we are interested in.

In this paper, firstly the measurement technique is outlined. A simulation of the method for a theoretical beam is then presented to demonstrate feasibility of the method. The finite difference error for infinite and finite beams is presented. A validation of the derived moment mobility error for finite beams is presented for the example used to highlight the feasibility of the method. A measurement of beam moment mobility by central difference is presented with comparison to what would be expected according to beam theory.

3 Measurement of moment mobilities by finite difference

By applying a force F_1 directly to a structure at a distance Δ_F from the point of interest a moment Γ is generated. Thus at the point of interest we have an effective force F_0 and moment Γ_0. Similarly, an effective velocity v_0 and angular velocity α_0 can be defined.

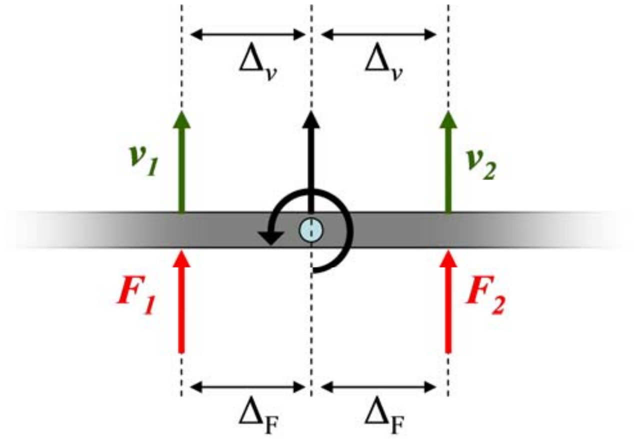

Fig. 1 Force excitation and velocity response measurement for the determination of force and moment mobilities.

These excitations and responses however will be coupled as forces and moments will have caused both translational and angular velocities. With a second excitation F_2 forces and moments can be separated and by measuring a second velocity v_2 angular and translational velocities can be extracted. The mobilities to be derived are defined as follows,

$$Y_{v/F} = \frac{v}{F} \approx v_0 \frac{1}{F_0} \tag{1}$$

$$Y_{\alpha/F} = \frac{\alpha}{F} \approx \alpha_0 \frac{1}{F_0} \tag{2}$$

$$Y_{v/\Gamma} = \frac{v}{\Gamma} \approx v_0 \frac{1}{\Gamma_0} \tag{3}$$

$$Y_{\alpha/\Gamma} = \frac{\alpha}{\Gamma} \approx \alpha_0 \frac{1}{\Gamma_0} \tag{4}$$

Here v, F, α and Γ correspond to velocity, force, angular velocity and moment respectively. Eq.2 and Eq.3 are referred to as cross mobilities as they relate rotational and translational degrees of freedom. Cross mobilities for the same point are equal by reciprocity. Eqs. 1 and 4 are force and moment mobilities respectively.

Referring to Fig. 1 it can be seen that applying the force F_2 will generate a positive moment and F_1 a negative moment. Thus the sum of the two will cancel the moment giving a force at the point of interest. The difference between F_1 and F_2 on the other hand will cancel forces, indirectly, giving a moment. Similar arguments lead to linear and angular velocities from v_1 and v_2.

It is possible therefore to derive force, moment and cross mobilties purely from force (point and transfer) mobilities.

$$Y_{v/F} = \left(\frac{v_1}{F_1} + \frac{v_2}{F_1} + \frac{v_1}{F_2} + \frac{v_2}{F_2} \right) \Big/ 4 \tag{5}$$

$$Y_{\alpha/F} = \left(-\frac{v_1}{F_1} + \frac{v_2}{F_1} - \frac{v_1}{F_2} + \frac{v_2}{F_2} \right) \Big/ 4\Delta_v \tag{6}$$

$$Y_{v/\Gamma} = \left(-\frac{v_1}{F_1} - \frac{v_2}{F_1} + \frac{v_1}{F_2} + \frac{v_2}{F_2} \right) \Big/ 4\Delta_F \tag{7}$$

$$Y_{\alpha/\Gamma} = \left(\frac{v_1}{F_1} - \frac{v_2}{F_1} - \frac{v_1}{F_2} + \frac{v_2}{F_2} \right) \Big/ 4\Delta_F\Delta_v \tag{8}$$

Where Eq. 8 is essentially the central difference equation [13]. Thus, force and moment mobilities can be found simultaneously from two excitations and two measured responses.

In a practical sense it is unlikely that one will deal with velocities and forces directly in this way. Rather, one will measure mobilities as transfer functions. In which case, we may replace the response v_1 to excitation F_2 with the mobility Y_{21} for example. We may then rewrite equations 5 to 8 in matrix form,

$$\begin{bmatrix} Y_{v/F} & Y_{v/\Gamma} \\ Y_{\alpha/F} & Y_{\alpha/\Gamma} \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 \\ -1/2\Delta_v & 1/2d\Delta_v \end{bmatrix} \begin{bmatrix} Y_{11} & Y_{21} \\ Y_{12} & Y_{22} \end{bmatrix} \begin{bmatrix} 1/2 & -1/2\Delta_F \\ 1/2 & 1/2\Delta_F \end{bmatrix} \tag{9}$$

The resulting matrix then conveniently includes all mobility terms. This formulation only applies to this single point two degree of freedom case but can be extended to further points and degrees of freedom relatively easily if required.

In comparison to other methods the ease with which the central difference technique can be applied is of significant benefit. Providing measurements of a reasonable quality are possible the method should provide a useful engineering tool. There follows a feasibility study for a specific case.

4 Feasibility

The errors associated with the proposed method can be determined to some extent through simulation. The range of individual situation specifics are however difficult to generalise

For mobility, direct validation is limited to predictable structures such as beams and perhaps the first few resonances of a plate where theoretical models can be used for comparison. Structures with a high modal overlap will not tax the measurement method sufficiently as modal behaviour is the most difficult to determine. Thus lightly damped resonant beams or plates make suitable validation structures.

As a preliminary validation a theoretical free-free beam model is used. The properties of a 50cm long steel beam of 1cm thickness were calculated. A point of interest was defined for which the force, moment and cross mobilities were calculated. For a separation Δ = 5mm the method was simulated. Shown in Fig. 2 is the mobility magnitude and phase calculated directly for the beam and compared to that found indirectly from Eq. 8.

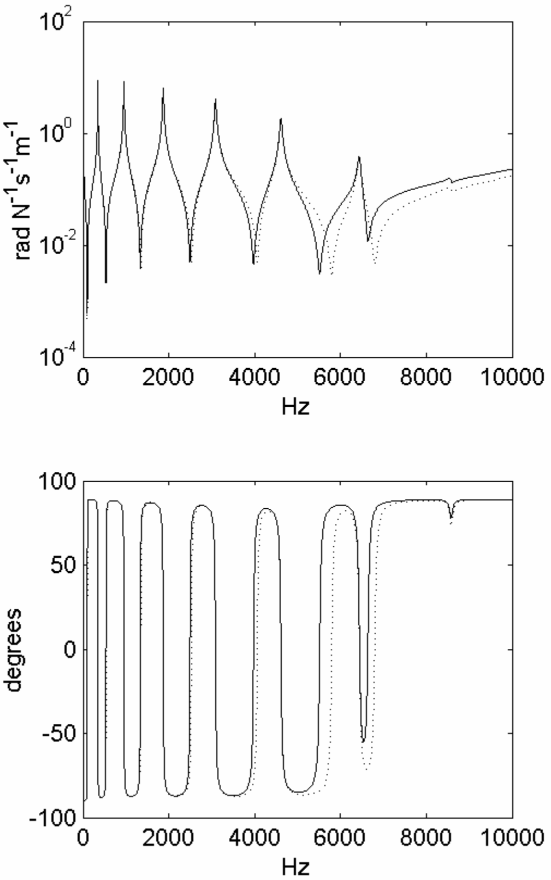

Fig. 2 Magnitude and phase of moment mobility derived using the central difference method. Solid line: theoretical moment mobility. Dotted line: calculated using central difference.

Similar plots can be made for the force and cross mobilities. However, for this small separation (Δ = 5mm) differences between theory and application are virtually indiscernible. The moment mobility simulation shown here is likely the best achievable result as transducer dimensions will not allow for a smaller Δ. Noise was not accounted for in this simulation. Shown later are moment mobilities found from real measurement data to demonstrate the reality of the method in terms of application.

Results of this simulation appear favourable over a large frequency range. Should such a large frequency range not be required it would be possible to reduce the sensitivity of the method to measurement errors by increasing the separation Δ. There follows a discussion and quantification of the finite difference error in relation to Δ. Discussions are limited to the moment mobility as errors in the force and cross mobilities will be small in comparison.

5 Error Analysis

Two errors types limit the use of the finite difference method. Firstly, the purity of the extracted quantities depends on the separation Δ and its significance in terms of wavelength. Counter to this, experimental errors such as noise dictate that Δ is sufficiently large to provide differences which can be resolved from the noise floor. The former error is due to the assumption of rigid body behaviour over the distance 2Δ. This error can therefore be controlled or perhaps minimised if one has some understanding of its nature. Other important sources of error include measurement chain phase and amplitude mismatches and inaccurate accelerometer placement or excitation position. As the finite difference error in moment mobilities is significantly greater than for force and cross mobilities we concentrate our error analysis on the moment mobility.

If we consider firstly an infinite beam, point force and moment mobilities may be given by,

$$Y_{v/F} = \frac{\omega}{4\bar{B}k^3}(1-j) \tag{10}$$

$$Y_{\alpha/\Gamma} = \frac{\omega}{4\bar{B}k}(1+j) \tag{11}$$

Where ω is angular frequency, *B* is bending stiffness and k is the wave number. Performing a power series expansion of point and transfer infinite beam mobilities in accordance with Eq. 8 we obtain the approximate point moment mobility $Y'_{\alpha/\Gamma}$.

$$Y'_{\alpha/\Gamma} = \frac{\omega}{4\bar{B}k}[1+j]\left[1-(1+j)\frac{2}{3}k\Delta+\frac{j}{3}k^2\Delta^2\ldots\right] \tag{12}$$

Referring to Eq. 12 it can be seen that the expanded terms are moment mobility errors due to the finite difference technique. The normalised moment mobility error can therefore be written.

$$\frac{Y'_{\alpha/\Gamma}}{Y_{\alpha/\Gamma}}-1 = (1+j)\frac{2}{3}k\Delta+\frac{j}{3}k^2\Delta^2\ldots \tag{13}$$

Plotted in Fig. 3 is the percentage error in the moment mobility modulus resulting from the central difference approximation, calculated using Eq. 13.

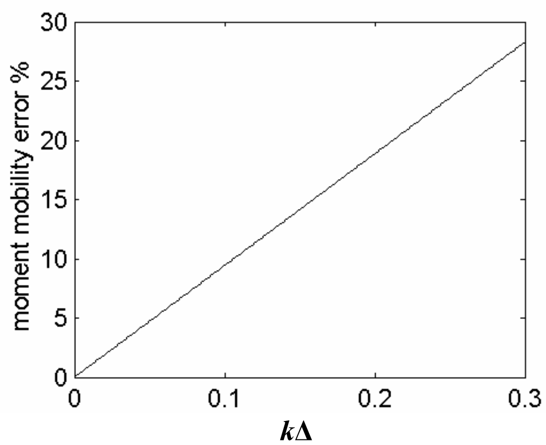

Fig. 3 Percentage error in moment mobility for any infinite beam.

Fig. 3 presents the first term approximation of the error plotted against Helmholtz number. For infinite beams the finite difference error is small where kΔ<<1. However, when kΔ<<1 test subjects rarely behave like infinite structures. The implications of the method relating to modal behaviour must therefore be accounted for.

The force [14] and moment mobilities of a simply supported finite beam may be written, in closed form,

$$Y_{v/F} = \frac{-j\omega}{2\bar{B}k^3}\left[\frac{\sinh(kx)\sinh k(l-x_0)}{\sinh(kl)} - \frac{\sin(kx)\sin k(l-x_0)}{\sin(kl)}\right] \quad (14)$$

$$Y_{\alpha/\Gamma} = \frac{j\omega}{2\bar{B}k}\left[\frac{\cosh(kx)\cosh k(l-x_0)}{\sinh(kl)} + \frac{\cos(kx)\cos k(l-x_0)}{\sin(kl)}\right] \quad (15)$$

Using Eq. 14 as a starting point, the finite difference errors associated with the method can be characterised in a more general way by using a Taylor series expansion in two variables. The resulting analysis is not shown here as the result can be easily verified using the free-free beam example from the previous section. Despite some rather drawn out analysis a simple first term approximation of the error can be obtained and normalised as before to give,

$$\frac{Y'_{\alpha/\Gamma}}{Y_{\alpha/\Gamma}} - 1 = \frac{j\omega\Delta}{3\bar{B}Y_{\alpha/\Gamma}} \quad (16)$$

Although derived using the simply supported beam as a starting point this result will apply to any beam. The generality of Eq.16 allows useful insight into the nature of the moment mobility finite difference error.

The generality of this result is demonstrated in Fig. 4. Consider again the moment mobility of the free-free beam shown previously in Fig. 2.

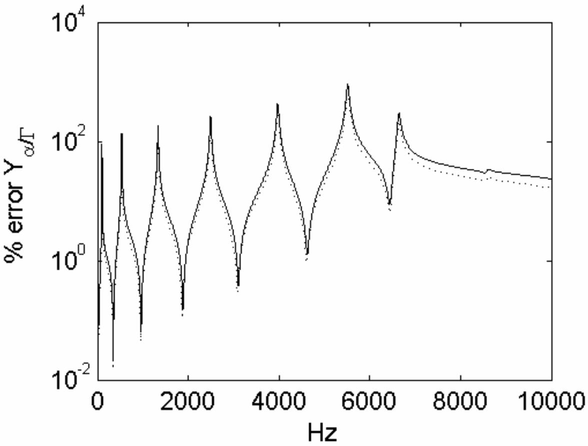

Fig. 4 Finite difference error in moment mobility. Solid line: percentage error in theoretical moment mobility shown in Fig.2. Dotted line: Estimated error in moment mobility from Eq. 16.

It is shown in Fig. 4 that Eq. 16 provides a good approximation to the finite difference error experienced in practice, providing $k\Delta \ll 1$. It can be seen that much larger errors are observed when trying to account for modal behaviour in comparison to infinite behaviour.

If we inspect Eq. 16 it can be seen that the finite difference error will increase with frequency and the separation Δ used for the measurement. Also where the moment mobility is small, as it will be at anti-resonance, the error in the moment mobility may be large. Maximum errors in the range 100 to 1000% are observed for the case given. These max errors are due to small shifts in frequency for position dependent anti-resonances and the large dynamic range in the given example. Only small errors are observed in the estimations of mobility peaks. This is because resonances will be found to be always at the same frequency regardless of measurement position.

6 Validation using measured data for a free-free beam

In order to verify the method in terms of real measurements a laboratory test was carried out. A beam was used as the validation structure as beam mobilities can be calculated and fitted to measured data visually. A 9mm thick steel beam was used to provide a challenging dynamic range.

Using measured beam dimensions/properties approximate theoretical beam mobilities were calculated. The wave speed and damping were then adjusted in the model to fit with measured beams point force mobilities. This was done by trial and error. Exact fitting over a large frequency range was not possible so a compromise was met which favoured the centre of the 0-3kHz range.

Shown in Figs. 5 and 6 are estimates of beam moment mobility magnitude and phase compared with those calculated from theory, respectively.

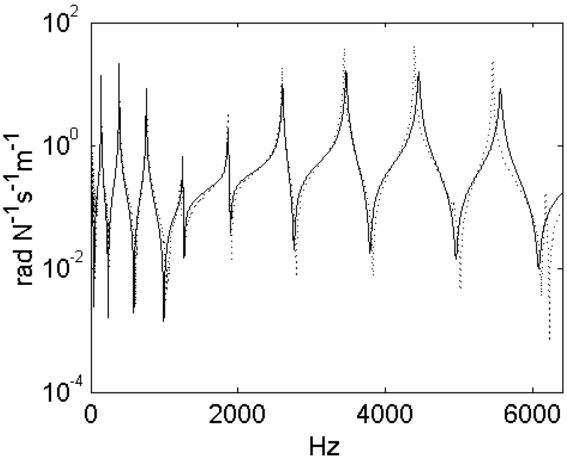

Fig. 5 Moment mobility magnitude. Solid line: Magnitude of moment mobility predicted from theory. Dotted line: Moment mobility from measured data using Eq. 8.

For the example shown in Figs. 5 and 6 a separation Δ of 1cm was used. As discussed previously it was not possible to fit measured data exactly to the theoretical. The frequency shift seen in the predictions of the resonant peaks would not be observed in reality. Thus there is a slight compression of the frequency axis for the measured data relative to the theoretical.

Although slightly distorted the measured moment mobility can be seen to compare well with that predicted from theory. As shown in Fig. 6, measurement noise can be clearly seen in the phase of the moment mobility below 1kHz.

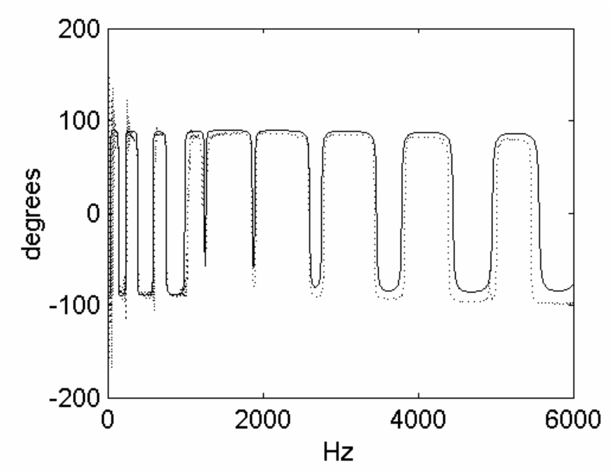

Fig. 6 Moment mobility phase. Solid line: Phase of moment mobility predicted from theory. Dotted line: from measured data using Eq. 8.

If measurement noise posed a problem for a specific case but a large frequency range was not required there is clearly scope for optimisation of the technique.

Measurements using this technique have been used in another study to investigate impedance and mobility relationships. The method was found to be of sufficient accuracy determine a requirement for the inclusion of moment mobilities in describing a single point coupling of two beams. Some related discussions are presented in [15].

7 Conclusions

A method for measuring force, moment and cross mobilities simultaneously was shown to be feasible as a technique. The method has been presented previously by other researchers but FFT analysers have since improved significantly. Now, given an analyser with a reasonable dynamic range, good results can be obtained with very little effort.

The error associated with the measurement of moment mobilities by central difference is seen to increase with frequency and the separation used. The error is inversely proportional to mobility magnitude. As a result, anti-resonances are affected most severely.

Errors associated with infinite structure behavior were found to be small. The errors presented are applicable to beams only.

Validation tests for the method using beam data measured in a laboratory showed good agreement with theory between 500Hz and 3kHz. The method could be further optimised for a particular frequency range if required.

Acknowledgments

This work was funded by the European Doctorate in Sound and Vibration Studies programme, a Marie Curie Scholarship.

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