# Mode transition of a flue organ pipe

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A flue organ pipe can be excited in various acoustic modes by changing the air pressure supplied to it. This research aims to reconstruct this behavior from the result of numerical flow simulation of a jet deflected by sound and from physical modeling simulation of the total sound production system. In the numerical flow simulation, motion of the jet in the pipe mouth was replicated: The jet emerges from a flue and travels in a space where the air oscillates laterally to the jet direction. As a result, the jet oscillates with the same frequency as the oscillation of the air i.e., sound. From the flow simulation, a model of the jet deflection was developed. This model was then used as a model of the sound source in the physical modeling simulation where not only the sound source but the resonance of the pipe is also modeled in a set of differential equations with delayed feedback. The mode transition observed in the physical modeling simulation was discussed by comparing with that experimentally observed.

# 1 Introduction

A flue organ pipe is normally operated in the fundamental resonance mode with foot pressure — air pressure supplied to a pipe — that is fixed in a range from 500 to 900 Pa. A pipe, however, makes sound with the pressure in a much broader range. If the pressure is changed from a few tens Pa to 10 kPa, a pipe sounds in several different oscillation regimes. As the pressure is increased, the second and the third resonance modes are excited. A pipe can sound with much smaller pressure. In this case, the second and the first resonance modes are excited. Transition among these oscillation regimes is called overblowing behavior.

In this paper, overblowing behavior of a flue organ pipe is simulated using a time-domain physical model of the instrument. The critical part of the physical model is a model of jet oscillation. This is developed from the result of numerical flow simulation of a jet oscillated by sound. How accurately overblowing behavior can be simulated would be a barometer of how much we understand the sounding mechanism of the flue instrument.

# 2 Theoretical model of jet oscillation

Fletcher and Thwaites[1] proposed a model of a jet oscillated by sound. First we suppose that a jet emerges from a flue and travels in a transversely directed acoustic flow field. The acoustic field deflects the jet laterally and make it oscillate with the same frequency. We define $\eta(x,t)$ as the lateral displacement of the jet at distance $x$ from the flue exit at time $t$. Assuming a uniform acoustic field oscillating with angular frequency $\omega = 2\pi f$, we denote the acoustic displacement along the $y$-axis perpendicular to the jet direction as $Y(t) = Y \exp(i\omega t)$. The jet displacement is modeled as

$$\eta(x,t) = Y(t)\left[1 - \exp(\mu x)\exp\left(-i\omega\frac{x}{v_{\mathrm{ph}}}\right)\right], \qquad (1)$$

where $\mu$ is the growth factor of an infinitesimal disturbance imposed on the jet and $v_{\mathrm{ph}}$ is the propagation velocity of the disturbance. The first term in the bracket in Eq. (1) indicates that the jet transversely oscillates as a lump with $Y(t)$. The second term implies that a small disturbance $-Y(t)$ is imposed on the jet at $x = 0$ (to meet the condition that no displacement of the jet is allowed at the flue exit) and it grows exponentially with factor $\mu$ and propagates downstream with velocity $v_{\mathrm{ph}}$. The growth factor $\mu$ and the propagation velocity $v_{\mathrm{ph}}$ are parameters of this model. The growth factor $\mu$ is determined by a stability analysis in Mattingly and Criminale[2]. Because there is ambiguity in determining $v_{\mathrm{ph}}$, we introduce a model parameter $\gamma$ in this paper, which is defined by

$$v_{\mathrm{ph}} = \gamma v_{\mathrm{jet}}, \qquad (2)$$

where $v_{\mathrm{jet}}$ is the jet velocity.

# 3 Flow simulation

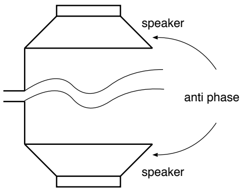
Coltman examined the motion of a jet oscillated by sound experimentally[3]. In his setup, a pair of speakers driven in anti-phase generate an acoustical cross flow in a channel where a jet travels. The same experiment was performed numerically in this paper with the method shown in Adachi[4]. The incompressible Navier-Stokes equations were solved in a two-dimensional domain representing the pipe mouth using the finite element method. Sound field, that is the oscillating lumped air, was realized by a condition imposed on the upper and lower boundaries. Instead of the pair of speakers in Coltman's experiment, sinusoidally oscillating acoustic velocity $v_{\mathrm{ext}}(t) = v_{\mathrm{ext}} \sin\omega t$ on these boundaries was imposed.

In the simulation, several parameters were specified as follows: The flue thickness $h$ was set to 0.25 or 0.5 mm. Tophat (or constant) velocity profile at the flue was assumed. The initial jet velocity $v_0$ was from 5 to 60 m/s and The sound frequency $f$ was changed among 200, 500 and 800 Hz. Figure 1 shows an example of a jet numerically simulated.

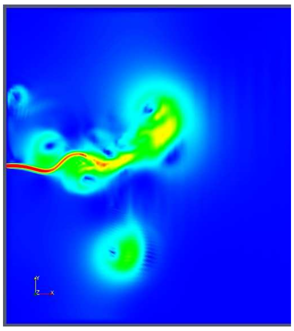
From the simulated jet, displacement $\eta$ normalized by acoustic displacement $Y$ was measured at several different distances of $x$ from the flue. The phase and amplitude of $\eta/Y$ are plotted in Fig. 2 as functions of two Strouhal numbers $St_x$ and $St_h$, which are defined by $St_x = fx/v_0$ and $St_h = 3\pi fh/2v_0$, respectively. The data of the phase and amplitude were fitted to surfaces also shown in Fig. 2 (a) and (b). The surfaces provide $\eta/Y$ as functions of $St_x$ and $St_h$. A jet deflection model was thus developed from the numerical experiment.

# 4 Sound synthesis

In physical modeling, sounding of a flue pipe is described by a few variables. These are the jet displacement at the




(a)



(b)

Figure 1: (a) Schematic illustration of a jet deflection experiment. (b) An oscillating jet numerically simulated.

upper labium $\eta(t) \equiv \eta(l,t)$, where $l$ is the distance between the flue and the upper labium or the cutup length, volume flow into the pipe caused by the jet $U_{\mathrm{jet}}(t)$, pressure $p(t)$ and volume flow $U(t)$ at the mouth, and those at the entrance of the pipe $p_{\mathrm{p}}(t)$ and $U_{\mathrm{p}}(t)$. The positive direction of the volume flow is here regarded to be towards the pipe. These variables are schematically illustrated in Fig. 3.

The resonance of a pipe is characterized by the input impedance defined by $Z_{\mathrm{in}}(\omega) = p_{\mathrm{p}}(\omega)/U_{\mathrm{p}}(\omega)$. This can be calculated with an acoustic transmission line model from the shape of the pipe. In time domain, the pipe resonance is characterized by a pressure reflection function $r(t)$ that is the inverse Fourier transformation of the reflection coefficient

$$R(\omega) = \frac{Z_{\mathrm{in}}(\omega) - Z_0}{Z_{\mathrm{in}}(\omega) + Z_0}, \tag{3}$$

where $Z_0 = \rho c/S_{\mathrm{p}}$ is the characteristic wave impedance of a pipe having cross sectional area $S_{\mathrm{p}}$. The pressure and the volume flow at the entrance of the pipe satisfy the following convolution integral:

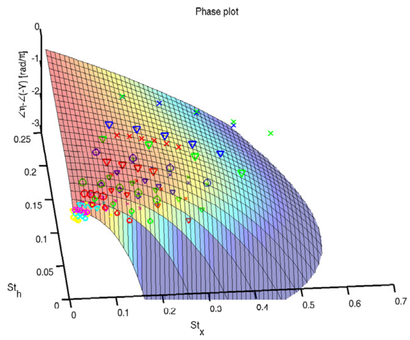
$$p_{\mathrm{p}}(t) = Z_0 U_{\mathrm{p}}(t) + \int_0^{\infty} ds\, r(s)\left\{p_{\mathrm{p}}(t-s) + Z_0 U_{\mathrm{p}}(t-s)\right\}. \tag{4}$$

At the entrance of the pipe, the volume flow $U_{\mathrm{jet}}(t)$ caused by the air jet is injected as well as the acoustic volume flow through the mouth $U(t)$. The flow and momentum conservations are satisfied here:

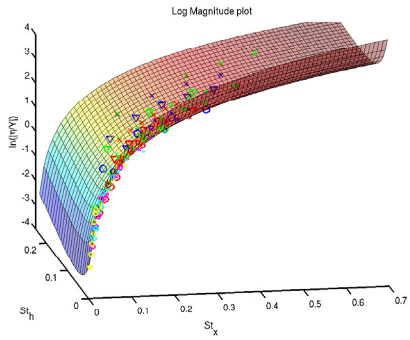
$$U_{\mathrm{p}}(t) = U(t) + U_{\mathrm{jet}}(t), \tag{5}$$

$$p_{\mathrm{p}}(t) = p(t) + \frac{\dot{M}_{\mathrm{jet}}(t)}{S_{\mathrm{p}}}, \tag{6}$$

where $\dot{M}_{\mathrm{jet}}(t)$ is momentum flux along the jet. Equation (6) implies that sudden deceleration of the jet results in force acting on the entrance. We assume that the jet has a bell-shaped velocity profile of $v(y) = v_{\mathrm{c}}(l)\,\mathrm{sech}^2(y/b(l))$ at the upper labium, where $v_{\mathrm{c}}(l)$ is the jet center velocity and $b(l)$ is the half thickness of the jet at




(a) Phase




(b) Amplitude

Figure 2: Data of $\eta/Y$ obtained in numerical simulations and fitting surfaces as functions of two Strouhal numbers $St_x$ and $St_h$.

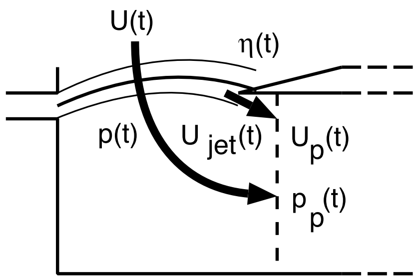



Figure 3: Flows and pressure near the pipe mouth.

$x = l$. $U_{\mathrm{jet}}(t)$ and $\dot{M}_{\mathrm{jet}}(t)$ are calculated as follows:

$$U_{\mathrm{jet}}(t) = W \int_{-\infty}^{h_{\mathrm{off}}} dy v(y) = W b(l) v_{\mathrm{c}}(l) \left(1 - \tanh \frac{\eta(t) - h_{\mathrm{off}}}{b(l)}\right), \tag{7}$$

$$\dot{M}_{\mathrm{jet}}(t) = \rho W \int_{-\infty}^{h_{\mathrm{off}}} dy v^2(y) = \frac{2}{3}\rho W b(l) v_{\mathrm{c}}^2(l) \left[1 - \left(1 + \frac{1}{2}\mathrm{sech}^2 \frac{\eta(t) - h_{\mathrm{off}}}{b(l)}\right) \times \tanh \frac{\eta(t) - h_{\mathrm{off}}}{b(l)}\right], \tag{8}$$

where $h_{\mathrm{off}}$ is the offset of the upper labium position towards the $+y$-direction from the center line along which the jet travels.

Sound is radiated from the pipe mouth. As pointed out in Fabre et al.[6], this process is nonlinear because of the high sound pressure level and of the existence of the upper labium, from which turbulence occurs. As shown in Skulina[7], the total radiation impedance i.e., the ratio of $p$ to $-U$ becomes the sum of the ordinary linear radiation impedance $Z_{\rm lin}$ and an additional non-linear contribution $Z_{\rm nl}$. $Z_{\rm lin}$ in the low frequency approximation becomes

$$Z_{\rm lin} = \frac{i\omega\rho\Delta L}{S_{\rm p}}, \tag{9}$$

where $\Delta L$ is the end correction of the pipe mouth. This can be $2.3S_{\rm p}/\pi\sqrt{S_{\rm m}}$, where $S_{\rm m}$ is the area of the mouth[8]. On the other hand, $Z_{\rm nl}$ is modeled in Disselhorst and Van Wijngaarden[9] as

$$Z_{\rm nl} = \hat{\beta} M St_m^{\frac{1}{3}} \frac{\rho c}{S_{\rm m}}, \tag{10}$$

where $\hat{\beta}$ is a model parameter we here set to 3.0, $M$ is a Mach number $U(t)/S_{\rm m}c$, and $St_m$ is a Strouhal number defined by $St_m = f\sqrt{4S_{\rm m}/\pi}/U(t)$. Because it is not easy to take the frequency dependence of $Z_{\rm nl}$ into consideration, we fix the frequency to 660 Hz in the simulation of a flue organ pipe. The time-domain representation of the radiation from the mouth then becomes

$$-\frac{\rho\Delta L}{S_{\rm p}}\dot{U}(t) = p(t) + Z_{\rm nl}U(t). \tag{11}$$

The jet in a flue organ pipe has a tophat velocity profile when it emerges from the flue. The tophat profile gradually changes to a bell-shaped profile as the jet travels. Also the jet gradually blends into the surrounding air. The center velocity $v_{\rm c}(x)$ becomes smaller and the half thickness $b(x)$ becomes larger in the mouth. The initial jet velocity $v_0$ is calculated from the pressure supplied to the pipe $p_0$ with $v_0 = \sqrt{2p_0/\rho}$. The initial half thickness $b_0$, supposing we may extend the $\mathrm{sech}^2$ velocity profile up to the flue exit, becomes $b_0 = 3h/4$ from the momentum conservation at the flue exit. Assuming a constant semi angle $\phi$ of the jet spreading, we have

$$b(x) = b_0 + x\tan\phi, \tag{12}$$

$$v_{\rm c}(x) = v_0\sqrt{\frac{b_0}{b(x)}}. \tag{13}$$

As discussed in Sec. 2, the theoretical model of the jet oscillation expressed by Eq. (1) assumes a jet having a constant velocity $v_{\rm jet}$ and half thickness $b$. To apply this model to the actual jet, we need averaging of $v_{\rm c}(x)$ and $b(x)$ over the cutup length $l$, which is defined here by

$$b = b(l/2), \tag{14}$$

$$v_{\rm jet} = l \Big/ \int_0^l \frac{dx}{v_{\rm c}(x)}. \tag{15}$$

If the jet velocity at the flue exit $v_0$, the cutup length $l$ and the flue thickness $h$ are specified, $\eta/Y$ can be expressed as a function of $\omega$ from the theoretical jet oscillation model [Eq. (1)] with Eqs. (2), (14) and (15). In the numerical model, the phase and amplitude of $\eta/Y(\omega)$ are given by fitting surfaces in Fig. 2, as the two Strouhal numbers $St_x$ and $St_h$ are now functions of $\omega$ only. Because $U(t) = -S_{\rm m}\dot{Y}(t)$, we have $\eta/U(\omega)$ in frequency domain in any cases. The time-domain representation of a jet oscillation model thus have a general form:

$$\eta(t) = \int_0^\infty ds G(s) U(t-s), \tag{16}$$

where $G(s)$ is a kernel function that is the inverse Fourier transformation of $\eta/U(\omega)$ provided by either model. The models were constructed with an assumption of the infinitesimal magnitude of $\eta(t)$. In the simulation with Eq. (16), the magnitude of $\eta(t)$ sometimes became unrealistically too large. To prevent this situation, we introduced a factor $\alpha\exp(-|\eta(t)|/b(l))$ to Eq. (16), where $\alpha$ was set to 0.2 after a preliminary sound simulation.

If all the geometrical parameters of the pipe are known and the blowing pressure $p_0$ is given, we can simulate sounding of a flue pipe. One time step in the simulation can be completed as follows: With Eq. (16), $\eta(t)$ can be calculated from the past data of $U(t)$. $U_{\rm jet}(t)$ and $\dot{M}_{\rm jet}(t)$ are calculated using Eqs. (7) and (8). $U(t)$ is calculated with Eq. (11). $U_{\rm p}(t)$ is obtained from Eq. (5). With Eq. (4), we can calculate $p_{\rm p}(t)$. Finally, $p(t)$ is obtained from Eq. (6).

# 5 Mode transition estimation

By changing the blowing pressure $p_0$ from 30 Pa to 10 kPa, sound generated by an E4 flue organ pipe examined in Fletcher[10] was synthesized with the method presented in Sec. 4, Table 1 lists the simulation parameters including the dimensions of this pipe. The offset of the upper labium was estimated from the measurement of the jet velocity profile shown in Fig. 6 of Fletcher[10]. The same jet spreading angle $\phi$ obtained experimentally in Thwaites and Fletcher[11] was used.

In the theoretical model, model parameter $\gamma$ should be determined. As $\gamma$ controls jet velocity $v_{\rm jet}$, it changes the blowing pressure at which transitions among the oscillation modes occur. After a preliminary simulation, $\gamma$ was determined to be 0.35 so that the transition between the normal oscillation regime and the overblow regime in the second mode occurs at around $p_0 = 1.6$ kPa.

Table 1: Simulation parameters

| Symbol | Parameter name | Value |
|---|---|---|
| $L$ | pipe length | 44 cm |
| $W$ | mouth width | 4 cm |
| $l$ | cutup length | 1 cm |
| $S_{\rm p}$ | pipe cross-sectional area | $W^2 = 16$ cm$^2$ |
| $S_{\rm m}$ | mouth area | $Wl = 4$ cm$^2$ |
| $h$ | flue thickness | 0.25 mm |
| $h_{\rm off}$ | upper labium offset | $-0.9$ mm |
| $\phi$ | jet spreading semi-angle | 6.3° ($\tan\phi = 0.11$) |
| $\alpha$ | factor to Eq. (16) | 0.2 |
| $\beta$ | coefficient in Eq. (**??**) | 0.625 |
| $\gamma$ | coefficient in Eq. (2) | 0.35 |

For comparison with the mode transition experimentally observed, the sound pressure level of each harmonic component at distance $R = 1$ m from the pipe mouth should be estimated. By assuming monopole radiation from the pipe mouth, the sound pressure amplitude and the level of the $n$-th harmonic component become

$$p_{\mathrm{out}} = \frac{\rho\omega U_n}{4\pi R}, \quad (17)$$

$$\mathrm{SPL} = 20\log(p_{\mathrm{out}}/20\mu\mathrm{Pa}), \quad (18)$$

where $U_n$ is the acoustic volume flow amplitude of the $n$-th harmonic at the mouth.

The fundamental frequency and the levels of the harmonic components were obtained as functions of $p_0$. The estimated mode transitions with the theoretical and numerical jet oscillation models are shown in Figs. 4 (a) and (b), respectively. The lower panel in each figure plots the fundamental frequency divided by the mode number of the oscillation. The upper panel shows the sound pressure levels of the harmonic components. The experimental result of the mode transition, that is Fig. 16.17 in Fletcher and Rossing[12], is also shown in Fig. 4 (c) for comparison. This is the target for the simulations. The general tendency of the mode transitions simulated with both the models is in good agreement with that observed experimentally. When the blowing pressure $p_0$ is increased from 30 Pa, an oscillation regime in the first resonance mode first appears. After a silence, a regime in the second mode appears. These two regimes are called underblows. The second mode oscillation suddenly turns to another regime in the first mode as $p_0$ is increased and it lasts until $p_0$ becomes larger than 1 kPa. This is called normal oscillation regime. In the higher $p_0$ region, there are two regimes: one in the second resonance mode and the other in the third mode. These are called overblows.

In the transitions among the normal and overblow regimes, hysteresis is observed and also simulated. This is the phenomenon in which two oscillation regimes are overlapped in their marginal ranges of the blowing pressure. Which oscillation regime actually appears in this range depends on the history of the generated sound. If an oscillation regime is excited at a certain $p_0$ and it is changed to go into a range where two regimes exist, this regime tends to remain. Transition to the other regime occurs after $p_0$ go beyond the range. Within each regime, the fundamental frequency is gradually increased as $p_0$ increases. This is well simulated with both the jet oscillation models. This phenomenon can be found commonly in flue instruments and is explained in Fletcher and Rossing[12] On the transition from the normal regime to the overblow in the second mode, the fundamental frequency of the sound jumps almost twice, but it is smaller than twice. Similarly, the frequency jump on the transition between the second and third modes is smaller than 1.5 that is expected from the ration of the mode numbers. These phenomena are well simulated both with the theoretical and numerical models.

Let us compare the simulated mode transitions with the experiment in detail. With the theoretical model, the normal oscillation regime is simulated for $p_0$ from 320 Pa to 1.6 kPa. With the numerical model, it is simulated for $p_0$ from 180 Pa to 890 Pa. As compared with the experiment, the lower limit of $p_0$ for the normal oscillation regime is better simulated with the numerical model. The upper limit of $p_0$ is better simulated with the theoretical model, but this is trivial because $\gamma$ was set to 0.35 so that the transition occurs near 1.6 kPa. If $\gamma$ is changed, the transition point is easily changed. The beginning of the overblow regime in the second resonance mode is well simulated with both the models. In the experiment, this regime lasts until $p_0$ reaches 10 kPa, but it is simulated to disappear at around 5 kPa with both the models. The overblow in the third mode is better simulated with the theoretical model. With the numerical model, it appears for $p_0$ as small as 2.8 kPa. The pressure ranges for the two underblows are better simulated with the numerical model. With the theoretical models, these are shifted towards the larger pressure side.

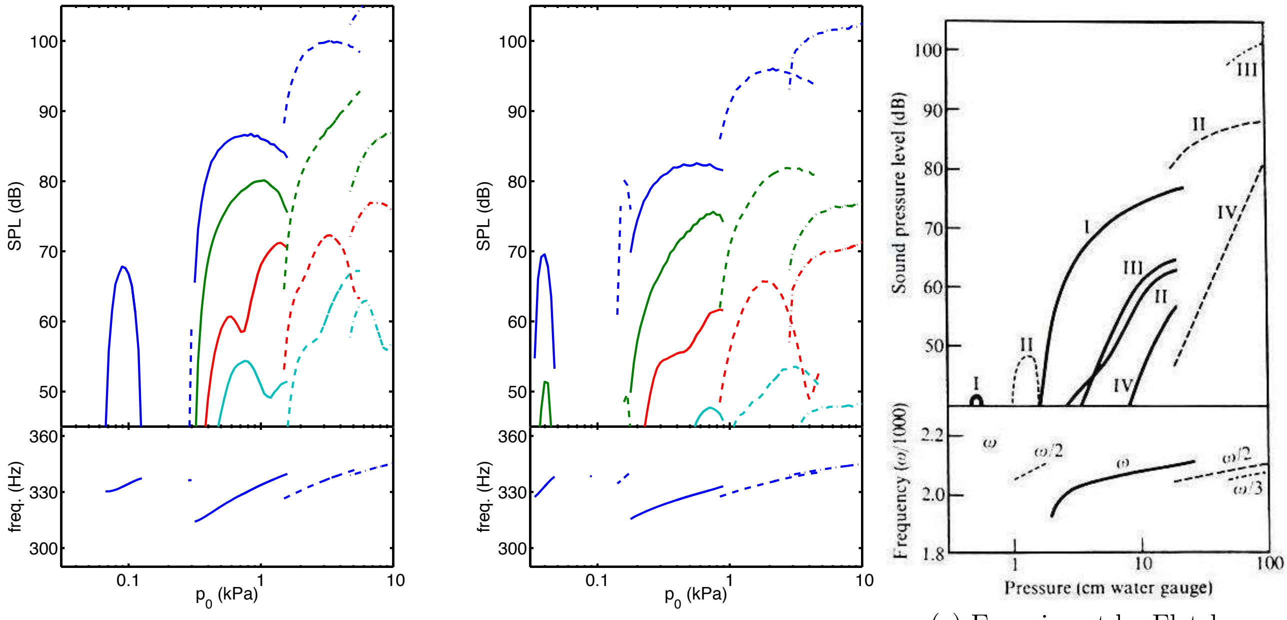
The sound pressure levels are much larger in the simulated mode transitions than in the experiment. Especially for the underblow regimes, the level difference between the simulations and the experiment is larger than 20 dB. This means that the oscillating jet excites the pipe oscillation much less efficiently in the underblow regimes. This may imply that the assumption of the jet velocity profile is no longer satisfied for the oscillating jet with a larger phase delay. In Nolle[13], it is pointed out that the oscillating jet is not a simply a time-delayed, broadened, and amplified version of the jet near the flue. In our numerical flow simulation, it is also observed that the velocity profile of the jet is considerably distorted and deviates from the shape of $\mathrm{sech}^2$ as it is oscillated by sound.

The synthesized sound spectra in Figs. 4 (a) and (b) are also different from that observed in the experiment in Fig. 4 (c). In the simulated normal oscillation regimes, the sound pressure levels of the lower harmonics are always larger than those of the higher harmonics. In the normal regime observed in the experiment, the level of the third harmonic is larger than that of the second harmonic for larger $p_0$. This may imply that the offset of the upper labium $h_{\mathrm{off}}$ is effectively smaller in the experiment than the assumed value of $-0.9$ mm, because the general tendency is that a smaller offset results in depression of the levels of the even harmonics[14].

# 6 Conclusion

Oscillations of a jet traveling in a transversely directed acoustic field were numerically simulated for various different values of the jet initial velocity, the sound frequency and the flue thickness. From the results of this numerical experiment, a new jet oscillation model was developed. Using physical modeling sound synthesis, the mode transition of an E4 organ flue pipe was estimated and compared with that simulated with an existing jet oscillation model theoretically developed and with that experimentally observed.

Although there are several discrepancies between the observed and estimated transitions, the basic properties of the mode transition can be successfully simulated both with the theoretical and numerical jet oscillation




(a) Theoretical model

(b) Numerical model

(c) Experiment by Fletcher

Figure 4: Mode transition diagrams: (a) that estimated with the theoretical jet oscillation model, (b) that estimated with the numerical model, and (c) that observed in experiment.

models. In particular, the blowing pressure ranges for all the oscillation regimes appearing in the experiment are fairly correctly estimated by the numerical model without any adjustment of the model parameters relating to the phase of the jet oscillation. The theoretical model can simulate the pressure ranges for the normal and overblow regimes correctly by virtue of adjustment of a model parameter, but it fails to simulate the correct ranges for the underblows.

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