Influence of temperature on sound transmission through viscoelastic sandwich plates

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A numerical study to investigate the effects of temperature on the diffuse sound transmission loss (TL) of sandwich plates is presented. The numerical prediction tool used is based on a finite element formulation for the sandwich plate coupled to a boundary element method for the acoustic medium. The plate formulation is derived from Kirchhoff's theory for the elastic faces and Mindlin's theory for the core. The frequency-temperature dependence of the viscoelastic material properties are taken into account using an experimentally derived viscoelastic constitutive law. The results presented deal with a laminated glass subjected to a diffuse sound field. It is found that the dip of the TL curve at the coincidence frequency of the plate is totally removed for temperatures where this frequency is in the transition region of the used viscoelastic material. Indeed, the relative low value of the storage modulus and the high value of the loss factor in this region, induce high transverse shear deformations of the viscoelastic interlayer and thus high energy losses.

1 Introduction

Vibration damping and noise control by means of sandwich structures with viscoelastic layer is commonly used in many industries [1]. In such structures, the main energy loss mechanism is due to the transverse shear of the viscoelastic core. However, the mechanical properties of viscoelastic materials vary with frequency and temperature. In this paper the transmission loss factor is used to analyze the effects of temperature and frequency on the vibro-acoustic response of sandwich plates. First, the variation of the mechanical properties of viscoelastic materials with temperature and frequency is described. Then, the numerical method used in this study is outlined. This method is based on a finite element formulation for the sandwich plate coupled to a boundary element method for fluid loading. Compared to the transmission loss computation based on infinite plate theory [2-4] or simply supported plate theory [5], the proposed approach has the following advantages: arbitrarily shaped three-layer plates with various boundary conditions can be handled; the acoustic excitation considered is a diffuse field noise which is the basis of standardized transmission loss measurements; fluid loading can be accounted for. The numerical results in term of noise transmission loss through a laminated glass are presented and the effects of temperature are discussed.

2 Characterisation of viscoelastic materials

A Viscoelastic material is characterized by possessing both energy storage and energy dissipation capability. For a linear viscoelastic material, the elastic and shear moduli are represented by complex quantities. The real part relates to the elastic behavior and the imaginary part relates to the material's viscous behavior and indicates its energy dissipation capability. The mechanical properties of viscoelastic materials vary considerably with frequency and temperature [6]. At high frequencies and/or low temperatures (glassy region) the material is stiff and relatively undamped. At low frequencies and/or high temperatures (rubbery region), the material is soft and the damping is small. At intermediate frequencies and temperature (transition region) the stiffness decreases rapidly with increasing temperature and the damping is highest. Different measurement techniques of the viscoelastic material properties are described in the literature [7-9]. Considering the temperature-frequency equivalence principle [10], measurements of the storage modulus and the loss factor as a function of frequency and temperature can be collapsed onto one master graph. This is achieved by plotting the data against a reduced frequency parameter, $\omega\alpha_T$, where ω is the actual frequency and α_T is an appropriate function of temperature. The viscoelastic material properties are thus fully characterized by a master curve at a nominal temperature and a law for the variation of the shift factor α_T with temperature. The core of the laminated glass used in this study is a PVB material. Its master curve, supplied by Saint Gobain Glass, is shown in Fig.1. A model of the following form is used for the temperature shift factor

$$\log \alpha_T = \frac{-c_1(T-T_0)}{c_2+T-T_0},$$

where c_1, c_2 and T_0 are material constants determined experimentally.

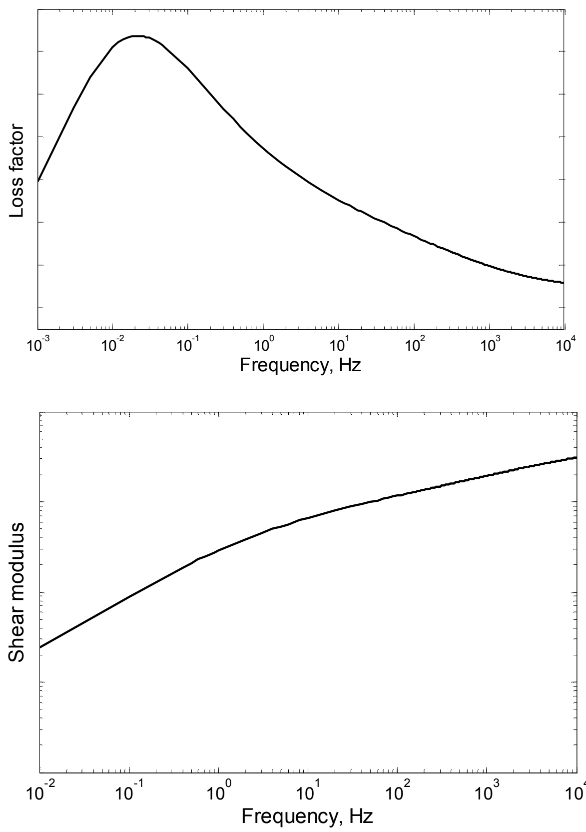

Fig.1 PVB material master curve

3 Formulation of sandwich plate equation

In the following, the finite element formulation for the transmission loss computation of the sandwich plate is outlined. Suppose that the sandwich plate is placed in an infinite rigid plane baffle and excited by an external acoustic field. The different layers are assumed to be in the (x, y) reference plane. The middle surface of the plate is assumed to be in $z=0$. The baffle separates two semi-infinite fluid domains, the excitation domain V^- ($z<0$) and the receiver domain V^+ ($z>0$). In order to determine the response of the sandwich plate certain assumptions are made in the formulation: (1) normal sections in each layer before deformation remain plane and continuous after deformation; (2) the transverse displacement remains constant throughout the thickness of the plate; (3) the face-layers are elastic and isotropic and they are subjected to extension, bending and in-plane shear deformations; (4) the core is subjected to extension, bending, in-plane shear deformations and transverse shear deformations; it is linearly viscoelastic with complex moduli in the frequency domain; (5) all displacements are assumed to be small and perfect continuity at the interfaces is assumed.

According to these assumptions, a linear displacement field is defined for each layer and the continuity of the displacement is enforced at the layer interfaces. The displacement field is thus given by

$$\begin{aligned} u_1 &= u_{\mathrm{m}} + \frac{u_{\mathrm{r}}}{2} - (z - z_1)\frac{\partial w}{\partial x}, \\ v_1 &= v_{\mathrm{m}} + \frac{v_{\mathrm{r}}}{2} - (z - z_1)\frac{\partial w}{\partial y}, \\ u_2 &= u_{\mathrm{m}} - \frac{u_{\mathrm{r}}}{2} - (z - z_2)\frac{\partial w}{\partial x}, \\ v_2 &= v_{\mathrm{m}} - \frac{v_{\mathrm{r}}}{2} - (z - z_2)\frac{\partial w}{\partial y}, \\ u_0 &= u_{\mathrm{m}} + z_{\mathrm{m}}\frac{\partial w}{\partial x} + z\varphi_x, \\ v_0 &= v_{\mathrm{m}} + z_{\mathrm{m}}\frac{\partial w}{\partial y} + z\varphi_y, \end{aligned} \tag{1}$$

where u_i, v_i are the in-plane displacements of the faces $(i=1,2)$ and of the core $(i=0)$, $w(x, y)$ is the transverse displacement of the plate, $u_{i0}, v_{i0} (i=1,2)$ are the in-plane displacements of the mid-surfaces of the top and bottom faces, φ_x and φ_y are the shear rotations of the normal to the core middle plane about x and y axes, z_i are the z coordinates of the mid-plane of the faces, h_0 is the thickness of the core, d is the distance between the mid-planes of the faces, and $z_{\mathrm{m}} = (z_1 + z_2)/2$. $u_{\mathrm{m}}, v_{\mathrm{m}}$ are the mean mid-plane displacements defined by

$$u_{\mathrm{m}} = \frac{u_{10} + u_{20}}{2}, v_{\mathrm{m}} = \frac{v_{10} + v_{20}}{2} \tag{2}$$

and $u_{\mathrm{r}}, v_{\mathrm{r}}$ are the relative mid-plane displacements defined by

$$u_{\mathrm{r}} = u_{10} - u_{20}, v_{\mathrm{r}} = v_{10} - v_{20} \tag{3}$$

The stress field is given by Hook's law as follows

$$\begin{cases} \boldsymbol{\sigma}_{\mathrm{i}} = \mathbf{E}_{\mathrm{i}}\boldsymbol{\varepsilon}_{\mathrm{i}} & \mathrm{i} = 0,1,2 \\ \boldsymbol{\tau}_0 = \mathbf{E}_{\mathrm{s}}\boldsymbol{\gamma}_0 \end{cases} \tag{4}$$

where $\boldsymbol{\varepsilon}_{\mathrm{i}}$ and $\boldsymbol{\sigma}_{\mathrm{i}}$ are respectively the in-plane strain and stress vectors in each layer, $\boldsymbol{\gamma}_0$ and $\boldsymbol{\tau}_0$ are the transverse shear strain and stress vectors in the core. $\mathbf{E}_{\mathrm{i}}$ and $\mathbf{E}_{\mathrm{s}}$ are the elasticity matrices. For isotropic materials, the components of the elasticity matrices are

$$\mathbf{E}_i = \begin{bmatrix} \frac{E_i}{1-\nu_i^2} & \frac{E_i\nu_i}{1-\nu_i^2} & 0 \\ \frac{E_i\nu_i}{1-\nu_i^2} & \frac{E_i}{1-\nu_i^2} & 0 \\ 0 & 0 & \frac{E_i}{2(1+\nu)} \end{bmatrix}; \mathbf{E}_{\mathrm{s}} = \begin{bmatrix} \frac{E_0}{2(1+\nu)} & 0 \\ 0 & \frac{E_0}{2(1+\nu)} \end{bmatrix} \tag{5}$$

where E_i is the Young's modulus and ν_i the Poisson's ratio of the layer i. For the linear viscoelastic material, the Young's modulus is represented in complex form and is frequency-temperature dependent

$$E_0(\omega,T) = E_v(\omega,T)(1 + \mathrm{i}\eta_v(\omega,T))$$

where E_v is the storage modulus, η_v is the loss factor of the viscoelastic material and $\mathrm{i} = \sqrt{-1}$.

The strain and kinetic energies of the sandwich plate can be written as

$$U = \frac{1}{2}\int_{\Omega} \boldsymbol{\sigma}_i^{\mathrm{T}}\boldsymbol{\varepsilon}_i \, d\Omega, \tag{6}$$

$$V = \frac{1}{2}\int_{\Omega} \rho_i(\dot{u}_i^2 + \dot{v}_i^2 + \dot{w}_i^2) d\Omega, \tag{7}$$

where ρ_i is the mass density for the *i-th* layer and the dot symbol (.) stands for time derivation. $\dot{u}_i, \dot{v}_i, \dot{w}_i$ are the components of the velocity vector. Substituting Eqs. (1) and (4) into Eqs. (6) and (7) and integrating through the plate thickness yields the strain and kinetic energies in terms of the mean mid-plane displacements of the faces u_m, v_m, the relative mid-plane displacements $u_{\mathrm{r}}, v_{\mathrm{r}}$ and the transverse deflection w.

The discrete forms of the strain energy and the kinetic energy, obtained by following conventional finite element procedure [11], give the stiffness ($\mathbf{K}$) and mass ($\mathbf{M}$) matrices of the sandwich plate as

$$U = \frac{1}{2}\mathbf{u}_{\mathrm{n}}^{\mathrm{T}}\mathbf{K}\mathbf{u}_{\mathrm{n}}, \quad V = -\omega^2\frac{1}{2}\mathbf{u}_{\mathrm{n}}^{\mathrm{T}}\mathbf{M}\mathbf{u}_{\mathrm{n}} \tag{8}$$

where $\mathbf{u}_{\mathrm{n}}$ is the nodal dof vector. The details of their derivation are given in the fourth chapter of reference [12].

The work done by the acoustic pressure on the plate is

$$W = \int_A (p^- - p^+) w ds + \int_A p_b w dS. \tag{9}$$

where A is the area of the plate, $p_b = p_i + p_r$ is the blocked pressure which is the sum of the incident pressure, p_{i}, and the reflected pressure, p_{r}, on the incident side when the plate is considered as a hard wall. The variables p^+ and p^- are respectively the radiated pressure in the half spaces $z > 0$ and $z < 0$. The radiated pressure is given by the classical Rayleigh integral

$$p^{+,-} = \int_A -\rho\omega^2 wG dS \quad \text{in } V^{+,-}, \tag{10}$$

where ρ is the density of the fluid and $G = \dfrac{e^{-ikr}}{2\pi r}$ is the half-space free-field Green's function satisfying $\dfrac{\partial G}{\partial n} = 0$ on the baffle. Substituting Eq. (10) into Eq. (9) gives

$$W = \int_A \int_A 2\rho\omega^2 wGwdS\, dS + \int_A p_b wdS. \tag{11}$$

The discrete form of the potential loading is obtained by using a boundary element method [13]

$$W = \omega^2 \mathbf{u}_{\mathrm{n}}^{\mathrm{T}} \mathbf{Z}_{\mathrm{R}} \mathbf{u}_{\mathrm{n}} + \mathbf{u}_{\mathrm{n}}^{\mathrm{T}} \mathbf{F}, \tag{12}$$

$\mathbf{Z}_{\mathrm{R}}$ and $\mathbf{F}$ are respectively the radiation impedance matrix and the source vector resulting from the acoustic excitation.

To derive the governing equation of motion for the sandwich plate, the variational formulation is applied in the following form

$$\delta V - \delta U + \delta W = 0, \tag{13}$$

where δV, δU and δW are respectively, the virtual variations of kinetic energy, strain energy and the work done by the external loads. Substituting Eqs. (8) and (12) into Eq. (13) leads to the following equation of motion

$$\left[\mathbf{K}(\omega,T) - \omega^2(\mathbf{M} + \mathbf{Z}_{\mathrm{R}}(\omega))\right]\mathbf{u}_{\mathrm{n}}(\omega,T) = \mathbf{F}(\omega). \tag{14}$$

The TL through the sandwich plate is defined by the ratio of the incident sound power to the transmitted sound power. The transmitted sound power is given by

$$\Pi_{\mathrm{t}} = \frac{1}{2}\mathrm{Re}\int_A p^+ v^* dS = \frac{\omega}{2}\mathrm{Im}\int_A p^+ w^* dS, \tag{15}$$

where v is the normal velocity of the sandwich plate (* denotes the complex conjugate). Substituting Eqs. (10) into Eq. (15) gives

$$\Pi_{\mathrm{t}} = \frac{\omega^3}{2}\mathrm{Im}\int_A\int_A -\rho wGw^* dSdS = -\frac{\omega^3}{4}\mathrm{Im}(\mathbf{u}_{\mathrm{n}}^{\mathrm{T}}\mathbf{Z}_{\mathrm{R}}\mathbf{u}_{\mathrm{n}}^*). \tag{16}$$

4 Numerical results

In this section, the acoustical response of a laminated glass is analyzed. The objective is to investigate the effects of temperature on the noise transmission loss. The laminated glass is $1.48\,\mathrm{m}$ long and $1.23\,\mathrm{m}$ wide with simply supported boundary conditions. The characteristics of the faces and of the viscoelastic core are as follows

$h_{1,2}$=4mm; $E_{1,2}$=72 GPa; $\nu_{1,2}$=0.22; $\rho_{1,2}$=2500 kg/m^3

h_0=0.76mm; ν_0=0.5; ρ_0=1000 kg/m^3

The PVB material properties are supplied by Saint Gobain Glass (Fig.2 and Fig.3). The surrounding fluid is air with density $\rho = 1.2\ \mathrm{kg/m^3}$ and sound speed $c = 340\,\mathrm{m/s}$. The TL of the sandwich plate is investigated in the frequency range [50-5000 Hz] and at the following values of temperature 15, 20, 50 and 60°C.

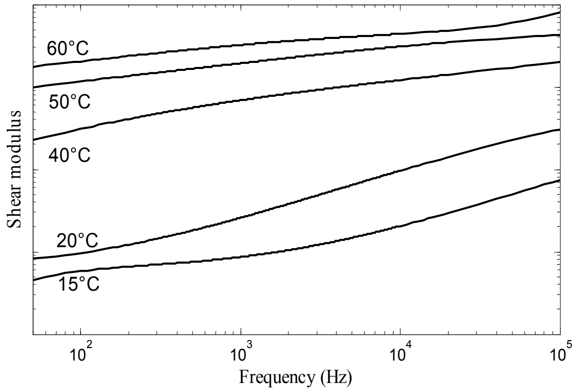

Fig.2 Shear modulus of the PVB material.

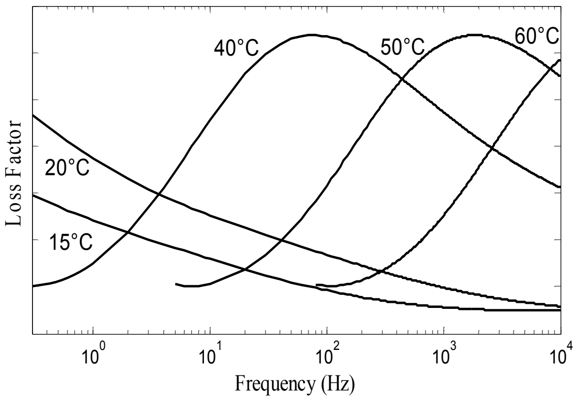

Fig.3 Loss factor of the PVB material.

The variation of the shear modulus and the loss factor of the PVB material with respect frequency and temperature is shown in Fig.2 and Fig.3. These figures show that with increasing temperature, the shear modulus decreases and the transition region moves toward the high frequencies.

The computed TL with the proposed formulation is presented in Fig.4. It is clearly seen that the variation of the temperature affects mainly the TL curve at the coincidence frequency region. At low temperatures (15, 20°C), the PVB material is characterized by low loss factor values in the frequency range of interest. Consequently, the dip of the TL curve caused by wave matching between incidence sound wave and structural wave at the coincidence frequency is clearly observed. At 50°C, this coincidence frequency is in the transition region of the viscoelastic material. In this transition region, the relative low value of the shear modulus increases the shear deformation in the viscoelastic core and the high value of the loss factor increases the energy dissipation. The consequence is that the TL dip is totally removed at the coincidence frequency. At 60°C the coincidence frequency becomes in the lower limit of the transition region and thus the dip reappears.

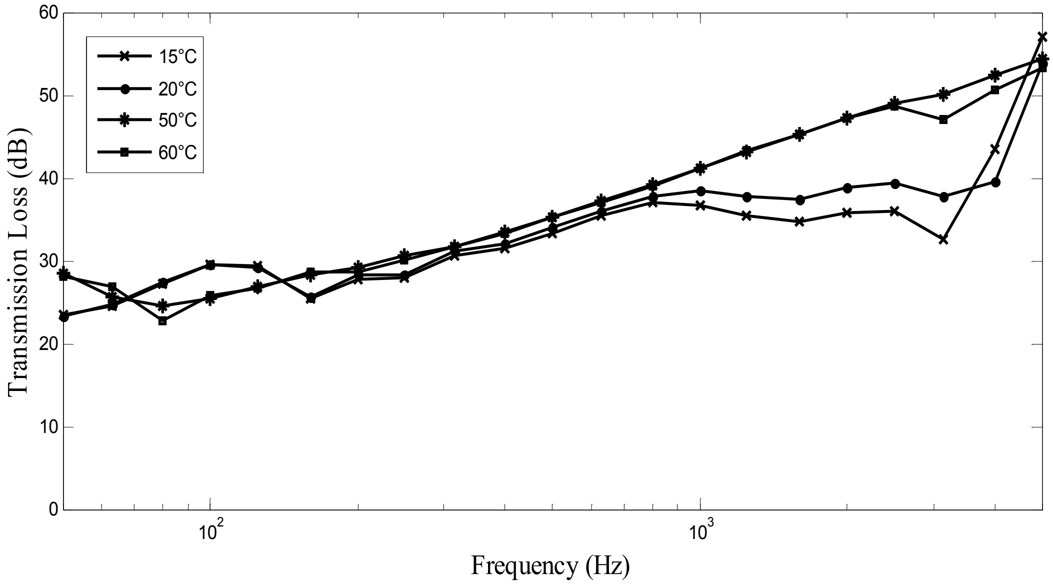

Fig.4 Transmission loss of the laminated glass

5 Conclusion

In this study, a laminated glass with viscoelastic core was investigated by using a numerical tool based on a finite element formulation for the sandwich structure coupled to a boundary element method for fluid loading. The frequency-temperature dependence of the viscoelastic material properties was taken into account using an experimentally derived master curve and using the complex moduli approach. The vibroacoustic response of the sandwich plate was analyzed in the frequency range [50-5000 Hz] and at 15, 20, 50 and 60°C. It was found that the dip of the TL curve at the coincidence frequency is totally removed when this frequency is in the transition region of the viscoelastic material.

The presented method could be used in a parametric study and/or optimization in order to improve the vibroacoustic characteristics of sandwich plates taking into account both frequency and temperature dependence of the viscoelastic material properties.

Acknowledgments

The authors gratefully acknowledge the financial support of this work by Saint Gobain Glass.

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