Acoustics'08 Paris
June 29-July 4, 2008
www.acoustics08-paris.org

A transversal substructuring modal method for the acoustic analysis of dissipative mufflers with mean flow

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This work presents a modal approach to evaluate the transversal modes and wavenumbers for dissipative mufflers with mean flow. The method is based on the division of the transversal section of the muffler into subdomains, for which two simple sets of modes are considered. The first set of modes satisfies the condition of zero pressure at the common boundary between subdomains, while the second fulfils the condition of zero derivative in the direction normal to this boundary. From these sets, a substructuring procedure is applied that provides the final modes of the complete cross section, considering the presence of absorbent material, a perforate and mean flow. The technique avoids iterative schemes associated with the nonlinear characteristic equation found, for instance, in the analytical modelling of perforated dissipative circular mufflers. Once the final transversal modes have been calculated, the mode matching technique is applied at the geometrical discontinuities to completely define the acoustic field inside the muffler. The acoustic attenuation is then predicted by means of the transmission loss. Comparison with results available in the literature shows good agreement. The attenuation of a dissipative muffler is presented, including the presence of a perforated duct and mean flow.

1 Introduction

Mufflers used in the exhaust system of internal combustion engines are subjected to gas flow that can be approximately modelled by uniform mean flow for some configurations. Due to the need to ensure adequate noise attenuation across a wide frequency band, it is advisable to incorporate both reactive elements (geometry changes) with satisfactory low frequency acoustic behaviour and dissipative elements (absorbent materials) that provide noise attenuation for higher frequencies.

The modelling of perforated pipes is usually carried out by an acoustic impedance [1]. If there is no mean flow, impedance models fitted from experimental data provide a fairly close reproduction of the muffler behaviour. The presence of grazing mean flow modifies the impedance in the absence of flow [1, 2] and the different models developed give a reasonable approximation to the problem. The presence of absorbent material on one side of the perforated surface also modifies the acoustic impedance, according to Kirby and Cummings [3], for which allowance should be made.

Due to their versatility, for a long time classical numerical methods such as the finite element method (FEM) and boundary element method (BEM) have been in use for the acoustic analysis of mufflers [4, 5, 6, 7]. However, their high computational cost for three-dimensional analysis prohibits their use in many practical applications. When sound wave propagation is three-dimensional, the use of modal methods has great advantages from the computational standpoint and is therefore a suitable analysis option. In particular, the mode matching method is especially useful, the technique being based on the transversal modal solution of each of the components forming the muffler and evaluating the amplitude of the waves by imposing the continuity conditions of the pressure and axial velocity at the geometrical discontinuities between the muffler components. The continuity required in the acoustic fields is imposed by the weighted residual method, in which the weighting functions are the transversal modes themselves [8]. The use of separation of variables allows the pressure field to be expressed in each component as a modal expansion in which each term is the product of an axial mode and a transversal mode that may be evaluated according to the geometrical complexity of the transversal section.

Many authors have used the mode matching method to study the acoustic behaviour of reactive mufflers with simple cross section, in which the transversal modes are defined analytically. Selamet et al. studied expansion and flow-reversing chambers, including the effect of different inlet/outlet positions and extended ducts, without considering the presence of mean flow [8, 9, 10].

With simple geometries such as the circular concentric resonator, the mode matching method has been used to solve problems that include a perforated duct. The solution of the associated characteristic equation allows transversal and axial wavenumbers to be calculated, although their highly nonlinear nature complicates the process (this method will hereinafter be referred to as the direct method). The same methodology can also be applied to the case of uniform mean flow with absorbent material in the chamber. However, obtaining all the necessary modes from the characteristic equation may on occasions be difficult [11, 12]. For more complex geometries, the direct method cannot be used to calculate the analytical transversal modes, but this is possible with the finite element method in two dimensions [13]. Additionally, a modal methodology has recently been presented that allows the calculation of the transversal modes, incorporating both perforated elements and absorbent materials [14]. The method is based on using two simple sets of modes for the subdomains associated with a cross section, and provides a good convergence. This paper presents its extension to the case of the presence of uniform mean flow. After the calculation of transversal modes for each of the components forming the muffler, the mode matching method is applied to assess the acoustic attenuation performance of the muffler.

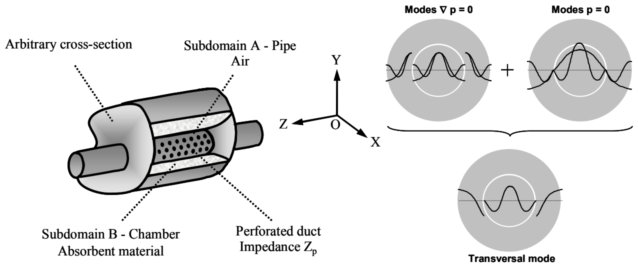
2 Substructuring method

2.1 Formulation of the acoustic problem

The following hypotheses are assumed: (1) An *OXYZ* Cartesian reference system will be used in which the *OZ* axis is the muffler axial direction; (2) the muffler can be divided into components with uniform cross section; (3) the transversal section of each component can also be divided into disjoint subdomains connected either directly or by perforated elements; (4) the medium of propagation in each transversal subdomain (air or absorbent material) is defined by its acoustic impedance and wavenumber; (5) the perforated duct will be modelled by acoustic impedance Z_p;

and (6) the uniform mean flow in each subdomain is known and it is associated with the axial direction.

To obtain the pressure field inside the muffler, the mode matching method is used. This requires the previous evaluation of the transversal modes associated with the muffler components, and then matching at the geometrical discontinuities. The solution of the differential equation in each component can be expressed in terms of the modes associated with the transversal section. In this paper a methodology is proposed based on modal synthesis which will allow the transversal modes to be obtained in the case of complex geometries that include absorbent materials and perforated ducts with uniform mean flow.



(a) (b)

Fig. 1 (a) Muffler and subdomains considered for the application of the component mode synthesis approach. (b) Substructuring scheme used to calculate transversal modes.

Although the methodology can be applied to other geometries, the scheme shown in Fig. 1 (a) is considered to facilitate the presentation, consisting of two subdomains connected by a perforated duct. The medium of propagation in the central duct is assumed to be air and in the duct there is uniform axial flow in the direction *OZ*. The lateral chamber contains an absorbent material. To expand the pressure in each of the two subdomains, two bases will be used, as seen in Fig. 1 (b). The first set of modes is obtained assuming that in the boundary associated with the perforated duct the derivative of the pressure in the normal direction is zero, while the second set is computed with zero pressure at the perforated interface. To establish the coefficients of the pressure expansion it will be necessary to satisfy the acoustic conditions associated with the perforated element.

If temporal variation is assumed to be harmonic, sound propagation in the subdomains *A* and *B* is given by equations [1]

$$\frac{\partial^2 p_A}{\partial x^2}+\frac{\partial^2 p_A}{\partial y^2}+\left(1-M^2\right)\frac{\partial^2 p_A}{\partial z^2}-2\,\mathrm{j}M\,k_0\frac{\partial p_A}{\partial z}+k_0^2\,p_A=0 \quad (1)$$

$$\nabla^2 p_B+\tilde{k}^2\;p_B=0 \quad (2)$$

where ∇^2 is the Laplacian operator, p_A and p_B are the complex acoustic pressure amplitudes in each subdomain, k_0 is the wavenumber defined by ω/c_0, c_0 being the speed of sound, ω the angular frequency, $\tilde{k}$ the complex wavenumber in the absorbent material and *M* the Mach number associated with the uniform axial flow, defined by U_0/c_0, U_0 being the mean flow velocity.

If the transversal section is uniform throughout the component, by using separation of variables the pressure amplitude at any point in the section can be expressed by:

$$p(x,y,z)=\Psi^{xy}(x,y)\,\mathrm{e}^{-\mathrm{j}k_z z};\;\Psi^{xy}(x,y)=\begin{cases}\Psi_A^{xy}(x,y) & (x,y)\in A\\ \Psi_B^{xy}(x,y) & (x,y)\in B\end{cases} \quad (3)$$

where Ψ^{xy} denotes a transversal pressure mode and k_z the axial wavenumber. If Eq. (3) is added to Eq. (1) and Eq. (2) the axial mode can be eliminated, so that

$$\nabla^2\,\Psi_A^{xy}+\left[k_0^2-2\,M\,k_0\,k_z-\left(1-M^2\right)k_z^2\right]\Psi_A^{xy}=0 \quad (4)$$

$$\nabla^2\,\Psi_B^{xy}+\left(\tilde{k}^2-k_z^2\right)\Psi_B^{xy}=0 \quad (5)$$

Eq. (4) can also be expressed as

$$\nabla^2\,\Psi_A^{xy}+k_{A,t}^2\,\Psi_A^{xy}=0 \quad (6)$$

where $k_{A,t}$ is the transversal wavenumber associated with the transversal mode in subdomain *A*, which is related to the axial wavenumber (of order *s*) by

$$k_{z,s}=\left(-M\,k_0\pm\sqrt{k_0^2-\left(1-M^2\right)k_{A,t,s}^2}\right)\Big/\left(1-M^2\right) \quad (7)$$

The pressure field in the component can be expressed by the series expansion

$$p(x,y,z)=\sum_{s=0}^{\infty}\left(C_s^+\,\mathrm{e}^{-\mathrm{j}k_{z,s}^+ z}\,\Psi_s^{xy+}+C_s^-\,\mathrm{e}^{-\mathrm{j}k_{z,s}^- z}\,\Psi_s^{xy-}\right) \quad (8)$$

where $k_{z,s}^+$ and $k_{z,s}^-$ are the progressive and regressive waves, respectively, in the component, and C_s^+ and C_s^- are the corresponding wave amplitudes.

2.2 Computation of transversal modes

Obtaining the modes of the complete transversal section requires imposing the conditions on all the boundaries of the subdomains of which it is composed. As all the modes utilised satisfy the conditions in the external boundary, except at the interface with the perforated element, to reach the solution it will only be necessary to impose the perforated impedance Z_p on the corresponding interface.

In this section, a formulation will be presented for obtaining the transversal modes of a component formed by various subdomains connected by a perforated element. Transversal pressure in subdomains *A* and *B* can be expressed by

$$\Psi_A^{xy}=\sum_{r=0}^{\infty}\phi_{A,r}^{xy,u}\,q_{A,r}^{u}+\sum_{r=1}^{\infty}\phi_{A,r}^{xy,p}q_{A,r}^{p} \quad (9)$$

$$\Psi_B^{xy}=\sum_{r=0}^{\infty}\phi_{B,r}^{xy,u}\,q_{B,r}^{u}+\sum_{r=1}^{\infty}\phi_{B,r}^{xy,p}q_{B,r}^{p} \quad (10)$$

where $\phi_{A,r}^{xy,u}$, $\phi_{B,r}^{xy,u}$ and $\phi_{A,r}^{xy,p}$, $\phi_{B,r}^{xy,p}$ are the pressure modes of the bases obtained for each subdomain with zero normal pressure gradient and zero pressure at the perforated surface, respectively, and $q_{A,r}^{u}$, $q_{B,r}^{u}$ and $q_{A,r}^{p}$, $q_{B,r}^{p}$ are the modal participation factors. From a practical point of view, the modal expansion must be truncated and only natural modes N_A^u, N_B^u and zero pressure modes at the interface N_A^p, N_B^p are used. Eqs. (9) and (10) can be expressed as

$$\Psi^{xy}(x,y)=\begin{cases}\Psi_A^{xy}=\displaystyle\sum_{r=0}^{N_A^u+N_A^p}\phi_{A,r}^{xy}(x,y)\,q_{A,r}=\mathbf{\Phi}_A^{\mathrm{T}}\cdot\mathbf{q}_A & (x,y)\in A\\ \Psi_B^{xy}=\displaystyle\sum_{r=0}^{N_B^u+N_B^p}\phi_{B,r}^{xy}(x,y)\,q_{B,r}=\mathbf{\Phi}_B^{\mathrm{T}}\cdot\mathbf{q}_B & (x,y)\in B\end{cases} \quad (11)$$

Each transversal mode is defined by its corresponding modal participation factors $\mathbf{q}_A$ and $\mathbf{q}_B$. To obtain these factors the weighted residuals method is considered. Applying this methodology to Eq. (4) we obtain [14]

$$\left[k_0^2-2Mk_0k_z-\left(1-M^2\right)k_z^2\right]\int_{\Omega_A}\mathbf{\Phi}_A\cdot\mathbf{\Phi}_A^{\mathrm{T}}\cdot\mathbf{q}_A\,\mathrm{d}\Omega \\ -\int_{\Omega_A}\nabla\mathbf{\Phi}_A\left(\nabla\mathbf{\Phi}_A\right)^{\mathrm{T}}\cdot\mathbf{q}_A\,\mathrm{d}\Omega=-\int_{\Gamma_I}\mathbf{\Phi}_A\,\partial\Psi_A^{xy}/\partial n\,\mathrm{d}\Gamma \tag{12}$$

All the modes used in the expansion of Ψ_A^{xy} satisfy the rigid wall conditions at the non-perforated boundaries, so that the integral to the right of Eq. (12) can be eliminated except in the case of the perforated element itself. To evaluate this integral, the coupling of both subdomains at their interface must be imposed, which satisfies the impedance condition Z_p in the perforated element, plus an additional continuity condition. The impedance condition in the perforated element is defined by

$$Z_p=\left(\Psi_A^{xy}-\Psi_B^{xy}\right)/u_{An} \tag{13}$$

The additional condition has been studied by various authors and at the present time it has not been perfectly established whether the continuity imposed should be acoustic velocity or displacement [12]. The pressure gradient in the normal direction to the perforated element can be obtained from Euler's equation. Both options can be combined in the following equation [12]

$$\partial\Psi_A^{xy}/\partial n=-\rho_0\,Du_{An}/Dt=-\mathrm{j}\rho_0\,\omega\left(1-Mc_0k_z/\omega\right)^{Cont}u_{An} \tag{14}$$

where *Cont* takes the value of 1 in the case of the continuity of velocity and 2 for continuity of displacement. The following equation is obtained by combining Eq. (13) and Eq. (14)

$$\partial\Psi_A^{xy}/\partial n=-\mathrm{j}\rho_0\,\omega\left(1-Mc_0k_z/\omega\right)^{Cont}\left(\Psi_A^{xy}-\Psi_B^{xy}\right)/Z_p \tag{15}$$

Adding the pressure gradient from Eq. (12), we get

$$\left[k_0^2-2Mk_0k_z-\left(1-M^2\right)k_z^2\right]\int_{\Omega_A}\mathbf{\Phi}_A\cdot\mathbf{\Phi}_A^{\mathrm{T}}\cdot\mathbf{q}_A\,\mathrm{d}\Omega \\ -\int_{\Omega_A}\nabla\mathbf{\Phi}_A\left(\nabla\mathbf{\Phi}_A\right)^{\mathrm{T}}\cdot\mathbf{q}_A\,\mathrm{d}\Omega \\ =\frac{\mathrm{j}\rho_0\,\omega}{Z_p}\left(1-\frac{Mc_0}{\omega}k_z\right)^p\int_{\Gamma_I}\mathbf{\Phi}_A\cdot\left(\mathbf{\Phi}_A^{\mathrm{T}}\cdot\mathbf{q}_A-\mathbf{\Phi}_B^{\mathrm{T}}\cdot\mathbf{q}_B\right)\mathrm{d}\Gamma \tag{16}$$

The following notation is defined

$$\mathbf{MA}^A=\int_{\Omega_A}\mathbf{\Phi}_A\cdot\mathbf{\Phi}_A^{\mathrm{T}}\,\mathrm{d}\Omega\,;\ \mathbf{KA}^A=\int_{\Omega_A}\nabla\mathbf{\Phi}_A\left(\nabla\mathbf{\Phi}_A\right)^{\mathrm{T}}\mathrm{d}\Omega \\ \mathbf{PA}^{AA}=\int_{\Gamma_I}\mathbf{\Phi}_A\cdot\mathbf{\Phi}_A^{\mathrm{T}}\,\mathrm{d}\Gamma\,;\ \mathbf{PA}^{AB}=\int_{\Gamma_I}\mathbf{\Phi}_A\cdot\mathbf{\Phi}_B^{\mathrm{T}}\,\mathrm{d}\Gamma \tag{17}$$

Using this notation and assuming continuity of velocities (which means adopting the value *Cont* = 1), Eq. (16) can be expressed by

$$\left[\mathbf{K}^{AA}+\mathbf{D}^{AA}k_z+\mathbf{M}^{AA}k_z^2\right]\cdot\mathbf{q}_A+\left[\mathbf{K}^{AB}+\mathbf{P}^{AB}k_z\right]\cdot\mathbf{q}_B=0 \tag{18}$$

where

$$\mathbf{K}^{AA}=k_0^2\,\mathbf{MA}^A-\mathbf{KA}^A-\mathrm{j}\rho_0\,\omega\mathbf{PA}^{AA}/Z_p \\ \mathbf{D}^{AA}=\mathrm{j}\rho_0\,Mc_0\,\mathbf{PA}^{AA}/Z_p-2Mk_0\,\mathbf{MA}^A \\ \mathbf{M}^{AA}=-\left(1-M^2\right)\mathbf{MA}^A\,;\ \mathbf{K}^{AB}=\mathrm{j}\rho_0\,\omega\mathbf{PA}^{AB}/Z_p \\ \mathbf{P}^{AB}=-\mathrm{j}\rho_0\,Mc_0\,\mathbf{PA}^{AB}/Z_p \tag{19}$$

From a similar procedure for Eq. (5) we obtain:

$$\mathbf{K}^{BA}\cdot\mathbf{q}_A+\left[\mathbf{K}^{BB}+\mathbf{M}^{BB}k_z^2\right]\cdot\mathbf{q}_B=0 \tag{20}$$

where the matrices that appear are defined by

$$\mathbf{K}^{BB}=\tilde{k}^2\,\mathbf{MA}^B-\mathbf{KA}^B-\frac{\mathrm{j}\tilde{\rho}\omega}{Z_p}\mathbf{PA}^{BB} \\ \mathbf{M}^{BB}=-\mathbf{MA}^B\,;\ \mathbf{K}^{BA}=\frac{\mathrm{j}\tilde{\rho}\omega}{Z_p}\mathbf{PA}^{BA} \tag{21}$$

and the matrices associated with the integrals are

$$\mathbf{MA}^B=\int_{\Omega_B}\mathbf{\Phi}_B\cdot\mathbf{\Phi}_B^{\mathrm{T}}\,\mathrm{d}\Omega;\ \mathbf{KA}^B=\int_{\Omega_B}\nabla\mathbf{\Phi}_B\left(\nabla\mathbf{\Phi}_B\right)^{\mathrm{T}}\mathrm{d}\Omega \\ \mathbf{PA}^{BB}=\int_{\Gamma_I}\mathbf{\Phi}_B\cdot\mathbf{\Phi}_B^{\mathrm{T}}\,\mathrm{d}\Gamma;\ \mathbf{PA}^{BA}=\int_{\Gamma_I}\mathbf{\Phi}_B\cdot\mathbf{\Phi}_A^{\mathrm{T}}\,\mathrm{d}\Gamma \tag{22}$$

Eqs. (18) and (20) can be grouped as

$$\left(\begin{bmatrix}\mathbf{K}^{AA} & \mathbf{K}^{AB}\\ \mathbf{K}^{BA} & \mathbf{K}^{BB}\end{bmatrix}+\begin{bmatrix}\mathbf{D}^{AA} & \mathbf{P}^{AB}\\ 0 & 0\end{bmatrix}k_z+\begin{bmatrix}\mathbf{M}^{AA} & 0\\ 0 & \mathbf{M}^{BB}\end{bmatrix}k_z^2\right)\begin{Bmatrix}\mathbf{q}_A\\ \mathbf{q}_B\end{Bmatrix}=0 \tag{23}$$

Eq. (23) represents an eigenvalue problem in which each axial wavenumber k_z is associated with a transversal mode (eigenvector) defined by the modal participation factors. This eigenvalue problem can be solved by conventional methods, which is an improvement in comparison with the analytical direct method, in which the transversal wavenumbers are obtained by solving the nonlinear characteristic equation. It also avoids the problem of modal jump and the subsequent loss of modes. With regard to the application of FEM to obtain the final transversal modes, the matrices considered to compute the initial sets of modes $\phi_{A,r}^{xy,u}$, $\phi_{B,r}^{xy,u}$ and $\phi_{A,r}^{xy,p}$, $\phi_{B,r}^{xy,p}$ area also valid for the problem expressed by Eq. (23), and therefore the computational effort is reduced [14].

The number of transversal modes that can be computed depends on the number of equations available, that is, on the number of basic modes used in each subdomain. As expected, the accuracy in the evaluation of the transversal modes improves as the number of modes is increased, thus leading to a suitable convergence.

2.3 Solution of the acoustic field in each component

When the transversal modes have been computed, Eq. (3) allows us to express the pressure field in a component s by

$$p_s=\sum_{r=1}^{N}C_{s,r}\,\Psi_{s,r}^{xy}\,\mathrm{e}^{-\mathrm{j}k_z^{(s,r)}z} \tag{24}$$

where the wave amplitudes $C_{s,r}$ are unknown and must be established from the continuity conditions at component interfaces by the mode matching method. The continuity conditions involve pressure as well as axial velocity, since *Cont* = 1 has been assumed previously [12]. The axial acoustic velocity can be computed by Euler's equation, leading to

$$u_z^{(s)}=\frac{1}{\rho_0\,c_0}\sum_{r=1}^{N}\frac{k_z^{(s,r)}}{k_0-M\,k_z^{(s,r)}}C_{s,r}\,\Psi_{s,r}^{xy}\,\mathrm{e}^{-\mathrm{j}k_z^{(s,r)}z} \tag{25}$$

2.4 Acoustic field in the muffler

Using the acoustic solution for each component given by Eq. (24) and Eq. (25), and weakly imposing the continuity of pressures and axial acoustic velocities at component interfaces by the mode matching method, all the wave amplitudes $C_{s,r}$ can be obtained, thus solving the acoustic problem in the muffler. When applying the mode matching method, the integration of the products between modes extended to the transversal subdomains must be evaluated. If the initial base modes have been obtained by the finite element method, these integrals can be evaluated by using the same element mass matrices [14].

Finally, the transmission loss (TL) can be calculated directly by

$$\mathrm{TL} = -20\log\left[\sum_{r=1}^{N} C_{TS,r}\,\Psi_{TS,r}^{xy}\,\mathrm{e}^{-\mathrm{j}k_z^{(TS,r)} z_{outlet}}\right] \qquad (26)$$

where TS refers to the component associated with the outlet duct and z_{outlet} is the coordinate of the muffler outlet section. In Eq. (26), an incident plane wave with unity amplitude is assumed as excitation, and an anechoic termination is imposed at the outlet.

3 Results

To validate the results obtained from the proposed method, these were compared to those obtained by Xu et al [15] by the direct method (modified to include mean flow), which allows the study of concentric resonators with absorbent material in the chamber and the presence of a perforated duct. Fig. 2 shows the geometry of the circular concentric resonator studied in two configurations. In Configuration 1 the muffler has a perforated duct that separates the central passage from the outer chamber filled with absorbent material, while in Configuration 2 this perforate duct is removed.

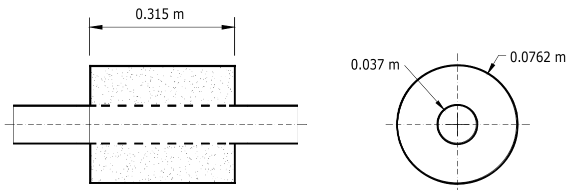


Fig. 2 Geometry of dissipative muffler.

The aim of the study is to establish the validity of the procedure both with and without perforated duct. The perforated pipe in Configuration 1 has the following properties: thickness $t = 0.001$ m, hole diameter $d_h = 0.0035$ m and porosity $\sigma = 0.263$. To model the perforated pipe, the impedance model with tangential mean flow proposed by Lee and Ih [2] was used, without including the effect due to the presence of the absorbent material [3], which was characterised by Delany and Bazley's two-parameter model [16], as defined by the following equations.

$$\frac{\tilde{k}}{k_0} = 1 + a_3\left(\omega/(2\pi R)\right)^{a_4} + \mathrm{j}a_1\left(\omega/(2\pi R)\right)^{a_2} \qquad (27)$$

$$\frac{\tilde{Z}}{Z_0} = 1 + a_5\left(\omega/(2\pi R)\right)^{a_6} + \mathrm{j}a_7\left(\omega/(2\pi R)\right)^{a_8} \qquad (28)$$

where the flow resistivity of the material used is R = 30716 rayl/m and the model is defined Table 1.

a_1	a_2	a_3	a_4
-0.2202	-0.5850	0.2010	-0.5829
a_5	a_6	a_7	a_8
0.0954	-0.6687	-0.1689	-0.5707

Table 1 Model for the absorbent material

To validate the proposed methodology, a uniform mean flow was considered in the central duct with M = 0.15. The wavenumbers of the first axial modes obtained from the two concentric resonators of Configurations 1 and 2 were compared with the direct method proposed by Xu et al. [15]. Figs. 3 and 4 show the results for both mufflers at the highest excitation frequency used in the computations, which could be expected to provide the maximum possible error. As can be seen from the graphs, there is close correspondence between both results, which demonstrates the validity of the proposed method, with and without perforated duct. The results were obtained with six modes in the central duct and seven in the chamber.

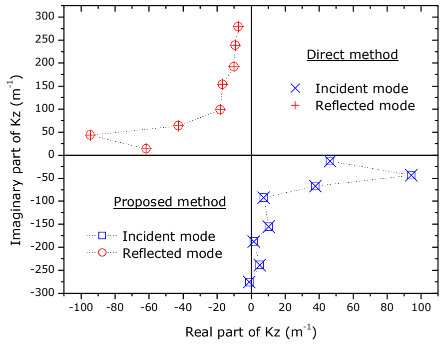


Fig. 3 Axial wavenumbers of dissipative muffler with perforated duct for an excitation frequency of 3.2 kHz.

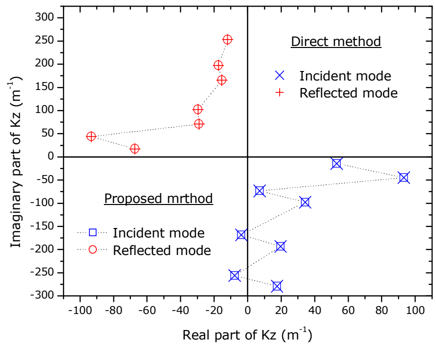


Fig. 4 Axial wavenumbers of dissipative muffler without perforated duct for an excitation frequency of 3.2 kHz.

Fig. 5 depicts the moduli of the transversal modes of progressive waves at an excitation frequency of 3.2 kHz in Configuration 1. The wavenumber can be seen to rise steadily. This means that none of the modes has been lost, as may happen with the direct method. The derivative of the pressure at the perforated interface can also be seen to jump, as expected.

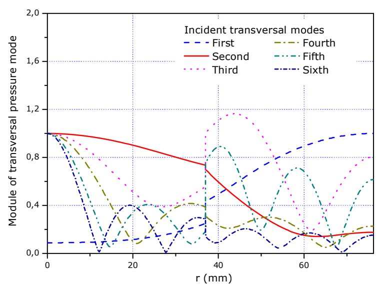


Fig. 5 Modulus of first transversal modes at a frequency of 3.2 kHz in Configuration 1.

Fig. 6 shows the transmission loss (TL) obtained by both methods for the mufflers studied. Computations were carried out with the modes indicated above. The results again show good agreement. Fig. 6 also shows the high attenuation obtained at high frequencies due to the presence of absorbent material and the disappearance of the typical pass bands associated with the propagation of transversal modes in reactive mufflers.

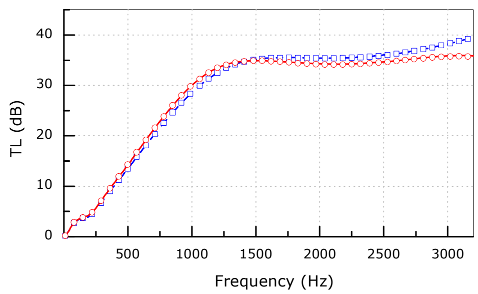


Fig. 6 TL of dissipative muffler: ——, direct method with perforations; ooo, proposed method with perforations; ----, direct method without perforations; □□□, proposed method without perforations.

4 Conclusions

A new method has been developed to compute the transversal modes of acoustic domains with uniform cross section including axial mean flow, absorbent material and a perforated pipe, thus permitting the subsequent application of the mode matching method to obtain the muffler behaviour. The procedure provides (with a sufficient number of modes) similar results to those obtained by the direct method (widely validated in the bibliography), both for axial wavenumbers and transmission loss. Although different analysis methodologies are usually validated by comparing transmission loss only, in this case axial wavenumbers were also compared, since any possible discrepancies between the methods are more likely to show up on a non logarithmic scale. In order to validate the methodology in relation to the direct method, circular concentric mufflers with absorbent material in the outer chamber and mean flow in the central duct were used. The results obtained confirm the satisfactory performance of the proposed method.

Acknowledgments

The authors wish to acknowledge the financial support received from Ministerio de Educación y Ciencia under project DPI2007-62635 and Conselleria d'Educació of Generalitat Valenciana (GV/2007/133).

References

[1] M. L. Munjal, *Acoustics of Ducts and Mufflers*, John Wiley & Sons (1986)

[2] S.-H. Lee, J.-G. Ih, "Empirical model of the acoustic impedance of a circular orifice in grazing mean flow", *J. Acoust. Soc. Am.* 114, 98-113 (2003)

[3] R. Kirby, A. Cummings, "The impedance of perforated plates subjected to grazing gas flow and backed by porous media", *J. Sound Vib.* 217, 619-636 (1998)

[4] D. F. Ross, "A Finite Element Analysis of Parallel-Coupled Acoustic System Using Subsystems", *J. Sound Vib.* 69, 509-518 (1980)

[5] R. K. Sigman, R. K. Majjigi, B. T. Zinn, "Determination of tubofan inlet acoustics using finite element", *AIAA* 16, 1139-1145 (1978)

[6] A. F. Seybert, C. Y. R. Cheng, "Application of the Boundary Element Method to Acoustic Cavity Response and Muffler Analysis", *J. Vib. Acoust. Stress Reliab. Design* 109, 15-21, (1987)

[7] Z. L. Ji, Q. Ma, Z. H. Zhang, "A Boundary Element Scheme for Evaluation of Four-Pole Parameters of Ducts and Mufflers with Low Mach Non-Uniform Flow", *J. Sound Vib.* 185, 107-117 (1995)

[8] A. Selamet, P. M. Radavich, "The Effect of Length on the Acoustic Attenuation Performance of Concentric Expansion Chambers: An Analytical, Computational, and Experimental Investigation", *J. Sound Vib.* 201, 407 426 (1997)

[9] A. Selamet, Z. L. Ji, "Acoustic Attenuation Performance of Circular Flow-Reversing Chambers", *J. Acoust. Soc. Am.* 104, 2867-2877 (1998)

[10] A. Selamet, F. D. Denia, A. J. Besa, "Acoustic behavior of circular dual-chamber mufflers", *J. Sound Vib.* 265, 967-985 (2003)

[11] A. Selamet, M. B. Xu, I.-J. Lee, N. T. Huff, "Analytical approach for sound attenuation in perforated dissipative silencers", *J. Acoust. Soc. Am.* 115, 2091-2099 (2004)

[12] R. Kirby, F. D. Denia, "Analytic mode matching for a circular dissipative silencer containing mean flow and a perforated pipe", *J. Acoust. Soc. Am.* 122, 3471-3482 (2007)

[13] R. Kirby, "Transmission loss predictions for dissipative silencers of arbitrary cross section in the presence of mean flow", *J. Acoust. Soc. Am.* 114, 200-209 (2003)

[14] J. Albelda, F. D. Denia, M. I. Torres, F. J. Fuenmayor, "A transversal substructuring mode matching method applied to the acoustic analysis of dissipative mufflers", *J. Sound Vib.* 303, 614-631 (2007)

[15] M. B. Xu, A. Selamet, I.-J. Lee, N. T. Huff, "Sound attenuation in dissipative expansion chambers", *J. Sound Vib.* 272, 1125-1133 (2004)

[16] M. E. Delany, E. N. Bazley, "Acoustical properties of fibrous absorbent materials", *Appl. Acoust.* 3, 105-116 (1970)