Acoustics'08 Paris
June 29-July 4, 2008
www.acoustics08-paris.org

Finite element modelling of thermoviscous acoustics in closed cavities

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A numerical methodology is presented to compute the acoustic field in a closed domain filled by a thermoviscous fluid, using the finite element method. The formulation based upon temperature variation and particle velocity is well suited for both *(i)* modeling the acoustic, thermal and viscous effects in the fluid bulk and *(ii)* accounting for the non-slip and the thermal boundary conditions on the solid. Due to the development of thin thermal- and viscous- boundary layers compared to the acoustic wavelength, very different scales are present in the field. The finite element mesh used is iteratively adapted to account for these different scales, and anisotropic because of boundary layers effects. Results are presented for the axisymmetrical back chamber of a miniature condenser microphone.

1 Introduction

Applications of acoustics in small devices need to model not only the acoustical field, but also the associated entropic and vertical fields. These effects are induced by the viscosity and the thermal conduction of the fluid ; they are more significant near the boundaries of the acoustic fluid where viscous- and thermal- boundary layers develop. Accounting for these phenomena clearly arises when dealing with acoustic transducers, inertial- or thermo-acoustics, capillary domains or porous materials. These systems often have complex geometry, such that the solution needs to be numerically computed. The standard description of acoustics by a scalar potential (like the pressure formulation) is not suited to represent the vertical movement, and a more complete formulation is required.

Publication related to the numerical modeling of thermoviscous acoustics include the work [1] for a non viscous but heat conducting fluid, the numerical models [2,3] for thin layers of viscous fluid between parallel plane walls and the modeling of viscous [4,5] or thermoviscous [6] boundary layers. These work are suited for only particular geometry, or limited by specific assumptions, like an acoustic domain large compared to the boundary layers thicknesses. The method presented below is a Finite Element application of a relevant complete formulation [7] for acoustics in thermoviscous fluids.

2 Basic formulation

The thermoviscous fluid is assumed homogeneous and Stokesian (stress proportional to rate of strain and heat flux proportional to temperature gradient), at rest (no mean movement), with a linear behavior. A small perturbation around the steady-state is considered, giving rise to acoustical propagation, but also to thermal and viscous diffusion, which cause dissipation of energy. These last diffusion processes are strongly excited near the boundaries, because of the non-slip and the isothermal boundary conditions, which differ from the free and quasi-adiabatic conditions in the bulk of the fluid.

Both vortical (shear) and entropic (potential) movements are described by the particle velocity v, and the thermal diffusion can be described by the temperature variation τ. This couple of variables (v,τ) is relevant to describe both *(i)* the propagative and diffusive phenomena in the bulk of the domain and *(ii)* to derive the boundary conditions.

Combining the linearized equations of conservation laws and of state of the fluid for a small perturbation in harmonic regime at angular frequency ω, the following coupled equation set is obtained [7] for the complex (v,τ) variables:

$$-\omega^2 \mathrm{v} - \left(\frac{c_0^2}{\gamma} + i\omega\, c_0\, l_v\right) \mathbf{grad}\ \mathrm{div}\ \mathrm{v} + i\omega\, c_0\, l_v^{'}\ \mathbf{curl}\ \mathbf{curl}\ \mathrm{v} + i\omega \frac{\hat{\beta}}{\rho_0} \mathbf{grad}\ \tau = 0 \quad (1.1)$$

$$i\omega\,\tau - \gamma\, l_h\, c_0\ \mathrm{div}\ \mathbf{grad}\ \tau + \frac{\gamma - 1}{\gamma\,\hat{\beta}} \rho_0\, c_0^2\ \mathrm{div}\ \boldsymbol{v} = 0 \quad (1.2)$$

where the properties of the fluid are the density ρ_0, the adiabatic speed of sound c_0, the increase in pressure per unit increase in temperature at constant density $\hat{\beta} = (\partial P / \partial T)_\rho$, the ratio γ of the heat coefficients at constant pressure and constant volume per unit of mass, and where the diffusion characteristic lengths are l_v' for shear viscosity, l_v for bulk viscosity and l_h for thermal diffusion [8]. In a bounded cavity filled by a thermoviscous fluid, the usual boundary conditions associated to the equation set (1.1-2) is a Dirichlet condition for the variables (v,τ): both the particle velocity and the temperature variation are prescribed on the boundary of the domain :

$$\tau = 0 \text{ (isothermal condition)}, \quad (1.3)$$
$$v = \bar{v} \text{ (prescribed velocity)} \quad \text{on the boundaries}$$

The other perturbation variables of the fluid (density variation, entropy variation, etc…) can be expressed from the solution (v,τ) of eq. (1) ; in particular the acoustical pressure variation is

$$p = \hat{\beta}\,\tau - \frac{\rho_0\, c_0^2}{i\omega\,\gamma}\, div\ v \quad (2)$$

3 Finite Element model

The operators of the coupled linear equation set (1) are usual Finite Element operators. Denoting the tensor $\underline{\underline{\mathbf{C}}}$ such that

$$\mathbf{div}\ \underline{\underline{\mathbf{C}}}\ \underline{\underline{\mathbf{grad}}} = -\left(\frac{c_0^2}{\gamma} + i\omega\, c_0\, l_v\right) \mathbf{grad}\ \mathrm{div} + i\omega\, c_0\, l_v^{'}\ \mathbf{curl}\ \mathbf{curl}$$

, this equation set takes the following matricial form:

$$\begin{bmatrix} -\omega^2 - \mathbf{div}\ \underline{\underline{\mathbf{C}}}\ \underline{\underline{\mathbf{grad}}} & \dfrac{i\omega\,\hat{\beta}}{\rho_0}\mathbf{grad} \\ \dfrac{\gamma-1}{\gamma\,\hat{\beta}}\rho_0 c_0^2\ \mathrm{div} & i\omega - \gamma\, l_h\, c_0\ \mathrm{div}\,\mathbf{grad} \end{bmatrix} \begin{Bmatrix} \mathrm{v} \\ \tau \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad (2)$$

Where the diagonal operators involve the usual mass- and stifnes- Finite Element matrices for vectorial and scalar variables, and where the non-diagonal operators concern first spatial derivatives (resp. div, **grad**) of the variables (resp. v,τ).

The set (2) is easily discretized by the finite element method to compute the solution of 2D, 3D or axisymmetrical systems. In a closed cavity, the field is excited by the conditions (1.3) prescribed on the boundaries, which are accounted for as usual Dirichlet conditions in the Finite Element Method.

4 Adaptive meshing

The physical phenomena occurring in in a cavity filled by a thermoviscous fluid develop at very different length scales: the thermal and viscous diffusion processes develop inside the boundary layers, whose thicknesses are several magnitude smaller than the propagative phenomena at the acoustic wavelength. Computing efficiently a numerical solution by the finite element method in such a multiscale model needs to take particular attention on the used mesh. Because of the thermal- and viscous- boundary layers, where the normal variations are much greater than the tangential ones, an anisotropic mesh is necessary. In some regions of the studied domain, the field is very smooth and presents variations at the only large acoustic wavelength, whereas diffusive transfers inside or near the boundary layers make the field strongly vary at length much smaller than the boundary layers thicknesses. The adaptive meshing technique [9] is a relevant method to refine the mesh exactly where needed. Starting with an initial coarse mesh, the outline of the method is:

1- to compute a (possibly approximate) solution of the set (2) on the given mesh by the Finite Element Method,
2- from this solution, to estimate the Hessian (2nd derivatives) of the field, and to build an anisotropic metric suited to minimize the interpolation error,
3- according to this metric, to build a new adapted anisotropic mesh,
4- go back to 1- for the next iteration.

Using this iterative procedure, the mesh is progressively adapted to the solution and refined exactly where needed by the solution.

5 Applications

The Finite Element modeling of acoustics in thermoviscous fluid is illustrated to the back chamber of an axisymetrical miniature condenser microphone. The first bending mode of the upper membrane (Ø 3mm) is excited by the incident field, and we study the field inside the back chamber filled with a thermoviscous fluid (air at atmospheric pressure). The incident acoustic signal is measured *via* the movement of the membrane, the condenser electrodes being the membrane and the facing backing electrode. Due to the design and the process used [10], the backing electrode has differents steps (Fig. 1 below):

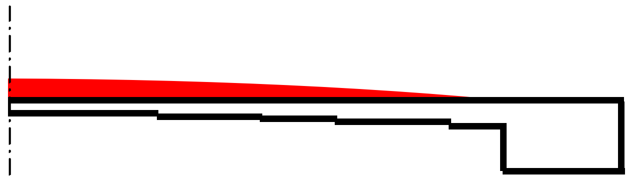

Fig.1 Axisymmetrical model of the back chamber of a miniature microphone
left : axisymmetrical axis, up: microphone membrane, right: peripheral cavity, down: backing electrode

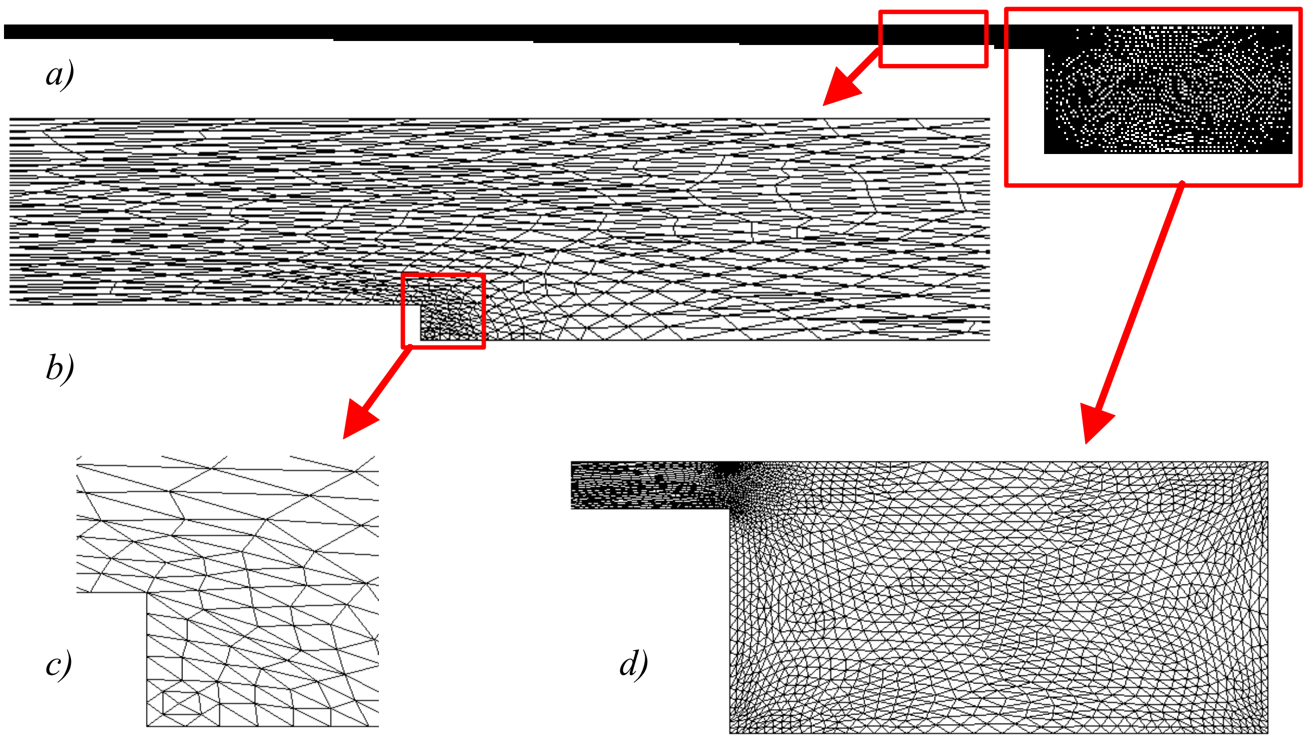

Fig.2 Optimized mesh of the back chamber after 16 mesh adapting iterations *a),*
detailed view near a step of the backing electrode *b) c),*
detailed view of the mesh in the peripheral reservoir *d).*

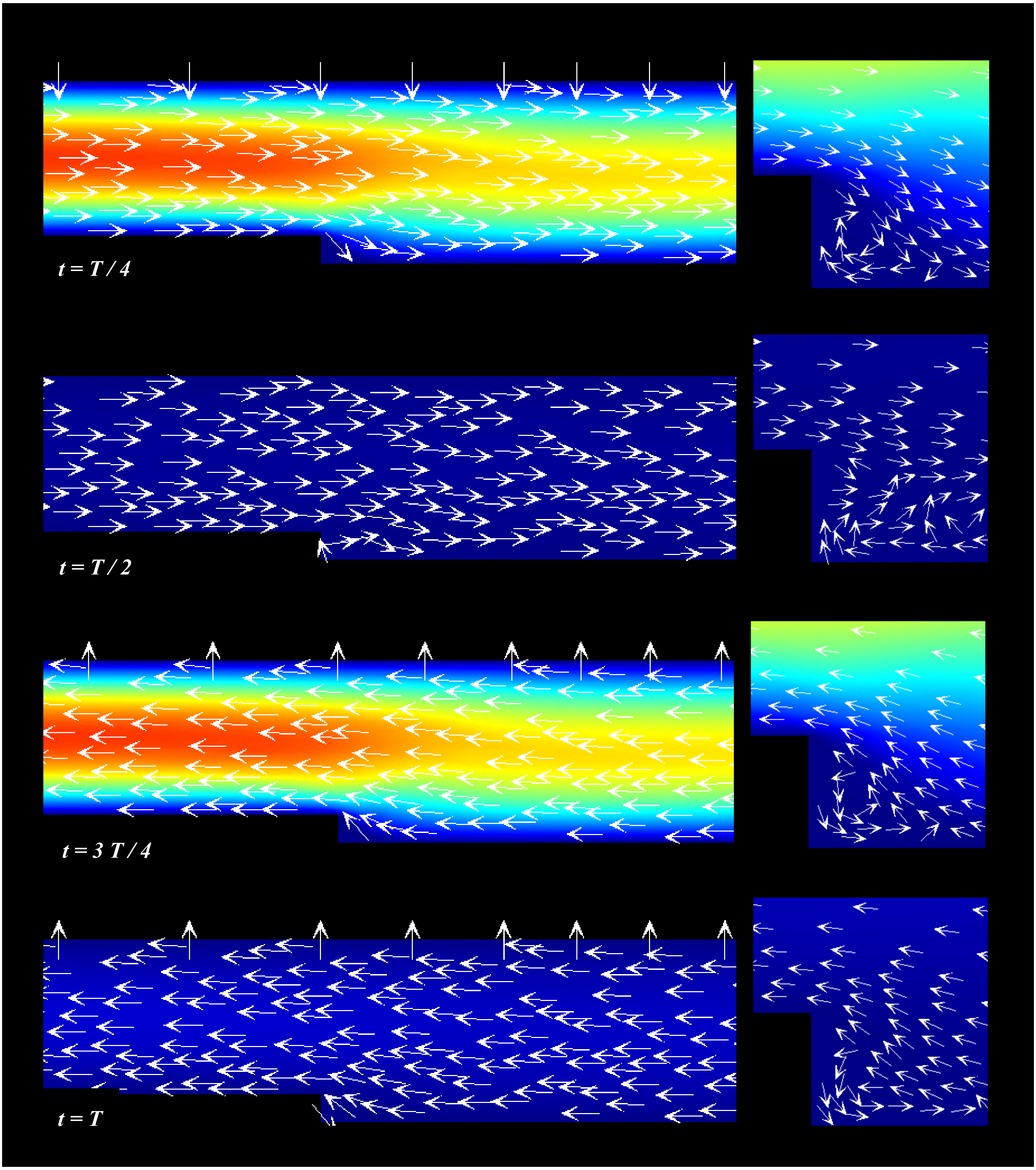

Figure 3 Particle velocity near the right step of the backing electrode.
(color map = magnitude, arrows = unitary orientation,
left : fluid domain between the upper membrane and the backing electrode (mesh of fig. 2b),
right : detailed view near the step (mesh of fig. 2c), illustrating the vortex (around *t=T/4* and *t=3T/4*))

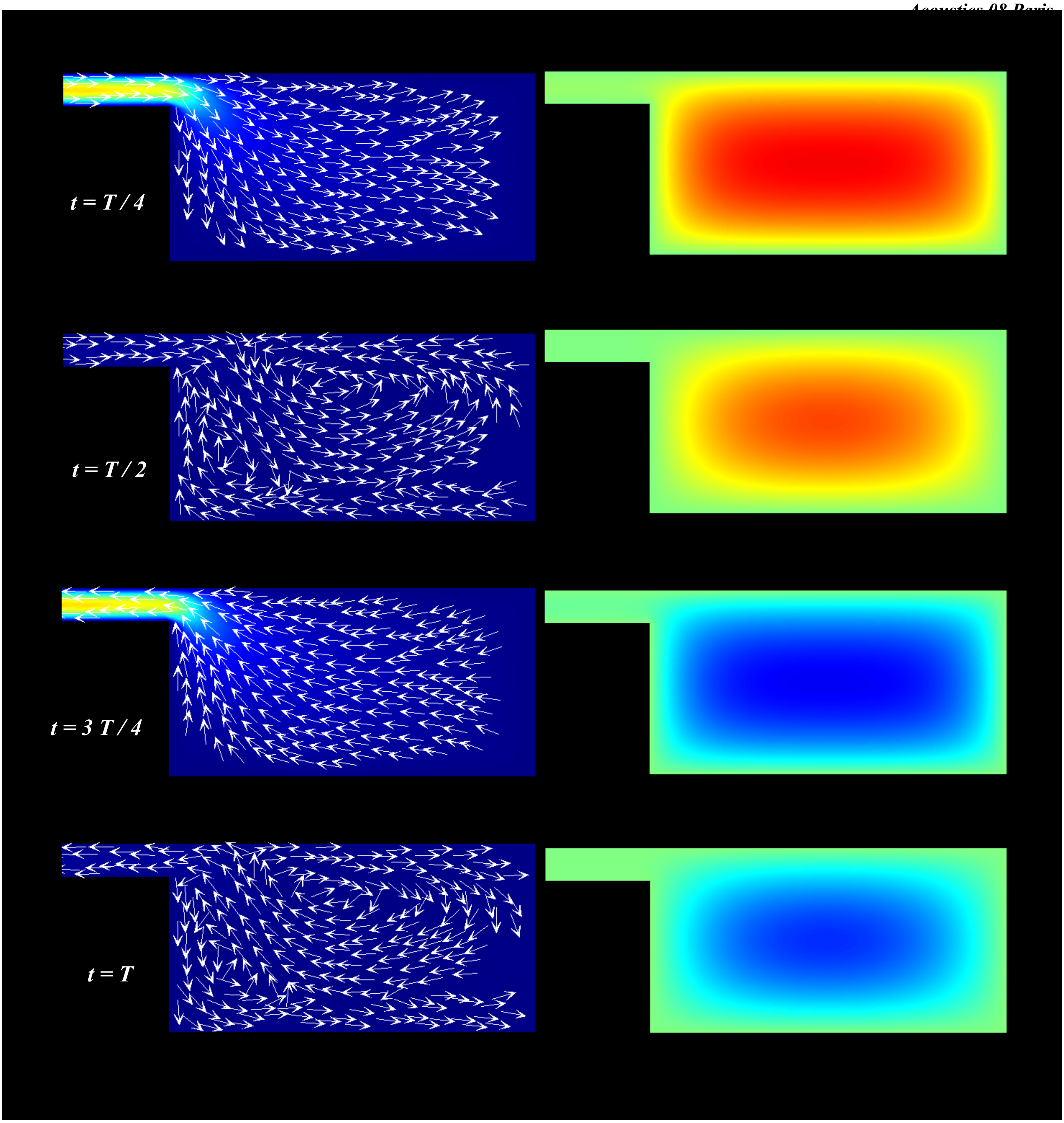

Figure 4 Particle velocity (left) and temperature variation (right) in the peripheral cavity (mesh of Fig. 2d).

(particle velocity (left) : color map = magnitude, arrows = unitary orientation,
temperature variation : color map = magnitude)

The boundaries of the cavity are isothermal; the solid boundary is rigid, except the upper membrane whose normal velocity is prescribed (Fig. 1, in red: input flow of the first bending mode of the microphone membrane). The flow generated by the normal movement of the upper membrane is stored in the surrounding reservoirs. This axisymmetrical device is presently modeled at a frequency of 1kHz, for a maximum deflection at the center of the membrane of 1µm. Figure 2 presents the mesh obtained after 16 iterations for mesh adapting. Due to the radial flow and the strong shear movement and thermal transfers, the adapted mesh is anisotropic in the tapered layer between the electrodes (Fig. 2b), whereas it is quasi isotropic in the peripheral reservoir (and coarse, fig. 2d) and near the steps of the backing electrodes (and very refined, fig. 2c).

The fluid flow generated by the oscillating membrane is accumulated and transferred to- (*t=0..T/2,* membrane moving down) or from- (*t=T/2..T,* membrane moving up) the peripheral reservoir, with an intensive radial velocity vector and intensive viscous shear stress, around times *t=T/4* and *t=3T/4* respectively (figure 3, left). Due to this driving flow, a small recirculating cell (vortex) is maintained just near the different steps of the backing electrode (figure 3, right). The fluid flow stored in- and back- in the peripheral reservoir is compressed / expanded, so that the temperature of the fluid oscillates (figure 4). However, due to thermal diffusion with the boundaries, the extrema for the temperature variation are delayed with respect to the input flow: the extrema for the fluid flow are *t=0.5T* and *t=T*, the extrema for the thermal power developed by the compressibility of the fluid occur at *t=0.25T* and *t=0.75T*, whereas the extrema for temperature variation can be observed at *t=0.4T* and *t=0.9T*. Due to the intensive heat transfer with the surrounding boundaries, the temperature variation remains very small in between the two facing electrodes. Unlike the maintained driven recirculating cells near the steps of the backing electrode (figure 3, right), the vortices observed in the peripheral reservoirs (figure 4) when the flow is inverting (at the times *t=T/2* and *t=T*) is very transient.

The acoustic pressure field, computed in a second time using equation (3), oscillates according to the input flow of the global back chamber, with a small delay induced by the thermoviscous properties of the fluid and the transfers with the boundaries.

6 Conclusion

When combined with the Finite Element Method and adaptive and anisotropic meshing techniques, the formulation based on the particle velocity and the temperature variables (v, τ) is well suited to compute numerically the harmonic thermoviscous acoustic field in closed cavities, accounting for both propagative and diffusive phenomena. Such multiphysics and multiscale numerical models give detailed information on the focal field ; it can be useful for applications in small acoustic devices, acoustical metrology, microfluidics or thermoacoustics.

Acknowledgments

The author gratefully thanks F. Hecht for the mesh adapting software BAMG [9] and P. Gentil for the graphic routine GRAME [11].

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