



**Acoustics'08  
Paris**  
June 29-July 4, 2008

[www.acoustics08-paris.org](http://www.acoustics08-paris.org)

## Determination of unknown parameters in impervious layers by inverse method

Eva Escuder Silla<sup>a</sup>, Jesús Alba Fernandez<sup>a</sup>, Jaime Ramis Soriano<sup>b</sup> and Romina Del Rey Tormos<sup>a</sup>

<sup>a</sup>Escola Politècnica Superior de Gandia, Universitat Politècnica de València, Crtra Natzaret-Oliva s/n, 46730 Gandia, Spain

<sup>b</sup>DFISTS. Univ. de Alicante, Carretera de Sant Vicent del Raspeig s/n, 03690 San Vicente del Raspeig, Spain  
evescude@fis.upv.es

In this work, it is showed a novel procedure for the determination of unknown parameters in impervious layers used in multilayer structures by inverse method and using scale models. Experimental pressure and velocity data are obtained by Nearfield Acoustic Holography (NAH) for the calculation of the Transmission Loss of the different multilayer structures mounted on the window of a wooden box designed for that end. These data are used as input data in the inverse method. The forecast model of acoustic insulation in multilayered structures used in this work was Trochidis&Kalaroutis model based on Spatial Fourier Transform (SFT). By applying Trochidis&Kalaroutis model and adjusting by numerical methods the variables that define the impervious layers of the system, the values of the unknown magnitudes of the layers are calculated. For validation purposes the results are compared to those obtained with Ookura&Saito model.

## 1 Introduction

Exist different models that describe acoustic behaviour in multilayer structures, among them the Ookura & Saito model [1], based on impedance coupling between layers; the Trochidis & Kalaroutis model [2] and the Bruneau model [3], based on a Spatial Fourier Transform. The model of Ookura & Saito analyzes a sound transmission index in multilayer structures by impedance transfer for inclined incident-angle waves and random sound fields. The method developed by Trochidis & Kalaroutis is based on a matrix that defines multilayer structures.

Most models that describe acoustic behaviour in multilayer structures are based on two types of structural materials: sound-impervious materials and sound-absorbing materials, such as mineral or organic wool, textile or glass fibres, open honeycomb-like layers. Absorption depends on the frequency and angle of the incident sound wave. Sound propagation through sound-absorbing materials is usually characterized for homogeneous and isotropous materials by two complex values: a complex propagation constant ( $\Gamma$ ) and complex characteristic impedance ( $Z$ ). Other methods can also be used to characterize sound-absorbing materials, e.g., Kundt's impedance tube; many such methods are described in the literature [4-5].

A thin, plane, and uniform surface is assumed with a given rigidity; it vibrates with small displacement amplitudes. The thin plate is characterized by a surface density  $m$  (kg/m<sup>2</sup>) and a given rigidity [6]. Within the plate, the restoring force is governed only by its rigidity. The general equation that governs the movement of symmetrical vibrations is as follows:

$$\nabla^4 w(x, y, t) + \frac{\rho(1-\sigma^2)}{Yh^2} \frac{\partial^2 w(x, y, t)}{\partial t^2} = 0 \quad (1)$$

where  $\rho$  (kg/m<sup>3</sup>) is the volume density of the material,  $\sigma$  is the Poisson coefficient,  $Y$  (N/m<sup>2</sup>) is the Young modulus, and  $h$  (m) is the turn radius of the surface, which has a value of  $h = L/\sqrt{12}$  where  $L$  is the thickness of the plate.

A number of mathematical models have been developed to estimate the coefficient of sound transmission of the impervious material or layer. These models generally require accurate data on the elastic properties of the material, as this is the most important factor in terms of vibration velocity. The surface density also affects transmission coefficients at low frequencies. Such models also require values for the flexural stiffness ( $D$ ), surface density ( $\rho$ ), thickness ( $h$ ), and loss factor ( $\eta$ ) of the layer.

Inverse analysis normally refers to the parameters of a system that provide the best fit between the calculated and

observed acoustic behaviour. Inverse analysis is more complex than direct analysis, as the mathematical problem consists of the minimization of a non-linear function. The function is an error function that is calculated as the difference between the calculated and measured data using any given combination of parameters. Such techniques are useful because the calculated parameters can be used in making estimates for future stages of the same project, thereby minimizing potential inaccuracies in the employed model. Various studies have made use of inverse methods in determining material properties based on finite element analysis and modal analysis [7-9].

Once the values of the parameters that characterize the layers and sound-absorbing material of the system are known, the Trochidis & Kalaroutis model can be used to estimate transmission loss in the multilayer structure. In inverse models, the values of experimental transmission loss can be used to obtain the system parameters that best fit the theoretical and experimental results.

In the present paper, we use an inverse method to determine unknown parameters in multilayered structures using as input data Nearfield Acoustic Holography (NAH) values and the multilayer prediction model described above.

The NAH technique is based on the measurement of sound pressure using an array microphone positioned on a plane that is both parallel and close to the measurement area. Using digital data-processing techniques, NAH values can be used to calculate the acoustic magnitudes on the object surface by back-propagation of the acoustic field. The main advantage of NAH is that the sound field of any other plane of the object can be reconstructed from 2-D values, termed a hologram [10].

This approach can be used for practical applications because the theoretical models are normally applied to ideal partitions with elastic properties that do not vary with the incidence angle of the sound wave. Therefore, complementary calculation techniques can be used to ensure the optimal application of theoretical models to real structures, with no need for real-time measurements

## 2 Fundamentals

### 2.1 The inverse method

The inverse method for the identification of parameters is based on iterative loops between the experimental data and the prediction model, using different parameter values of the structural materials to optimize the results and minimize model error. To this end, we use an error function that shows the minimum value of the most suitable parameter of the plate (Fig. 1).

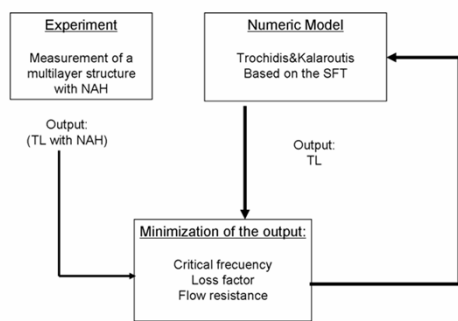


Fig. 1. Diagram of the inverse method

Parameter-identification techniques are used to obtain the parameters of the model that best fit real-time measurements and model predictions. The problem of estimating parameters using real-time measurements can be solved by using an explicitly formulated model that relates a number of measurements  $x$  with a certain number of parameters of which we have no a priori knowledge:

$$x=Z(y) \tag{2}$$

where  $Z$  represents the model. The relation expressed by  $Z$  is non-linear. The inverse problem consists of finding a set of parameters “ $y$ ” such that the variables calculated using such parameters “ $\hat{x}_i$ ” via Eq. (2) provide a better fit to real-time measurements “ $x_i$ ” [7-9]. Data fitting is mathematically performed based on an identification criterion. The selection of the criterion determines the function whose maximum and minimum correspond to the solution of the problem. There are different identification criteria, but the most widely used are the least squares and maximum likelihood criteria. In the present study, we use the mean square error function, expressed as follows:

$$\varepsilon = \sum_{i=1}^n (\tau_i - \hat{\tau}_i)^2 \tag{3}$$

where  $\tau_i$  represents the value of the transmission coefficient obtained from experimental measurements at frequency  $i$  and  $\hat{\tau}_i$  is the theoretical value given by the model.

The model adjusts to the characteristics of the materials. The model uses three variables that have been measured, surface density, Young modulus and shear modulus. The loss factor is a parameter that depends on the frequency and in the model a constant total loss factor is used.

## 2.2 Trochidis & Kalaroutis model

The theoretical model consists of two infinite, thin, elastic plates with no connection between them. A sound-absorbing material is placed between them [2] in such a way that a gap exists between the plates and the intervening material (Fig. 2).

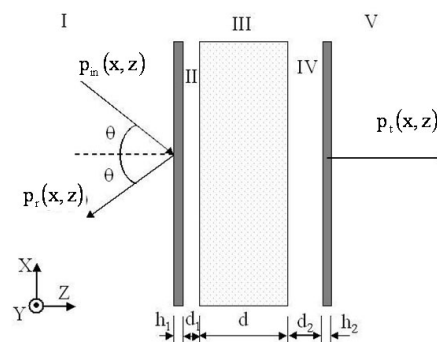


Fig. 2. Multilayer model under study

The multilayer structure is excited with a plane wave front that is incident to the structure at an angle  $\theta$  from the direction normal to the structure. The time dependence is assumed to be  $e^{-j\omega t}$ , where  $\omega$  is the angular frequency. Zones I, II, IV, and V are described by a Helmholtz scalar equation, representing the propagation of sound in air:

$$[\nabla^2 + k_0^2] p_i(x, z) = 0 \quad i = I, II, IV, V \tag{4}$$

where  $k_0 = \omega/c_0$  is the sound wavenumber.

The equation that describes the movement of Plate I is expressed as

$$[D_1 \nabla^4 - \rho_1 h_1 \omega^2] w_1(x) = p_I(x, 0) - p_{II}(x, 0) \tag{5}$$

where  $w_1(x)$  is the plate displacement in the normal direction,  $\rho_1$  is the density of the plate material,  $h_1$  is the thickness of the plate,  $D_1$  is the flexural stiffness of the plate, and  $p_I(x, z)$  and  $p_{II}(x, z)$  are the sound pressures in Zones I and II respectively. The space filled with the sound-absorbing material (zone III) is represented by a complex wavenumber,  $k_b$ , and a complex density,  $\rho_b$ . The wave equation for the sound-absorbing material is as follows:

$$[\nabla^2 + k_b^2] p_{III}(x, z) = 0 \quad h_1 + d_1 < z < d + h_1 + d_1 \tag{6}$$

The equation that describes the movement of outer Plate II is

$$[D_2 \nabla^4 - \rho_2 h_2 \omega^2] w_2(x) = -p_V(x, d + h_1 + d_1 + h_2 + d_2) + p_{IV}(x, d + d_1 + h_1 + d_2) \tag{7}$$

where  $p_{IV}(x, z)$  is the sound pressure in zone IV (air) and  $p_V(x, z)$  is the sound pressure in zone V.

This method generates a matrix for the different multilayer structures, which is obtained by directly applying the boundary conditions of the materials. This results in different equations with partial derivatives that can be transformed into algebraic equations via the Spatial Fourier Transform.

Once the transmitted sound pressure,  $p_t(x, z)$ , is known, the transmission coefficient can be obtained by

$$\tau_d = \frac{\int_0^{\theta_{lim}} \tau(\theta) \cos \theta \sin \theta d\theta}{\int_0^{\theta_{lim}} \cos \theta \sin \theta d\theta} \quad (8)$$

where  $\theta_{lim}$  represents the limit angle from which any contribution to the sound field is negligible. From Eq. (9), transmission loss can be obtained from the following expression:

$$TL = -10 \log \tau_d \quad (9)$$

### 2.3 Nearfield Acoustic Holography

Nearfield Acoustic Holography is a technique that reconstructs the sound field and vibration velocity of an object or sound source from measurements taken with microphones placed on a plane that is both parallel to and close to the sound source (see Fig. 3). The nearfield measurements enable the capture of the evanescent waves (subsonic waves that exponentially decay with increasing distance from the sound source) that are generated by the sound source and that contain high-resolution details about the source [11].

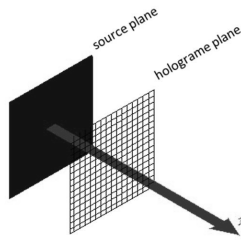


Fig. 3. Source plane and hologram plane

Based on Green's theorem, an integral can be derived that describes the sound pressure at any point in space between the sound source and the measurement plane. The complex pressure at any point in the free space can be expressed as a function of the complex pressure ( $\bar{p}$ ) on the source plane  $z_s$ , where  $\bar{p}_s(x', y', z_s)$  is the distribution of the complex pressure on  $z_s$  and  $\bar{G}'(x-x', y-y', z-z_s)$  is the normal derivative of Green's function that satisfies the Dirichlet eigenvalue limit condition on  $z_s$ .

$$\bar{p}(x, y, z) = - \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \bar{p}_s(x', y', z_s) \times \bar{G}'(x-x', y-y', z-z_s) dx' dy' \quad (10)$$

If all points are assumed to be located on the same measurement plane, termed the hologram,  $z_h$ , and as  $z_h - z_s$  is a constant, Eq. (10) describes a 2-D convolution between the complex pressure on plane  $z_s$  and the modified Green's function, which becomes a simple product in the wavenumber domain:

$$\bar{p}_h(k_x, k_y, z_h) = \bar{p}_s(k_x, k_y, z_s) \cdot \bar{G}'(k_x, k_y, z_h - z_s) \quad (11)$$

$$(12)$$

### 3 Development

The test setup used for the measurements is shown in Fig. 4. The setup consists of a rod with holes in which four 1/4" microphones were placed 1.5 cm apart. The rod was mounted on a robot that moved the linear array microphone by the box. The materials used were 49 x 61 cm in size and were mounted on the window of a wooden box of dimensions 110.4 x 69.9 x 47.3 cm. The wooden box used is inwardly recovered of polyester fiber that has a average absorption coefficient of 0.6. The layers were settled to ensure that there were not air-gaps between the layers and the sound-absorbing material. All the panels were mounted with elastic contour. Two broadband speakers were located on each side of the box, and the room was half-lined with sound-absorbing wedges.

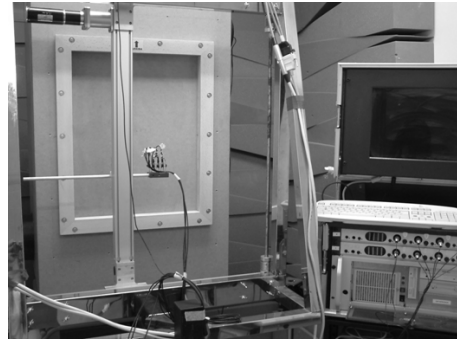


Fig. 4. Experimental setup and the robot with the array microphone

A white noise of 3 sec in duration was generated by the speakers located inside the box, being maintained at the same intensity for all measurements. The noise generated inside the box was transmitted through the material of the box window and the response was recorded by the microphones over the measurement area and in the near field. A total of 1064 recordings were taken, distributed in a 28 x 38 matrix. The materials used in the study were analyzed individually and in combination, including 3 and 5 mm wooden boards, 2.5 cm polyester wool plates, and a 1 mm steel sheet. Measurements were taken at a distance of 2 cm from the box window that contained the materials of interest. The data were then analyzed using NAH to calculate the acoustic pressure and the vibration velocity on the surface of the material outside the window. The filter parameters were  $kc = 0.6$  kmax and  $\alpha = 0.2$ .

The amount of Transmission Loss (TL) from the interior to the exterior of the box can be calculated according to the following Eq. (12):

$$(L_S - L_R) = TL \quad (12)$$

where  $L_S$  is the sound pressure level (measured in dB) in the source region, i.e., inside the box, and  $L_R$  is the sound pressure level (dB) in the receiver zone, i.e., on the material

surface. Diffusers were placed inside the box to enhance the degree of homogeneity of the acoustic field.

Knowing the plate's parameters, adjusts the flow resistance of the sound-absorbing material such that the resulting TL fits the TL obtained via NAH [12,13]. Funcapa algorithm calculates the flow resistance using as input data the critical frequencies of the plates,  $f_{c1}$  and  $f_{c2}$ , the surface densities of the plates,  $m_1$  and  $m_2$ , and TL obtained using NAH. The following Matlab© function is then run:

$$y = \text{fminsearch}(\text{'funcapa'}, X) \quad (13)$$

where  $y$  is the minimum value of the error function, calculated in Funcapa using the criteria of the function "fminsearch", of the difference between experimental measurements and the theoretical value of TL. Funcapa calculates the error in each iteration, which is retained in order to represent it.  $X$  is the value that starts the function and yields a local minimum  $y$ , close to  $X$ ; the function 'funcapa' accepts the input  $y$  and returns a scalar function value. The "fminsearch" function is based on the Nelder-Mead algorithm. The function minimizes a non-linear function of  $n$  real variables using only function values, with no additional data [14]. The function minimizes the error function calculated in Funcapa and generates an optimal value that ensures a better fitting of TL to NAH values.

$$\varepsilon = \sum_{i=1}^n (TL_i - \hat{TL}_i)^2 \quad (14)$$

All of the other parameters for the plates, such as loss factors and critical frequencies, can be evaluated following the same procedure to obtain a better fit of TL to NAH values. The algorithm identifies the parameter frequency per frequency and calculates the average in 1/3 octave bands.

A multilayer structure was evaluated, fixing a range of values for the critical frequency for the layers ranging from 500 to 1500 Hz. The error is obtained where the minimum value yields the approximate value of the critical frequency of the wooden layers.

Fig. 5 shows the error (measured in dB) obtained for the critical frequency of the wood of the multilayer structure. Note that the minimum error gives the optimal frequency for fitting, approximately 1070–1090 Hz.

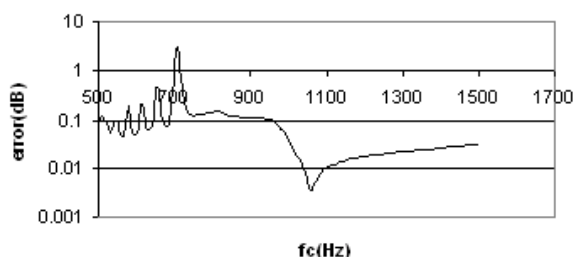


Fig. 5. Error function (dB) dependence on critical frequency of wood obtained from fminsearch function in case of a multilayer structure

TL with NAH was calculated as described in the previous section. The TL values for the 5 mm wood and 1 mm steel obtained from the experimental measurements were used as input data for the SFT model (Table 1); the inverse method was then used to calculate the specific data for each material (Table 2).

|                                      | Steel | Wood  |
|--------------------------------------|-------|-------|
| Thickness (mm)                       | 1     | 5     |
| Surface density (kg/m <sup>2</sup> ) | 10    | 3.8   |
| Critical frequency (Hz)              | 12500 | 1000  |
| Loss factor                          | 0.004 | 0.024 |

Table 2. Specifications of the analyzed materials

The measured value of critical frequency of the steel plate is of 12500 Hz. This can be observed in the measurement, since the critical frequency corresponds with a diminution of the isolation.

The data obtained using the inverse method are used to calculate TL for multilayer structures. The input data for the absorbing-material model is flow resistance, which in the present case is 1200 Rayls/m.

Fig. 6 shows TL for the "5 mm wood + wool + steel" multilayer structure obtained using the Trochidis & Kalaroutis (based on SFT) and Ookura & Saito models, as well as the TL obtained using NAH. Both of the models and the NAH technique yield similar trends in TL. Our measures are carried out with Fast Fourier Transform.

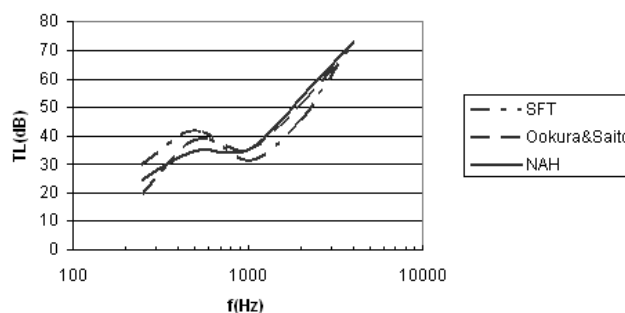


Fig. 6. Comparison of the SFT, Ookura&Saito and NAH methods to obtain TL of the 5mm wood+wool+steel structure

## 4 Conclusion

With the inverse method employed in the present study it is possible determining unknown parameters for impervious layers with good results using the Trochidis & Kalaroutis prediction model for the Transmission Loss in multilayer structures and experimental data obtained by NAH as input data.

## Acknowledgments

This work was supported by the Ministry of Education and Science (BIA2007-68098-C02-02-AR07).

Method in low dimensions. SIAM J. OPTIM, 9, 1 (1998) 112-147

## References

- [1] Ookura K, Saito Y. Transmission Loss of multiple panels containing sound absorbing materials in a random incidence field. Proceedings 1978 International Conference on Noise Control Engineering, Inter-Noise 78, San Francisco, California, May 1978.
- [2] Trochidis A, Kalaroutis A. Sound transmission through double partitions with cavity absorption, Journal of Sound and Vibration 107 (1986) 321.
- [3] Bruneau M, Manual d'Acoustique fondamentale, Edition Hermès, 1998.
- [4] Ingard KU, Dear TA. Measurement of acoustic flow resistance. J. Sound Vib. 103, 4, (1985) 567-572.
- [5] Acoustics-Determination of sound absorption coefficient and impedance or admittance by the impedance tube. Part I (1996): Standing wave ratio method. Part II (1998): Transfer-Function method.
- [6] Kinsler, Lawrence E.; Frey, Austin R.; Coppens, Alan B.; Sanders, James V. "Fundamentals of Acoustics". John Wiley & Sons, New York (etc.), 2000
- [7] Alba J, Ramis J. Modeling of impervious layers from measurements of the sound reduction index. Applied Acoustics 64 (2003) 385
- [8] Shi Y, Sol H, Hua H. Material parameter identification of sandwich beams by inverse method, Journal of Sound and Vibration 290, (2006)1234-1255
- [9] Szeliga D, Gawad J, Pietrzyk M. Parameters identification of material models based on the inverse analysis. Int. J. Appl. Math. Comput. Sci. 14, 4, 2004, 549-556
- [10] Williams EG, Maynard JD. Numerical evaluation of the Rayleigh integral for planar radiators using the FFT. Journal of the Acoustical Society of America, 72, 6, Diciembre (1982), 2020-2030.
- [11] Veronesi WA, Maynard JD. "Nearfield Acoustic Holography (NAH) II. Holographic reconstruction algorithms and computer implementation", Journal of the Acoustical Society of America, 81, 5, may 1987, 1307-1322.
- [12] Delany ME, Bazley EN. Acoustical properties of fibrous absorbent materials. Applied Acoustics 3 (1970), 105 -116.
- [13] Miki Y. Acoustical properties of porous materials – modifications of Delany-Bazley models-. J. Acoust. Soc. Jpn (E) 11, 1 (1990) 19-24.
- [14] Lagarias JC, Reeds JA, Wright MH, Wright P.E. Convergence properties of the Nelder-Mead Simplex