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Doppler effect in aeroacoustics

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The prediction of acoustic fallout caused at the ground level by aircraft in flight or, in some cases, by launchers at take-off, sets the problem of the distortion of the emitted sound spectrum by the Doppler effect.

Former studies applied to this problem use a geometrical formalism based on simplifying hypotheses such as homogeneous atmosphere, rectilinear trajectory and constant velocity of the sound source. Other studies consider the supersonic case using a model of spherical waves inside the Mach cone. These approaches are in fact not well adapted to the real cases, and the literature does not give many practical examples concerning this topic.

In this study, we propose a new time approach of the Doppler effect that allows to avoid the complexity of the geometrical models and to take into account real atmosphere, curvilinear trajectories or accelerations of the moving sound sources. This model allows to calculate the frequency shift and the change of acoustic level of the signal recorded at a given hearing point, in both cases of subsonic and supersonic sound sources.

We present a first validation of this method applied to signals recorded in Kourou during the atmospheric phase of Flight 521 of launcher “Ariane 5”.

1 Introduction

Although the noise pollution caused by a launching vehicle in flight arouses less interest than in the past, because of a relative geographical isolation of launch pads, the prediction of “acoustic fallout” constitutes a very interesting problem from theoretical and practical points of view. In this paper, we deal with the “jet noise” side, the effects of which cannot be neglected, in particular in the infrasound range, although acoustic levels generated are in general lower than levels from a sonic boom.

In the framework of the “Ariane 5” program supported by the CNES, the ONERA had recorded the launching vehicle noise from the ground station “Toucan” located at Kourou, French Guiana. Recordings were made during Flight 521 on February 12, 2005. It was of great interest to check to what extent the available tools – related to jet aerodynamics, jet noise, and propagation – allowed to simulate the noise recorded and more particularly its frequency spectrum. A particular problem was posed by taking the Doppler effect into account for a source in movement on a curved trajectory at subsonic, then supersonic speed.

The first step in the simulation consisted in taking the characteristics of “Ariane 5” motors into account, with the help of a semi-empirical aerodynamics model [1], in order to determine the acoustic reference length for every jet, namely the laminar core length at altitudes from 0 to 13,000 m. The aerodynamics model that has been developed allows to calculate the characteristics of the fully expanded jets according to the ambient conditions at a given altitude.

The second step consisted in adapting a classical jet noise model, generally applied and validated at sea level [2], to high altitude conditions. The aerodynamic characteristics of a jet flow are closely linked to surrounding air conditions, because the lengthening of the jet gets as predictable effect a shift of the emitted sound spectrum towards the low register. Furthermore, in order to calculate the sound efficiency of jet flows at considered altitudes, a formula including the ambient sound celerity has been used [3].

Taking the Doppler effect into account in order to predict the jet noise of an aircraft or a launching vehicle in flight is a subject that has been seldom taken up in the literature [4, 5]. The ONERA has developed an original method that allows to calculate the Doppler effect and the resulting modification of spectrum levels for any flight path and any speed of the sound source. This calculation may consider real or simplified conditions of atmospheric propagation. The introduction of the Doppler factor proves decisive in order to simulate correctly the sound spectrum taken from recordings. In the case of Flight 521 of “Ariane 5”, results obtained from recordings performed at the “Toucan” listening station constitute a first validation for methods that have been used.

In this paper, we mainly present the method which has allowed us to estimate the distortion of the noise spectrum by the Doppler effect. The computations are related to the part of the flight path included between 0 and 13,000m above sea level, an altitude where the speed of the launching vehicle is close to Mach 2. Flight trajectory and atmospheric data were provided to the ONERA by the CNES (Division of Launching Vehicles at Evry, Meteorological Station of Kourou).

2 Aerodynamics and jet noise models

For a rocket-engine loaded with a given propellant, the physical characteristics and the chemical composition of jet flow at the exhaust can be computed using complex aerothermodynamics computer codes. Nevertheless, it is possible to approach the jet characteristics by using a “perfect gases” formalism and an “averaged molar mass” of the combustion products possibly including solid or liquid components [6]. Furthermore, by means of relatively simple models for real gases, it is possible to compute how evolves the specific heats ratio of the jet as a function of its temperature, assuming that chemical reactions in the jet are brought to a standstill (“frozen flow” hypothesis). In the jet aerodynamics model set out in Ref. [1], this calculation is performed along the jet for the mixture made up by combustion products and surrounding air, by considering that all aerodynamic parameters are constant in a given jet cross-section. The overall formalism, in addition, is based on the equations of conservation from fluid dynamics, with the nuance that the mass flow rate of gases is not constant, since a progressive mixing between the jet flow and the surrounding fluid is taken into account by introducing a mass flow ratio gas-to-pulled-air. Of course, temperature and air pressure at the considered altitude are taken into account, in particular by calculating the fully expanded jet diameter, that is the main reference length for the model: so, results of this calculation are linked up with chamber conditions and initial mass flow rate, but they are independent of the nozzle shape.

The main results from our computer code "Jedi" are the fully expanded jet data, but also the laminar core length and the sound peak position along the jet axis, which constitute the main imput data for semi-empirical jet noise models such as the "improved NASA model" described in Ref. [2]. This model gives the spectrum of sound power of the jet, the distribution of this power along the jet and the acoustic directivity of local sources. The fully expanded jet diameter, that governs the aerodynamic reference lengths, directly acts on the range of frequencies emitted by the jet.

The corresponding jet noise code "Minotaure" has been used until now in order to calculate sound emissions of jets from static installations, rocket-engines at test bench or also from launching vehicles during their lift-off phase, therefore moving with a low speed and remaining at an altitude next to sea level. In order to be applied to an atmospheric flight, a subroutine taking account of acoustic effects generated by the vehicle speed has been added. The vehicle speed indeed acts on the acoustics of the jet by modifying its speed in relation to ambient environment.

Finally, knowing the rocket engine characteristics and the flight data (speed, altitude, trajectory) of a launcher, we are able to calculate the sound field in the moving space reference linked up with the vehicle.

3 Flight 521 of launcher "Ariane 5"

During Flight 521 in the evening of February 12, 2005, a listening station allowed to record the sound signal from a flying "Ariane 5" during near 2 minutes at an acquisition frequency of 5 kHz. The station, called "Toucan", was implemented by the ONERA and placed at about 4 km from the Ela3 launch pad at Kourou, French Guiana. Almost immediately after the lift-off, the flight path of "Ariane 5" bent, heading for East, while the angle between jet flow axis and direction towards the listening station passed from 90° near the ground to 50° when the launching vehicle was at 4,000 m above sea level, then to 22° at 13,000 m. We see that the listening point was amply inside the Mach cone when "Ariane 5" speed became supersonic: by this very fact measurements are neither disrupted nor interrupted by the sonic boom. The flight path of the launching vehicle and weather data at the time of the launch were obligingly provided to the ONERA by the CNES. It should be noted that temperature profiles in real and in standard atmospheres are similar within a gap of about 15° Celsius (Fig. 1). This fact allows to do the approximation of a constant temperature gradient between 0 and 13,000 m. Wind was light, blowing more or less from West to East (in standard atmosphere, wind speed is considered as null). It should be noted however that there was a wind gradient between 7,000 and 8,000 m above sea level able to disadvantage sound propagation towards the listening station. In the continuation of this paper, three reference altitudes were taken into consideration along the flight path:

- 3,750 m, the launching vehicle speed is still subsonic;
- 8,000 m, the launching vehicle speed is transonic;
- 13,000 m, the launching vehicle speed is supersonic.

According to calculation performed, the launching vehicle is flying above this last altitude when the recording ends. From an acoustical point of view, the increase in jet length between 0 m and 13,000 m due to the decrease of air pressure and air density corresponds to a shift of the spectrum towards lower frequencies: the energy finally tends to concentrate in the infrasound frequency range (f < 20 Hz). On Fig. 2, showing sound levels calculated at the listening point with no account of the Doppler effect, the curve *"static firing"* does not include effects from speed or effects from upper air conditions. The curve *"with speed, ground level"* only takes into account the speed of the launcher that is transonic in this example. At last the curve *"with speed, in altitude"* takes into account the ambient conditions and the acoustic efficiency calculated for the altitude of 8,000 m. Let us note that according to our model this efficiency increases with altitude slightly. It appears that effects of altitude act in the same way than effects of the vehicle speed, but in a much more distinct way.

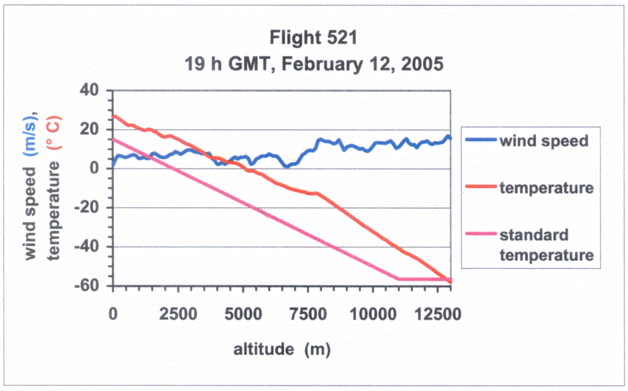

Fig.1 Weather conditions during Flight 521 of "Ariane 5".

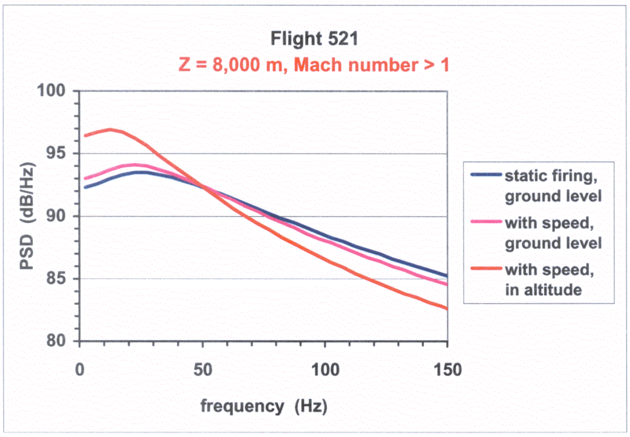

Fig.2 Sound power spectral densities at the ground level according to several calculation hypotheses.

4 What about the Doppler effect ?

The following representation of the Doppler effect in aeroacoustics is often found in the literature: a succession of spherical waves emitted at regular time intervals, say the period of a harmonic or pulse source S moving at speed V. Fig. 3 gives such a representation in the case of a supersonic source. The geometric envelope of these spheres is called "sound cone", since the designation "Mach cone" is rather kept for the shock wave generated by a real moving body, the area of which is not equal to zero. This distinction is often lost from sight by authors who therefore class the moving body as a harmonic source in motion, a definition which can be applied to components of the jet noise, but in no case to the "sonic boom" from aircraft. The half-angle at vertex α of the sound cone is such as:

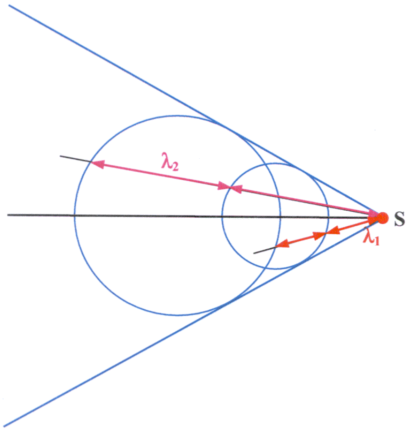

Fig. 3 Sound cone of a supersonic sound source.

$$\sin \alpha = c/V = 1/M \tag{1}$$

where c is the celerity of sound in the surrounding air and M is the Mach number equal to c/V. Thus, the angle α decreases while the celerity increases, which reduces the volume in which the signal is detected. The geometric representation in Fig. 3 seems to be convenient, because it allows to observe an apparent modification of the signal wavelength depending on a listening direction in relation to the source: let us call it λ_1 for the wave fronts moving towards the source, λ_2 for the wave fronts moving away from the source. Thus, an observer who is inside the Mach cone should detect two different Doppler frequencies, but the same observer on the cone of Mach should detect only one Doppler frequency, the analytic expression of which is given in the literature.

In fact, this representation seems to be unconfirmed: for instance, Ernest Esclangon [7] remarked that an observer on the ground always perceives only one frequency in the whistling caused by movements of rotation and nutation of a shell and he identifies it as being the "backward frequency" of the sound cone, i.e. the frequency of fronts moving away from the source. If we define the Doppler factor D.F. as the ratio between the frequency f perceived at the listening point (linked to the fixed space reference) and the frequency f_0 emitted by the source, the expression of D.F. commonly admitted is:

$$\text{D.F.} = f / f_0 = 1 / (1 + M \cos \theta) \tag{2}$$

The expression is found just as it is in [4], whereas in other references [5, 8] its denominator includes an empirical exponent. Whether the source speed is subsonic or supersonic, it is important to observe that the angle θ is defined in accordance with conventions of Fig. 4 at the top, i.e. in relation to the position of the source S when the signal is emitted (Fig. 4 shows the position of the source at the reception time). In particular, when M = 1, this angle is equal to π for a listening point put on the "sound barrier" and not to π / 2 as we should be tried to think (see Fig. 4 at the bottom). That is the reason why the Doppler frequency listen in E is infinite in theory. Furthermore, for some Mach number > 1, an immediate consequence of Eqs. (1) and (2) is a Doppler frequency which is infinite if the listening point is put on the Mach cone, the angles α and θ being supplementary (see Fig. 5). Let us notice that this observation is obviously in contradiction with the representation of Fig. 3, where the wavelength λ_0 seen on the sound cone (distance between two points tangential to spheres emitted at frequency f_0) is not null obviously.

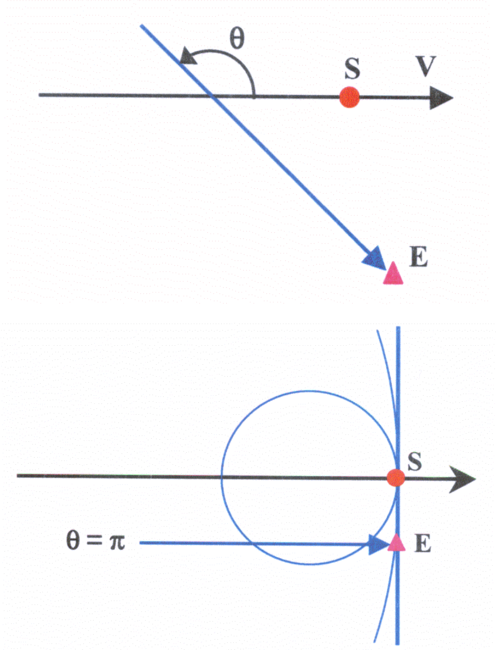

Fig. 4 Geometric conventions for calculation of Doppler effect: M ≠ 1 (at the top), M = 1 (at the bottom).

From the point of view of physics, this result is interpreted by the fact that the Mach cone is, like the sound barrier, the geometric locus of points on which all signals emitted on the trajectory of the source arrive simultaneously, the ray path difference between sound rays being compensated for the source speed. This is obvious in the sonic case of Fig. 4 (only rays emitted perpendicularly to the shock front must be taken into consideration), and the demonstration is easy in general case of Fig. 5 by making the same hypothesis, the listening point being then rejected to infinity.

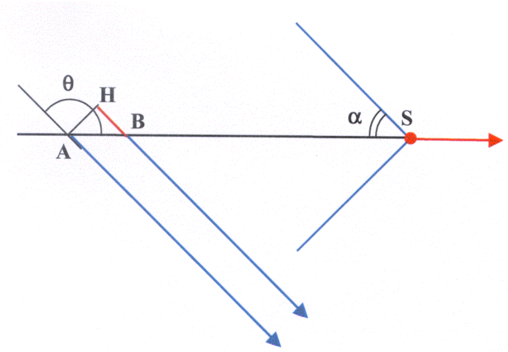

Fig. 5 Calculation of ray path difference between two successive pulses.

Note that, if segment AB is the distance covered between the emission of two successive pulses with a period T_0 and if Δt is the time-lag of pulses at the listening point due to the ray path difference HB, trigonometric relations in the right-angled triangle AHB lead to the Eq. (3) which proves Eq. (2) in addition:

$$f / f_0 = T_0 / (T_0 + \Delta t) = 1 / (1 + M \cos \theta) \tag{3}$$

If the direction of emission is perpendicular to the Mach cone, we have $\Delta t = -T_0$ and f is infinite. The fact that Δt is negative has to be connected with the "time inversion" of the literature, even though it is a question of a terminology which applies to the "direct frequency" from Fig. 3.

In addition, a basic notion, most of the time omitted in the literature even targeted at this subject [9], is the result of Eq. (3): the Doppler effect can be defined as the ratio between the duration of a signal at emission and its duration at the listening point. We have done a generalization of this notion to a continuous signal taken between two arbitrary points on the source trajectory, with a not inconsiderable advantage: our purely “time-dependent” approach excludes any geometric approximation and so can be applied to any trajectory (Fig. 6). Between points A and B, the signal is emitted between the successive instants t_1 and t_2. Say t'_1 and t'_2 the arrival instants corresponding respectively to the beginning and the end of the considered signal at the listening point E. If these instants can be calculated, then the average value of the Doppler factor in the interval is given by:

$$\mathrm{D.F.} = f / f_0 = (t_2 - t_1) / (t'_2 - t'_1) \qquad (4)$$

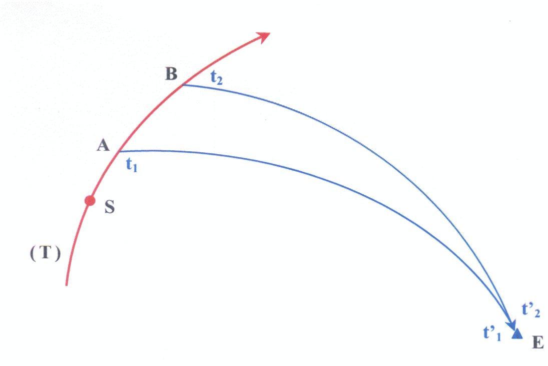

Fig. 6 Time approach of the Doppler effect.

It is obvious that the formula above is applied to the whole spectrum of frequencies f_0 emitted by the source S. Strictly speaking, nothing allows to assert that the inequality $t'_2 > t'_1$ is true, in a first analysis we can deal with as well a negative “direct frequency” as a positive “backward frequency”. Propagation is shown on Fig. 6 in real atmosphere (paths of curvilinear sound rays). Eq. (4) remains valid in the subsonic case where its denominator is positive of course.

Another aspect of the problem is that, if the hypothesis of constant energy is considered setting aside effects of atmospheric absorption, the level of spectrum received S_R (power spectral density PSD, here in dB/Hz) is inferred from the level of spectrum emitted S_E by the relation below:

$$S_R = S_E + 10 \log \left[\left| (t_2 - t_1) / (t'_2 - t'_1) \right| \right] \qquad (5)$$

In fact, depending on whether the received signal is shorter or longer than the emitted signal, its mean power must increase or decrease. The absolute value of duration ratio is justified by the fact that it is possible to have the inequality $t'_2 < t'_1$ in the supersonic case. We also see that if the listening point is on the sound cone ($t'_2 = t'_1$), then the Doppler frequency f tends towards infinity and the power spectral density (PSD) too. Eq. (5) acts as a substitute for the “convective amplification factor” quoted in the literature as an application of Eq. (2) and of its derived semi-empirical formulae.

5 Recorded spectrum simulation

In the case of “Ariane 5” launching vehicle, to calculate a modification of the frequencies spectrum due to the Doppler effect necessarily comes through a computation of propagation. Hypotheses have to be done on the duration of the selected signal around reference altitudes and on the emission direction from the source - strictly speaking, the considered emission must come from the sound power peak of every jet, but a very small error is made by considering a unique emission coming from the base of the launching vehicle. Although it is recommended to use a computing 3-D rays code, such as our “Simoun” model, in order to calculate sound trajectories and real duration of propagation, the approximation of a propagation in straight line can be done with the reserve that both emission points are close enough each other. Actually, as it is shown on Fig. 1, we are in light wind conditions with a temperature gradient relatively steady, therefore next to conditions of the standard atmosphere. The ray paths may then be considered like arcs of cycloid and analytic formulae can be applied to calculate the duration of curvilinear trajectories. Furthermore, the average sound celerity on a straight trajectory, between a point at sea level 0 and a point at an altitude z above sea level can be calculated by the formula apparently trivial:

$$\langle c \rangle = (c_z + c_0) / 2 \qquad (6)$$

In fact, the formula above is the result of an integration of the expression c(z) linked to the local temperature and its constant gradient. The theoretical duration on a vertical or oblique path is inferred from this formula. Calculations show that in the most unfavorable case (grazing incidence at ground level) the error made is 10 % at the most. In fact, as we consider a time shift between next rays, the margin of error appears to be very small compared to the possible spectrum shift due to a change of the acoustic directivity of the sound sources, in particular in the supersonic case. The advantage of the hypothesis of a rectilinear propagation is of course the direct calculation of the emission angle according to the jets, knowing the position of the listening point and the flight path of the launching vehicle.

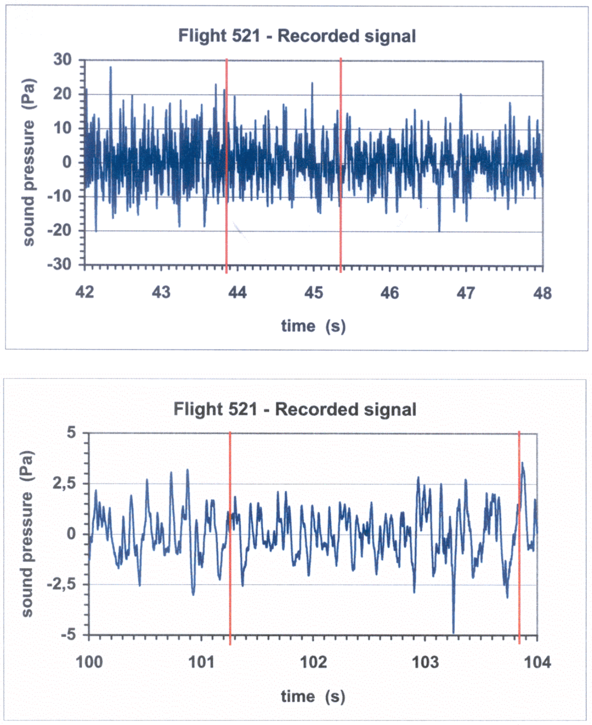

Fig. 7 Signal duration corresponding to 1 s of emission, launching vehicle at z = 3,750 m and at z = 13,000 m.

We give two examples of calculation, the former when the launching vehicle is at 3,750 m above sea level flying at subsonic speed, the latter when it is at 13,000 m flying at a supersonic speed. The process consists in:

1) choosing two emission points on the trajectory on either side of the reference altitude, such as the translation of the launching vehicle between these two points lasts one second;

2) calculating duration of both trajectories up to the listening point to deduce the arrival pips of the signal onto the recorded tape (red markers in Fig. 7). The tape has been synchronized at booster ignition;

3) inferring the value of Doppler factor from the time-lag of arrival pips (just divide emission time of one second by reception time included between the two markers);

4) applying inferred Doppler factor value, in frequency and in acoustic level, to the spectrum of jet noise simulated at the listening point - see §4, Eqs. (4)-(5);

5) applying to the calculated sound level a variation of 3 dB due to the fact that, in the vicinity of the "Toucan" station, the ground is covered with low-height vegetation. Such a ground, indeed, cannot be considered either as perfectly reflective (hypothesis called "*with ground*") or perfectly absorbent (hypothesis called "*free field*"): the real spectrum is assumed to be between the two curves;

6) analyzing the signal spectrum recorded between two arrival pips and comparing it with calculated spectra, without taking the Doppler factor into account (see §3, Fig. 2) on the one hand, by taking the Doppler factor into account in both hypotheses "*with ground*" and "*free field*" on the other hand.

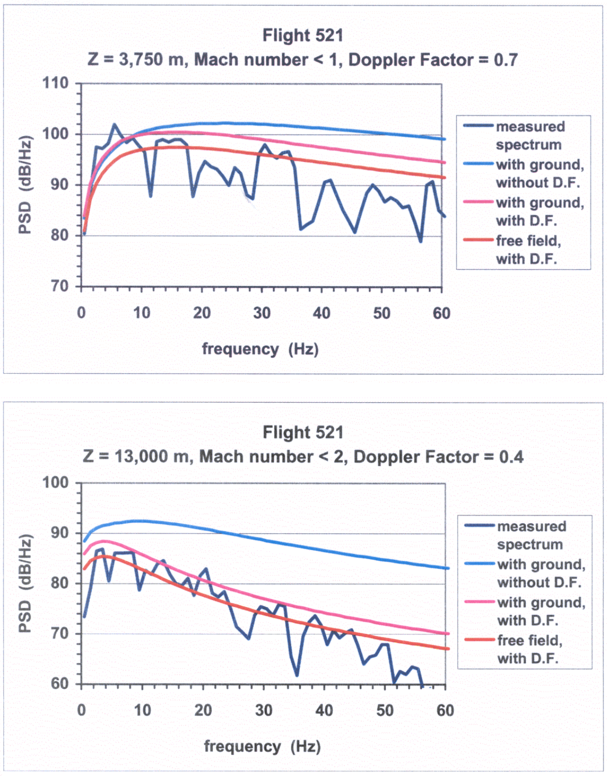

Fig. 8 Recorded spectrum and calculated spectra, launching vehicle at z = 3,750 m and at z = 13,000 m.

It appears that the Doppler factor, equal to 0.67 for z = 3,750 m and to 0.39 for z = 13,000 m, plays a considerable role in the frequency shift of the spectrum as well as in the sound level at the listening point. As shown in Fig. 8, the similarity between calculated and measured spectra is satisfactory eventually, more particularly in the infrasonic range (f < 20 Hz). In case of a transonic speed of the launching vehicle (z = 8000 m), this similarity is a bit less satisfactory, but the fact that the launching vehicle has just crossed the zone where the wind gradient is higher (Fig. 1) must be taken into account. More precise calculations of propagation should be perhaps necessary in this case.

6 Conclusion

Despite a number of listed uncertainties, the proposed example of application can be considered as a first experimental validation of the models described in this paper. The simulation of propagation might be improved by calculating sound rays and the atmospheric attenuation might be taken into account. Such an improved simulation has the predictable effect of reducing the differences observed between calculated and measured spectra.

Our original approach for calculating the Doppler effect described above seems to be more precise and more reliable than calculations from semi-empirical models found in the literature. This approach is simpler and more general than theoretical models, the formulae of which can generally be applied only in the case of a source moving at a constant speed and in line with the observation point.

Finally, this approach can be coupled with classic tools of propagation and applied in the case of any atmosphere, which gives it a practical value undoubtedly.

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