Acoustics'08 Paris
June 29-July 4, 2008
www.acoustics08-paris.org

Sound radiation induced force vibration of Submerged and Fluid Filled multi-layered spherical shell

Sadeq Malakooti[a], Seyyed M. Hasheminejad[a] and Nader Mohammadi[b]

[a]Acoustics Research Laboratory, Dept. of Mechanical Engineering, Iran University of Science and Technology, Narmak, 16844 Tehran, Iran
[b]Univ. of Tehran, School of Mechanical Engineering, 1439957131 Tehran, Iran
s_malakooti@mecheng.iust.ac.ir

Nowadays sound radiation from spherical shells is attracted for aviation industry. In this paper, the general approach is presented for predicting a coupled fluid solid interaction problem. Therefore, a fully elasticity solution is applied in conjunction with laminated shell model for considering of the sound field over the shell. For each plies the elasticity coefficients are assumed to be constant. The main objectives of this work are investigating the influence of internal acoustic mediums and also radially inhomogeneity on acoustic performance of the multilayered spherical shell. Numerical results are presented for a three layered spherical shell. For each excitation frequency, sound pressure over zenith angle in polar coordinate is plotted. Also radiated sound pressure levels against excitation frequency for the scope of resonance spectroscopy are investigated.

1 Introduction

In the recent years, Interaction of an incident sound field with spherical shells is a problem of long-standing interest in underwater acoustic [1, 2]. Experiments as well as theoretical studies are continued and reported in the literature [3, 4, and 5]. Against the acoustic scattering, radiation of the sound by elastic structures caused by mechanical excitation had less considered. In this way, Wu et al. [6] predicted the sound radiation of coated cylindrical shell and also Chen [7] could formulate submerged elastic structures using in-vacuo vibrational mode expansions with which the acoustic impedance loading is derived based on radiation mode theory. Vibration and acoustic radiation of reinforced spherical shell was studied by C. Junming [8]. Pathak et al. was addressed the problem of harmonically excited spherical shells surrounded by a fluid medium by the use of the finite Legendre transform [9].

Above review indicates that, in contrast with the mono layer spherical shells, there seem to be no rigorous investigations on acoustic radiation induced forced vibration of a multi layered spherical shell. The primary purpose of the current work is to fill this gap. Hence, the Legendre Transform is used to expand the field variables for achieving an exact analysis for radiation of acoustic waves by a isotropic multilayered spherical shell submerged in and filled with compressible ideal fluid mediums.

2 Theory

The problem geometry is depicted in Fig. 1, where (x,y,z) is the Cartesian coordinate system with origin at O and (r,θ) is the corresponding spherical polar coordinate system.

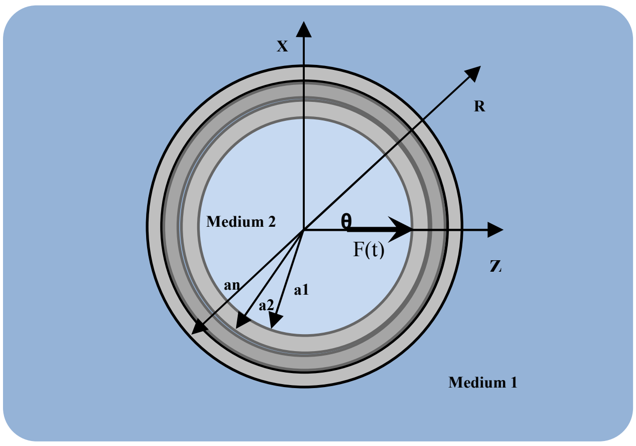


Fig. 1 Problem geometry

A multilayered spherical shell which immersed in an infinite static ideal fluid space and filled with another ideal fluid is excited harmonically by a point load. Both shell layers and fluids are assumed to be homogeneous and isotropic. Taking advantage of problem axisymmetry (i.e., ignoring all Φ-dependencies) the irrotational and rotational elastic displacement potentials for ith layer of the shell are denoted by $\Phi_i(r,\theta)$ and $\chi_i(r,\theta)$ respectively and the irrotational displacement potential for the fluids is denoted by $\Phi^f(r,\theta)$. The Lame constants for the ith layer of the shell materials are denoted by λ_i and μ_i and that for the fluids by λ^f. Harmonic distributed force per unit area $p(r,\theta)\exp(+j\omega t)$ is assumed to be acting normally on the inner surface of the shell. In the following development the factor $\exp(j\omega t)$ will be suppressed.

The potentials for the coupled shell-fluid system $\Phi(r,\theta)$, $\chi(r,\theta)$ and $\Phi^f(r,\theta)$ must satisfy the following reduced wave equations in the spherical polar coordinate system,

$$\nabla^2 \Phi^i(r,\theta) + k_d^{i\,2} \Phi^i(r,\theta) = 0 \tag{1}$$

$$\nabla^2 \chi^i(r,\theta) + k_s^{i\,2} \chi^i(r,\theta) = 0 \tag{2}$$

$$\nabla^2 \Phi^f(r,\theta) + k^{f\,2} \Phi^f(r,\theta) = 0 \tag{3}$$

, where $k_d^{i\,2} = \omega^2 / c_d^{i\,2}$, $k_s^{i\,2} = \omega^2 / c_s^{i\,2}$, $k^{f\,2} = \omega^2 / c^{f\,2}$ and c_d^i is the dilatational wave speed, c_s^i is the shear wave speed in the shell materials, c^f is the acoustic wave speed in the fluids, and ω is the excitation frequency. The point force is expressed by

$$P(a_0,\theta) = \frac{F_0 \delta(\theta)}{2\pi a_0^2 \sin\theta} \tag{4}$$

, where δ is Dirac Delta function. By expanding of eq. 4 in spherical coordinate, we have

$$P(a_0,\theta) = \sum_{n=0}^{\infty} \frac{2n+1}{2} \frac{F_0}{2\pi a_0^2} P_n(\cos\theta) \tag{5}$$

, which $P_n(\cos\theta)$ is Legendre function with argument cosθ. Solutions of Eq. 1 to 3 in spherical coordinate are

$$\Phi^i(\mathrm{r},\theta) = \sum_{n=0}^{\infty} \left[A_n^i j_n\left(k_d^i r\right) + B_n^i y_n\left(k_d^i r\right)\right] P_n(\cos\theta) \tag{6}$$

$$\chi^i(\mathrm{r},\theta) = \sum_{n=0}^{\infty} \left[C_n^i j_n\left(k_s^i r\right) + D_n^i y_n\left(k_s^i r\right)\right] P_n(\cos\theta) \tag{7}$$

$$\Phi^f(r,\theta)=\sum_{n=0}^{\infty} E_n^f h_n\left(k^f r\right)P_n(\cos\theta) \tag{8}$$

where A_n^i, B_n^i, C_n^i, D_n^i and E_n^i are constants, j_n and y_n are nth-order spherical Bessel functions of the first and second kind, respectively and h_n the nth-order spherical Hankel functions of the second kind. By applying boundary conditions for each interface, the unknown constants will determine. Continuity normal/shear stresses, displacements for interfaces of the each layer is formed set of required boundary conditions. Quantities related to left side of each layer is superscripted with (-) and ones related to right side of each layer is superscripted with (+).

Normal stress continuity for the ith layer

$$\begin{matrix}\left.\sigma_{rr}^{-}\right|_{r=a_{i-1}}\\ \left.\sigma_{rr}^{+}\right|_{r=a_{i}}\end{matrix}=-\lambda^i k_d^{i\,2}\Phi^i+2\mu^i\left[\frac{\partial^2}{\partial r^2}\left(\Phi^i+\frac{\partial}{\partial r}\left(r\chi^i\right)\right)+k_s^{i\,2}\frac{\partial}{\partial r}\left(r\chi^i\right)\right] \tag{9}$$

Shear stress continuity for the ith layer

$$\begin{matrix}\left.\tau_{r\theta}^{-}\right|_{r=a_{i-1}}\\ \left.\tau_{r\theta}^{+}\right|_{r=a_{i}}\end{matrix}=\mu^i\left[2\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(\Phi^i+\frac{\partial}{\partial r}\left(r\chi^i\right)\right)\right)+k_s^{i\,2}\frac{\partial\chi^i}{\partial\theta}\right] \tag{10}$$

Displacement continuity between i-1th and ith layer

$$\begin{aligned}u_r^{i-1}&=\frac{\partial}{\partial r}\left[\Phi^{i-1}+\frac{\partial}{\partial r}\left(r\chi^{i-1}\right)\right]+rk_s^{i-1\,2}\chi^{i-1}=\\ &\frac{\partial}{\partial r}\left[\Phi^{i}+\frac{\partial}{\partial r}\left(r\chi^{i}\right)\right]+rk_s^{i\,2}\chi^{i}=u_r^i\end{aligned} \tag{11}$$

Full derivation for a three layered sandwich spherical shell submerged and filled with ideal fluids is derived (Appendix).

3 Numerical results

Numerical results are now presented for the case of a sandwich (Steel-Copper-Steel) spherical shell surrounded by water. The shell is considered that filled with three types of fluid (Air, Freon and Glycerin). Each layer has the same thickness. Ratio of shell thickness over shell radius is 0.01. Also outer radius of the shell is 1 meter. The numerical values of the material constants are listed in table 1.

	λ (GPa)	μ (GPa)
Steel	103.16	77.82
Copper	92.77	43.66
	ρ (kgm^{-3})	**Speed of sound (ms^{-1})**
Air	1	340
Water	1025	1460
Freon	1880	655
Glycerin	1250	1910

Table 1 Fluids and structure parameters used in the calculations

All quantities are presented as a function of nondimensional frequency (k_1a_s) which is equal to multiply of acoustic wave number in surrounding on outer radius of the shell. Pressure on the shell and far field pressure directionality patterns are plotted (Fig. 2) for different internal fluids.

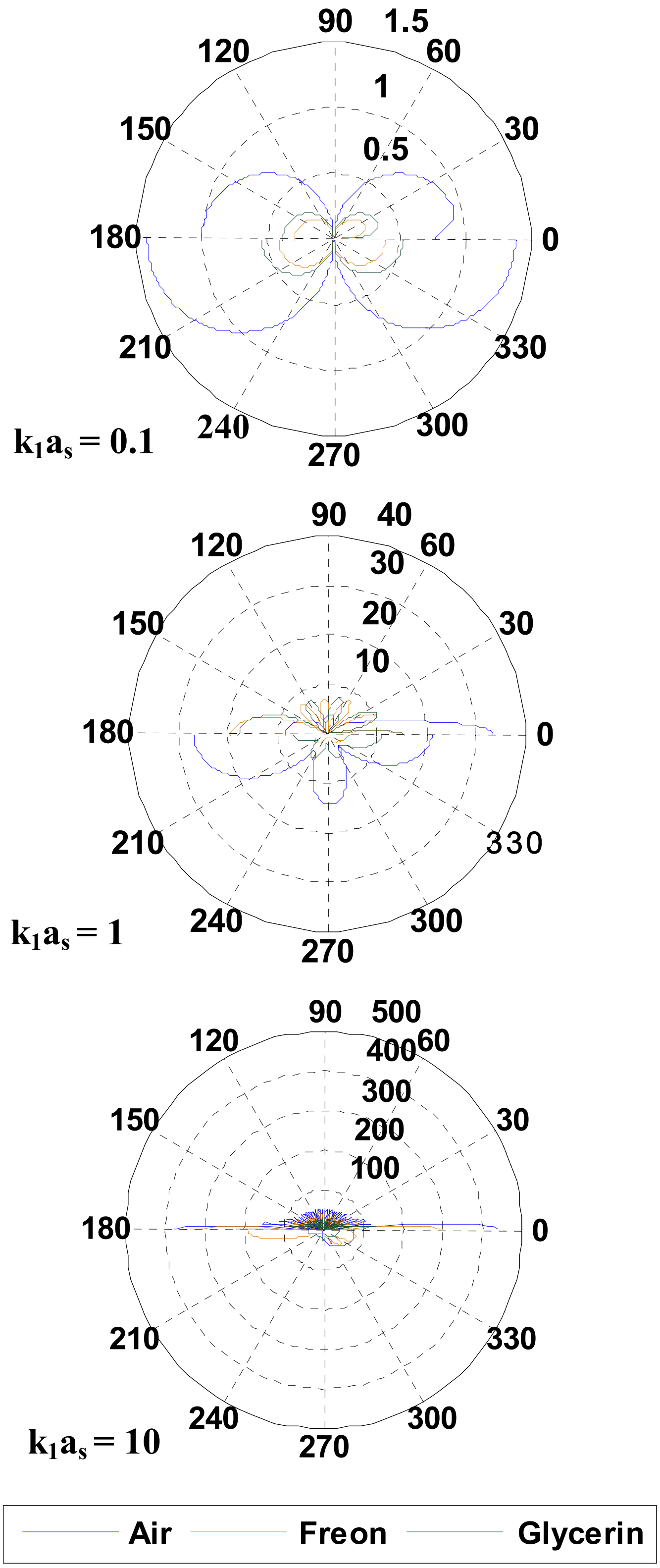
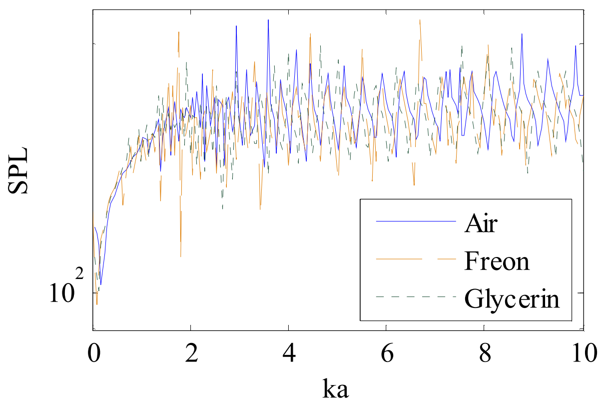
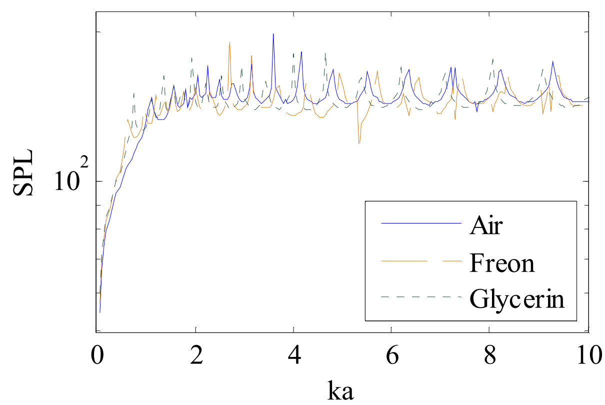


Fig. 2 Near field (@ r = a_s, $0 < \theta < \pi$) and Scaling Far field (@ r = 10 a_s, $\pi < \theta < 2\pi$) pressure radiated by the sandwich (Steel-Copper-Steel) spherical shell



(A)



(B)

Fig. 3 Sound pressure level on the shell, (A) @ θ = 0, (B) @ θ = π/2 versus frequency

Also sound pressure level against excitation frequency for two points on the shell is plotted (Fig. 2). Finally, in order to check overall validity of the work, we computed pressure pattern on the evacuated single layer steel spherical shell surrounded in water. Numerical result, as shown in Fig. 3 show excellent agreement with Fig. 7 of [9].

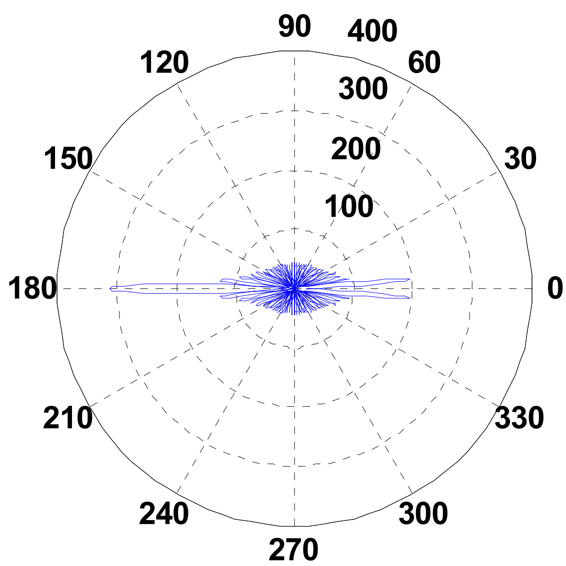


Fig. 4 Pressure on the point excited evacuated single layer spherical shell for ka=10

4 Discussion and conclusion

At the first, consider near field pressure patterns. For the excitation in low frequency, the dominant contribution to the response comes from the rigid body motion mode. Hence the pressures on the shell vary as cosθ except in the vicinity of the force. But in high frequency, higher shape functions are advent in the response. Indeed in higher mode shape, numbers of nodes are increased. Therefore pressure patterns in high frequency convert as a multi pick structures. Far field region in low frequency is behaved dipole like radiated pressure which (Like near field) indicates the domination of rigid body mode in shell response.
Another aspect that can be founded in fig. 2 is the role of internal fluid density on the radiation of the shell on outer acoustic field. High dense fluid increases the damping in the system. Therefore the shell with light interior fluids radiates high value of sound pressure as a heavy interior fluid. So near and far field sound radiated in the case air is higher than glycerin and also Freon.
Resonance behavior of the system in two cases is shown in fig. 3. In the case of direct point measurement (the load and response point are in one direction), between each two resonance points, the antiresonance point is existed. But in the case of indirect measurement the antiresonance points are absent.

Acknowledgments

The authors would like to sincerely thank to the people of Acoustics Research Laboratory of Iran University of Science and Technology. Also I dedicate my thanks to Mr. S. Abbasion, Mr. A. Rafsanjani, Dr. M. Vahedi and Dr. Y. Mirzaei for their advices.

Appendix

Continuity of normal stresses on each interface

$$\sigma_{rr}^{1}(a_1,\theta)=F_{\text{int}}(\theta)-P_{f_1}(a_1,\theta)=$$

$$\left.\left(-\lambda^{1}k_d^{1\,2}\Phi^{1}+2\mu^{1}\left[\begin{array}{l}\dfrac{\partial^2}{\partial r^2}\left(\Phi^{1}+\dfrac{\partial}{\partial r}\left(r\chi^{1}\right)\right)\\+k_s^{1}\dfrac{\partial}{\partial r}\left(r\chi^{1}\right)\end{array}\right]\right)\right|_{r=a_1}$$

$$\sigma_{rr}^{2}(a_2,\theta)=\sigma_{rr}^{1}(a_2,\theta)=$$

$$\left.\left(-\lambda^{2}k_d^{2\,2}\Phi^{2}+2\mu^{2}\left[\begin{array}{l}\dfrac{\partial^2}{\partial r^2}\left(\Phi^{2}+\dfrac{\partial}{\partial r}\left(r\chi^{2}\right)\right)\\+k_s^{2}\dfrac{\partial}{\partial r}\left(r\chi^{2}\right)\end{array}\right]\right)\right|_{r=a_2}$$

$$\sigma_{rr}^{3}(a_3,\theta)=\sigma_{rr}^{2}(a_3,\theta)=$$

$$\left.\left(-\lambda^{3}k_d^{3\,2}\Phi^{3}+2\mu^{3}\left[\begin{array}{l}\dfrac{\partial^2}{\partial r^2}\left(\Phi^{3}+\dfrac{\partial}{\partial r}\left(r\chi^{3}\right)\right)\\+k_s^{3}\dfrac{\partial}{\partial r}\left(r\chi^{3}\right)\end{array}\right]\right)\right|_{r=a_3}$$

$$\sigma_{rr}^{3}(a_4,\theta)=F_{ext}(\theta)-P_{f_2}(a_4,\theta)=$$

$$\left.\left(-\lambda^3 k_d^{3\,2}\Phi^3+2\mu^3\left[\begin{array}{l}\frac{\partial^2}{\partial r^2}\left(\Phi^3+\frac{\partial}{\partial r}\left(r\chi^3\right)\right)\\+k_s^3\frac{\partial}{\partial r}\left(r\chi^3\right)\end{array}\right]\right)\right|_{r=a_4}$$

Where

$$P_{f_1}(a_1,\theta)=-\lambda_{f_1}^2 k_d^{f_1\,2}\Phi^{f_1}$$

$$P_{f_2}(a_4,\theta)=-\lambda_{f_2}^2 k_d^{f_2\,2}\Phi^{f_2}$$

Zero shear stress on each interface

$$\sigma_{r\theta}^{1}=\mu^1\left.\left(\begin{array}{l}2\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial\theta}\left(\Phi^1+\frac{\partial}{\partial r}\left(r\chi^1\right)\right)\right]\\+k_s^{1\,2}\frac{\partial\chi}{\partial\theta}\end{array}\right)\right|_{r=a_1,a_2}=0$$

$$\sigma_{r\theta}^{2}=\mu^2\left.\left(\begin{array}{l}2\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial\theta}\left(\Phi^2+\frac{\partial}{\partial r}\left(r\chi^2\right)\right)\right]\\+k_s^{2\,2}\frac{\partial\chi}{\partial\theta}\end{array}\right)\right|_{r=a_2,a_3}=0$$

$$\sigma_{r\theta}^{3}=\mu^3\left.\left(\begin{array}{l}2\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial\theta}\left(\Phi^3+\frac{\partial}{\partial r}\left(r\chi^3\right)\right)\right]\\+k_s^{3\,2}\frac{\partial\chi}{\partial\theta}\end{array}\right)\right|_{r=a_2,a_3}=0$$

Continuity normal displacement

$$\left.\frac{\partial\Phi_{f_1}}{\partial r}\right|_{r=a_1}=\left.\left(\frac{\partial}{\partial r}\left(\Phi^1+\frac{\partial}{\partial r}\left(r\chi^1\right)\right)+rk_s^{1\,2}\chi^1\right)\right|_{r=a_1}$$

$$\left.\frac{\partial\Phi_{f_2}}{\partial r}\right|_{r=a_4}=\left.\left(\frac{\partial}{\partial r}\left(\Phi^3+\frac{\partial}{\partial r}\left(r\chi^3\right)\right)+rk_s^{3\,2}\chi^3\right)\right|_{r=a_4}$$

Above equations are generated the system of equation for the unknown coefficients in the matrix form as

$$[C]_{12\times12}\left[A_n^1\quad B_n^1\quad C_n^1\quad D_n^1\quad A_n^2\quad B_n^2\quad C_n^2\quad D_n^2\quad A_n^3\quad B_n^3\quad C_n^3\quad D_n^3\right]^T=$$
$$\left[F_{ext}\quad F_{\mathrm{int}}\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0\right]^T$$

By solving obtained system of equation, the outer normal velocity and also acoustic pressure on the shell can be determined as follow.

$$v(a_4,\theta)=\left.\left[\frac{\partial}{\partial r}\left(\Phi^3+\frac{\partial}{\partial r}\left(r\chi^3\right)\right)+rk_s^{3\,2}\chi^3\right]\right|_{r=a_4}=$$

$$\sum_{n=0}^{\infty}v_n P_n(\cos\theta)$$

And consequently the radiated pressure by the shell can be

$$p(r,\theta)=\sum_{n=0}^{\infty}v_n\times\left(-i\rho_{f_2}c_{f_2}\frac{h_n\left(k_{f_1}r\right)}{h_n'\left(k_{f_1}r\right)}\right)\times P_n(\cos\theta)$$

References

[1] R. Goodman, R. Stern, Reflection and transmission of sound by an elastics shell, *J. Acoust. Soc. Am.* 34, 338-334 (1962)

[2] R. Hickling, Analysis of echoes from a hollow metallic sphere in water, *J. Acoust. Soc. Am.* 36 1124-1137 (1964)

[3] G. Kaduchak, P. L. Marston, Backscattering of chirped bursts by a thin spherical shell near coincidence frequency, *J. Acoust. Soc. Am.* 93 2700-2706 (1993)

[4] Ho JM, Geometrical theory of acoustic scattering by thin elastic shells, *J. Acoust. Soc. Am* 96 (5) 3115–3125 (1994)

[5] Ho JM, Near-field ray acoustic response of submerged elastic spherical shells, *J. Acoust. Soc. Am.* 96 (10) 525–535 (1994)

[6] Wu C, Chen H, X. Huang, Theoretical prediction of sound radiation from a heavy fluid-loaded cylindrical coated shell, *Chinese J. Mech. Eng.* 11 (4) 249–256, (1998)

[7] P. T. Chen, Acoustic radiations for submerged elastic structures using natural mode expansions in conjunction with radiation modes approach, *J. Sound Vib.* 246 (2) 245–263 (2001)

[8] C. Junming, H. Yuying, Vibration and acoustic radiation from submerged stiffened spherical shell with deck-type internal plate, *Acta Mechanica Solida Sinica* 16 (3), (2003)

[9] G. A. Pathak, P. R. Stepanishen, Acoustic harmonic radiations from fluid-loaded spherical shells using elasticity theory, *J. Acoust. Soc. Am.* 96 (4), (1994)