



Acoustics'08
Paris
June 29-July 4, 2008

www.acoustics08-paris.org

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Acoustically invisible cylinder

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Coatings of new type recently proposed in (Acoustical Physics, 2007, vol. 53, N5, pp. 535-545) are applied to bodies of the cylindrical geometry to reduce reflection or scattering of sound and thus to make them undetectable by imaging systems. It is shown by computer simulation that a rather simple coating of this type can reduce the back-scattered pressure amplitude more than 40 dB with respect to the rigid cylinder practically at all frequencies. Considerable reduction of the scattered power can also be achieved but in a low frequency range. The width of this range and the reduction index depend on the number of couplings introduced into the coating.

1 Introduction

Most acoustic imaging systems are based on insonification of an object and analysis of the sound energy that is reflected or scattered by the object [1]. If the object does not reflect or scatter sound it cannot be detected by such systems. Acoustical invisibility is understood, in this paper, just as nonreflecting or nonscattering. The purpose of the paper is to show how to make an arbitrary body acoustically invisible with the help of a thin passive coating recently proposed in [2], to illustrate its work on bodies of cylindrical geometry and compare its efficiency with that of existing methods. But before this, a brief account of the state-of-the-art of the acoustical invisibility problem is given.

Pulse-echo techniques that provide cross-sectional images of objects are the simplest techniques used in sonar (and radar) systems as well as in Nature by certain animals (bats, dolphins). Information about the object is derived from the signals reflected from the object back to the sound source. To make the object invisible means, in this case, to suppress backscattering. As commonly accepted, the best way to do so is to make the object surface absorptive like the surface of the black body. By definition, the black body, introduced by Kirchoff in 1859, absorbs all the wave energy that falls upon its surface and, as a consequence, does not reflect back. There are some bodies and coatings that behave like the black body, e.g. the matched coating (that has the local impedance ρc) and the so-called body of Macdonald. At high frequencies they are almost invisible with respect to pulse-echo systems [3,4].

Much more difficult is the problem of making a body invisible with respect to more sophisticated imaging systems that use the scattered field in a wide range of angles. An invisible body should, in this case, be indiscernible from the fluid displaced by the body, i.e. acoustically transparent (nonscattering). In particular, such a body should absorb the incident wave on the insonified side of the surface and reradiate it from the opposite (shadow) side of the surface. Note, that since the black body casts the shadow it is visible in transmission field.

In the literature, there are several solutions to the problem of suppressing the scattered field. The most elaborated are the *active* methods of Maluzhinets, JMC (Jessel-Mangiant-Canevet) and others that are based on the Huygens principle. In these methods, the total acoustic field is measured around the object and, by the use of the integral operator of Helmholtz-Huygens, is decomposed into the incoming (incident) component and the outgoing (scattered) component. The scattered component is then suppressed by a set of acoustic actuators that envelope the object and sensors. The methods are verified in laboratory experiments. Reviews of the literature on the subject can be found elsewhere [5,6]. One more active method is

described in paper [6]. It differs from the methods mentioned above by the use of structural sensors and actuators.

Principally different method of suppressing the scattered field is proposed in paper [2]. Here the problem is solved with the help a thin *passive* coating called as a coating of extended reaction (CER). The coating represents a periodic surface structure of identical small wave size elements. The distinguishing feature of a CER is the introduction of couplings between neighboring elements. Owing to these correctly chosen couplings the efficiency of CERs can be made much higher than that of the existing coatings. In the case of the plane geometry high efficiency of CERs is shown theoretically in paper [2]. In the present paper, CERs are applied to bodies of the cylindrical geometry and its high efficiency is demonstrated in computer simulation examples.

In conclusion of the review, it is worth mentioning that the problem of an invisible cloak, i.e. of a nonscattering coating, is now intensely studied in optics. But the majority of the scientific papers here develop one idea: to construct a matching coating-waveguide that makes the light rays trace round the object without reflection or scattering. Existence of such a solution is mathematically proved, but no substantial implementation has been reported so far [7,8].

2 Impedance solution to the scattering problem

Let in a fluid medium ρc be an infinite circular cylinder of radius a which is insonified by a plane wave of the complex amplitude p_i at an angle $\pi/2-\theta$ to the cylinder axis x . In the cylindrical coordinates (x, r, φ) the incident wave is written as

$$\begin{aligned} p_i(x, r, \varphi) &= p_i e^{ik_x x} \sum_{m=0}^{\infty} p_{im} \frac{J_m(k_r r)}{J_m(k_r a)} \cos(m\varphi), \\ v_i(x, r, \varphi) &= \frac{p_i}{\rho c} e^{ik_x x} \sum_{m=0}^{\infty} v_{im} \frac{J'_m(k_r r)}{J'_m(k_r a)} \cos(m\varphi), \\ p_{im} &= \varepsilon_m i^m J_m(k_r a), \quad v_{im} = -p_{im} / z_{im}, \\ z_{im} &= -\frac{iJ_m(k_r a) k}{J'_m(k_r a) k_r}. \end{aligned} \quad (1)$$

Here v_i is the radial component of the fluid particle velocity, ε_m is the Neumann's function, $k_x = k \sin \theta$, $k_\varphi = k \cos \theta$, and z_{im} is the specific surface impedance of the cylindrical volume of the fluid with respect to an external excitation in the form of m -th circular mode

$$\cos(m\varphi) \exp(ik_x x). \quad (2)$$

These impedances will be called as the internal modal impedances of the fluid. Similarly, the field component scattered by the cylinder can be written as

$$\begin{aligned} p_s(x, r, \varphi) &= p_i e^{ikx \sin \theta} \sum_{m=0}^{\infty} p_{sm} \frac{H_m(k_r r)}{H_m(k_r a)} \cos(m\varphi), \\ v_s(x, r, \varphi) &= \frac{p_i}{\rho c} e^{ikx \sin \theta} \sum_{m=0}^{\infty} v_{sm} \frac{H'_m(k_r r)}{H'_m(k_r a)} \cos(m\varphi), \\ p_{sm} - z_{rm} v_{sm} &= 0, \quad z_{rm} = \frac{iH_m(k_r a)}{H'_m(k_r a)} \frac{k}{k_r}, \end{aligned} \quad (3)$$

where z_{rm} is the specific radiation impedance of m -th mode, i.e. the specific impedance of the exterior fluid with respect to the excitation (2) applied to the surface $r=a$.

As the cylinder is assumed to be uniform and axially symmetric it is fully characterized by a set of the modal impedances in vacuo with respect to the excitation (2). These impedances, normalized with ρc , are designated as z_m .

The scattering problem for such a cylinder is tractable analytically and its solution can be found in the literature, e.g., in [9]. Here, this solution is represented in an impedance form in accordance with the new (impedance) theory of scattering [10] used in the present work. Following the theory, two modal scattering coefficients, S_m for the pressure and Q_m for the radial velocity, are defined on the cylinder surface

$$v_{sm} = Q_m v_{im}, \quad p_{sm} = S_m p_{im}. \quad (4)$$

These coefficients can be found from the following generalized Fresnel's equations

$$\begin{aligned} Q_m &= \frac{z_{im} - z_m}{z_{rm} + z_m}, \quad S_m = \frac{y_{im} - y_m}{y_{rm} + y_m}, \\ z_{rm} Q_m + z_{im} S_m &= 0, \end{aligned} \quad (5)$$

where $y_j = 1/z_j$ ($j=m, im, rm$) are the specific modal mobilities (admittances). Equations (4), (5) express the solution to the problem via the incident field and three impedances – the cylinder impedance z_m and two fluid impedances, z_{im} and z_{rm} .

As an index of transparency TI of a cylinder we use the normalized scattering cross-section. It is defined as the ratio of the power of sound scattered by 1m of the cylinder to the power of sound incident on 1m of the cylinder: the smaller TI the lower scattering and the more transparent the cylinder. Using equations (1) and (3) one can derive the following formulae for TI :

$$\begin{aligned} TI &= \frac{4}{k_r a} \left[\frac{|Q_0|^2 J_1^2}{2|H_1|^2} + \sum_{m=1}^{\infty} \frac{|Q_m|^2 (J'_m)^2}{|H'_m|^2} \right] \\ &= \frac{4}{k_r a} \left[\frac{|S_0|^2 J_0^2}{2|H_0|^2} + \sum_{m=1}^{\infty} \frac{|S_m|^2 J_m^2}{|H_m|^2} \right], \end{aligned} \quad (6)$$

As a measure of the cylinder ability to reflect sound back to the source accepted is the so called backscattered form

function BFF [11], i.e. the normalized backscattered pressure amplitude at $r=R$, $\varphi=\pi$:

$$BFF = \frac{|p_s|}{|p_{ref}|} = \sqrt{\frac{2R}{a}} \left| \sum_{m=0}^{\infty} (-1)^m v_{im} z_{rm} \frac{Q_m H_m(k_r R)}{H_m(k_r a)} \right|. \quad (7)$$

The reference amplitude is $p_{ref} = p_i (a/2R)^{1/2}$. In the numerical examples below, in the frequency range $ka < 10$ the number of modes is restricted by $m < 100$ and $R=100a$.

3 Coating of extended reaction (CER)

Fig.1 presents a schematic of one of the simplest CERs for a cylinder. This is a two dimensional periodic structure consisting of identical elements of size $l_x \times l_\varphi$. Each element is characterized by impedance Z_0 and is coupled with the first neighbors by impedance Z_1 in the φ -direction and by Z'_1 in x -direction. It means that the neighbors are acting on each other with forces

$$\pm Z_1 (v_{n+1,j} - v_{n,j}), \quad \pm Z'_1 (v_{n,j+1} - v_{n,j}),$$

so that forced vibrations of the whole coating are described by a set of the following linear difference equations

$$\begin{aligned} f_{n,j} &= Z_0 v_{n,j} + Z_1 (2v_{n,j} - v_{n+1,j} - v_{n-1,j}) \\ &\quad + Z'_1 (2v_{n,j} - v_{n,j+1} - v_{n,j-1}), \end{aligned} \quad (8)$$

where $v_{n,j}$ denotes the complex amplitude of the radial velocity of the coating element with indexes n, j ; $n=1, 2, \dots, N$; $j \in (-\infty, \infty)$.

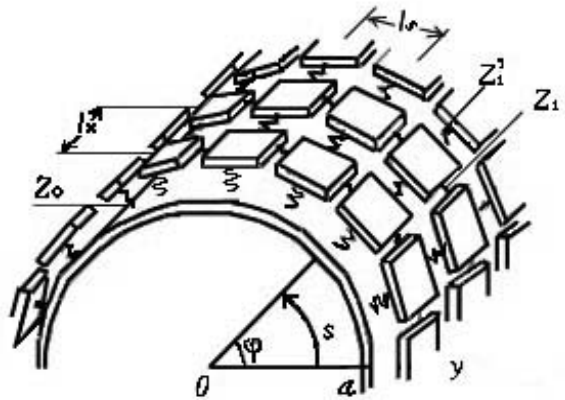


Fig.1. Schematic of a cylindrical coating of extended reaction.

Since the wave size of the coating element is assumed small it is convenient to pass from the difference equations (8) to the following continuous equation

$$s_0 v(\varphi, \xi) - s_1 \alpha \frac{\partial^2 v}{\partial \varphi^2} - s'_1 \alpha^{-1} \frac{\partial^2 v}{\partial \xi^2} = q(\varphi, \xi), \quad (9)$$

where $q=f/l_\varphi l_x$ is the surface density of the external force (pressure), $\xi=x/a$, and

$$\alpha = \frac{l_s}{l_x}, \quad s_0 = \frac{Z_0}{l_s l_x}, \quad s_1 = \frac{Z_1}{a^2}, \quad s'_1 = \frac{Z'_1}{a^2}. \quad (10)$$

The modal specific impedance of the cylinder with the coating is computed from equation (9) where the external force q is taken in the form of a cylindrical mode (2):

$$\rho c z_m = s_0 + m^2 \alpha s_1 + (k_x a)^2 \alpha^{-1} s_1'. \quad (11)$$

The modal impedance of the coated cylinder is, thus, linearly dependent on three impedance structural parameters Z_0 , Z_1 and Z_1' . If the coating elements are coupled to the second, third, etc. neighbors the order of differential equation (9) becomes 4, 6, etc., and the number of the structural parameters increases.

In what follows, the optimal values of the structural parameters which render minimum to the goal function (6) or (7) are first found. Unfortunately, this variational problem is not tractable analytically even in the simplest cases, so it is solved here numerically. After that the efficiency of such optimal CERs is computed and compared to that of some existing ones.

4 Efficiency of CERs

Fig.2 represents the optimal values of impedance $Z_0/\rho c$ of an element of the best local coating that minimizes the backscattered pressure amplitude (7): the real part of the impedance (resistance) is shown by a solid line, and the imaginary part – by a dashed line. At high frequencies it is close to the impedance of the matched coating. At low and middle frequencies it demonstrates low quality resonances. The efficiency of this optimal local coating is shown in Fig.3 (curve 3). As is seen from the figure, optimization of a local coating leads practically to full suppression of back scattering. In a wide frequency range ($ka > 0.4$), the amplitude of the backscattered pressure is more that 40 dB lower than the amplitude reflected from the rigid cylinder (curve 1). It is also much better than the efficiency of the matched coating (curve 2).

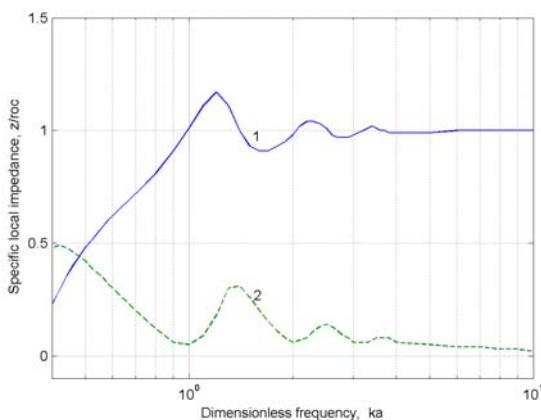


Fig.2. Specific impedance $Z_0/\rho c$ of a local coating that minimizes the backscattering pressure amplitude: 1 – resistance, 2 – reactance.

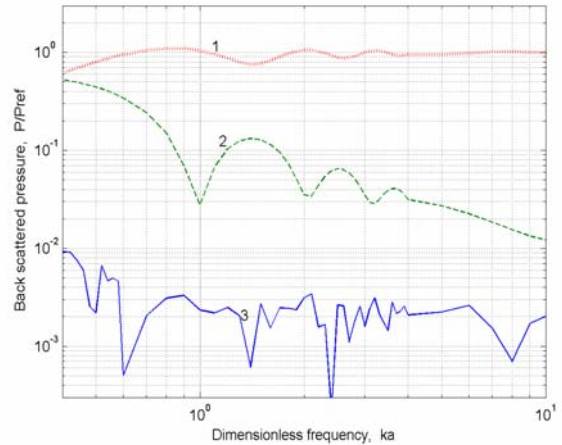


Fig.3. Backscattering form function for: 1 – rigid cylinder, 2 – matched coating, 3 – coating with the impedance shown in Fig.2.

Fig.4 shows the transparency index (6) of several cylindrical structures: a rigid cylinder (curve 1), a cylinder with the matched coating (curve 2), and of two optimal CERs (curves 3, 4) that render minimum to the goal function (6). Curve 3 corresponds to the local optimal coating. At low frequencies it is more transparent than the rigid or matched cylinder.

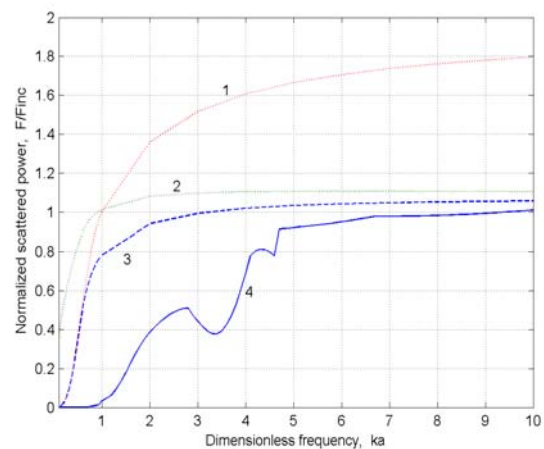


Fig.4. Acoustical transparency of: 1 – rigid cylinder, 2 – matched coating, 3 – optimal local coating, 4 – optimal CER with coupled first neighbors

At higher frequencies it is a little more efficient than the matched coating. Curve 4 corresponds to the coating with optimal coupling between the first neighbors. As it is clearly seen, the coupling makes the efficiency higher than that of the optimal local coating as well as of the matched coating at low and middle frequency range ($ka < 4$). At higher frequencies the curve 4 approaches curves 3 and 2. The author also verified that introduction of couplings between the second and third neighbors widens the frequency band of transparency and increases the transparency index in the band.

5 Conclusion

The main results of the paper are the following. An optimal coating of a very simple structure is sufficient for making a body invisible with respect to pulse-echo systems – see Fig.3. Much more complex coatings are needed to make a body nonscattering. Such a coating should have a certain number of couplings between its elements, depending on the required efficiency and width of the frequency band of transparency. Physically, the couplings provide the propagation of vibration along the coated surface for further reradiating sound in the forward direction and reducing the shadow.

As for implementation of the proposed coatings, the problem is confined to construction of elements and couplings with given impedances (like shown in Fig.2) using available materials and mechanical devices. Here, the experience gained in designing electric and numerical filters may be useful.

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