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Functionally graded plates studied by laser ultrasonic technique

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In this work, a theoretical model is proposed to predict the dispersion and the transient displacement field generated by laser in a functionally graded plate (FGP). After the laser line source is assumed as an ideal transient force, and the FGP is considered as an inhomogeneous plate along its thickness direction, the dispersion curves and the transient displacement waveforms are numerically solved. The model is first demonstrated on an aluminum plate with the comparison of the result on a homogeneous model. Laser ultrasonic measurement was carried on a FGP sample, and the direct arrivals of longitudinal and shear waves were observed in the experimental displacement field. The agreement between experimental and calculated theoretical waveforms provides a promise for both experimental and theoretical methods.

1 Introduction

Functionally graded materials (FGMs) have spatial variations in composition and structure resulting in corresponding changes in material properties. FGMs can be applied in various ways to improve material performance including: enhancing the strength of interfacial bonding and eliminating the presence of an abrupt interface, redistributing and minimizing thermal stress, suppressing micro crack damage and improving impact resistance, and reducing the driving force for crack formation at interfaces. While FGMs show great potential, few experiments has been reported on the study of ultrasound propagation in FGMs, due to the coupling difficulty of generating and detecting ultrasounds by conventional transducers. However, laser ultrasonics [1] is a technique through which the ultrasound is excited and detected by lasers, it delivers a high spatial and temporal resolution in a noncontact way (a large k and omega technique). With the advantage of this technique, it is possible to study FGMs in a noncontact way, which may provide a nondestructive testing tool on site.

Very recently, Baron et al. have theoretically studied the impact of inhomogeneity on the dispersion of surface wave for a solid [2]. Matsuda and Glorieux have obtained the dispersion of surface wave for a medium with continuously or discontinuously varying elastic property and mass density profiles for a plate [3]. The authors have found the dependence of the surface wave dispersion on power-law profiles for functionally graded coatings on a cylinder [4]. To author's knowledge, the transient response of a functionally graded plate has not been reported. The objective of this work is to develop a model that could not only provide the dispersion equation, but also predict the transient displacement response of a functionally graded plate excited by a laser line source. Numerical results on dispersion curves and transient displacement responses are presented to demonstrate the model. Laser ultrasonic experiment was carried out on FGMs samples, and experiment waveforms are further compared with theoretical predictions.

2 Theoretical model and solution

Consider the acoustic wave propagation in a functionally graded plate of thickness d, and the density ρ(z) and two lamé coefficients λ(z) and μ(z) vary along the thickness direction z. The governing wave equation is

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \sigma_{ij,j} \tag{1}$$

with stress-strain relation as

$$\sigma_{ij} = \lambda(z) u(k,k)\delta_{ij} + \mu(z)(u_{i,j} + u_{j,i}). \tag{2}$$

The displacement **U** and the stress vector **σ** are sought in the form :

$$\begin{cases} \mathbf{U}(x,z,t) = \mathbf{u}(z)\exp[j\omega(t-sx)], \\ \Sigma(x,z,t) = \boldsymbol{\sigma}(z)\exp[j\omega(t-sx)]. \end{cases} \tag{3}$$

where s being the horizontal slowness, ω the angular frequency, the horizontal wave number $k_x=\omega s$. Generally no explicit analytical solution exists for the corresponding wave equation of a second order differential equation with varying coefficients. The state vector approach [5] is applied to obtain the following matrix differential equation:

$$\frac{\partial}{\partial z}\begin{pmatrix} i\omega u_x \\ i\omega u_z \\ \sigma_{xz} \\ \sigma_{zz} \end{pmatrix} = i\omega \begin{pmatrix} 0 & s & 1/\mu & 0 \\ s\lambda/(\lambda+2\mu) & 0 & 0 & 1/(\lambda+2\mu) \\ \rho - s^2\xi & 0 & 0 & s\lambda/(\lambda+2\mu) \\ 0 & \rho & s & 0 \end{pmatrix} \begin{pmatrix} i\omega u_x \\ i\omega u_z \\ \sigma_{xz} \\ \sigma_{zz} \end{pmatrix} \tag{4}$$

where $\xi = 4\mu(\lambda+\mu)/(\lambda+2\mu)$, and state vector **B** is the displacement–traction as:

$$\mathbf{B}(z) = \begin{pmatrix} i\omega u_x & i\omega u_z & \sigma_{xz} & \sigma_{zz} \end{pmatrix}^{\mathrm{T}}, \tag{5}$$

and Eq. (4) could be rewritten as

$$\partial \mathbf{B}(z)/\partial z = \mathbf{Q}(z)\mathbf{B}(z) \tag{6}$$

The propagator matrix $\mathbf{M}(z, z_0)$ is the fundamental solution of Eq. (4) with the form :

$$\mathbf{B}(z) = \mathbf{M}(z, z_0)\mathbf{B}(z_0) \tag{7}$$

where z_0 is a reference point on the z-axis. The matrix can be calculated by the Peano expansion :

$$\mathbf{M}(z, z_0) = \mathbf{I} + \int_{z_0}^{z} \mathbf{Q}(\zeta_1)d\zeta_1 + \int_{z_0}^{z} \mathbf{Q}(\zeta_1)\int_{z_0}^{\zeta_1} \mathbf{Q}(\zeta_2)d\zeta_2 d\zeta_1 + \ldots \tag{8}$$

The explicit form of the system matrix $\mathbf{Q}(z)$ allows a convenient factorization by ω. In Eq. (4), $\mathbf{Q}(z)$ can be rewritten as a series in ω with coefficients depending only on z (through the upper integration limit) and s. This factorization can lead to a matrix polynomial form of the propagator matrix.

Let the laser line source excite the plate at (x, z)=(0, 0) in ablation regime. Assuming a delta source in time [6], we have

$$\mathbf{B}(z=0) = \begin{pmatrix} i\omega u_x & i\omega u_z & -F_0 & 0 \end{pmatrix}^{\mathrm{T}} \qquad (9)$$

where F_0 represents the intensity of the laser source in unit N·μs·m^{-1}. The other side of the plate is traction-free which leads to the P-SV wave dispersion equation in the form:

$$\det \mathbf{M}_3(d,0) = 0 \qquad (10)$$

where $\mathbf{M}_3(d, 0)$ is the left-down off-diagonal block of $\mathbf{M}(d, 0)$. The left hand side of Eq. (10) can be arranged as a polynomial, whose zeros are the eigen frequencies for a prescribed value of s. Thus the obtained set of pairs (ω, k_x) describe the dispersion curves. And the laser excited transient displacement at $z=d$ is:

$$\begin{pmatrix} i\omega u_x & i\omega u_z \end{pmatrix}^{\mathrm{T}} = \left(\mathbf{M}_2 - \mathbf{M}_1\mathbf{M}_3^{-1}\mathbf{M}_4\right)\begin{pmatrix} -F_0 & 0 \end{pmatrix}^{\mathrm{T}} \qquad (11)$$

Where $\mathbf{M}_1$, $\mathbf{M}_2$, $\mathbf{M}_3$, $\mathbf{M}_4$ correspond to the left-up, right-up, left-down, and right-down off-diagonal block of $\mathbf{M}(d, 0)$ as four 2×2 sub-matrix.

For the comparison with experiment, the transient displacements are obtained in time and space domain by a method suggested by Weaver et al [7]. The Fourier transform is generalized by replacing ω by a complex variable ω-$i\delta$ with a small, constant and imaginary part δ, and we have

$$\tilde{u}_{x,z}(x,z,t) = \frac{e^{\delta t}}{4\pi^2}\iint_{-\infty}^{+\infty} u_{x,z}(k_x, z, \omega - i\delta)e^{-i(k_x x - \omega t)}dk_x d\omega \qquad (12)$$

The benefit of this method is twofold: (i) it preserves the application of the fast Fourier transform algorithms for the final inversion, and (ii) the integrand is a nonsingular function that may now be integrated numerically. To perform the numerical integration, the value δ=0.4 rad·μs^{-1} has been chosen for the auxiliary parameter in the following numerical calculations.

3 Result on a homogeneous plate

To demonstrate the theoretical model and solution, the dispersion spectrum has been calculated for a 1 *mm* thick aluminum plate, and it is compared with that obtained by an analytical method for a homogeneous plate [8]. The dispersion curves of various P-SV wave modes are clearly observed in Figs. 1(a) and 1(b) obtained on the homogeneous model (the analytical method) and the proposed inhomogeneous one respectively. As shown in Fig. 1(a), the first order modes S0 and A0 are the basic symmetric and anti-symmetric mode of a plate, and S1, A1, … are the high order modes for a homogeneous plate. The density, and the longitudinal and shear wave velocities for aluminum, are chosen to be ρ=2700 kg/m^3, V_L=6400 m/s and V_T=3110 m/s through this paper. The dispersion spectrum obtained by the presented theoretical model and solution in Fig. 1(b) is identical to that by the analytical method, this shows the capability of an inhomogeneous model in dealing with a homogeneous plate.

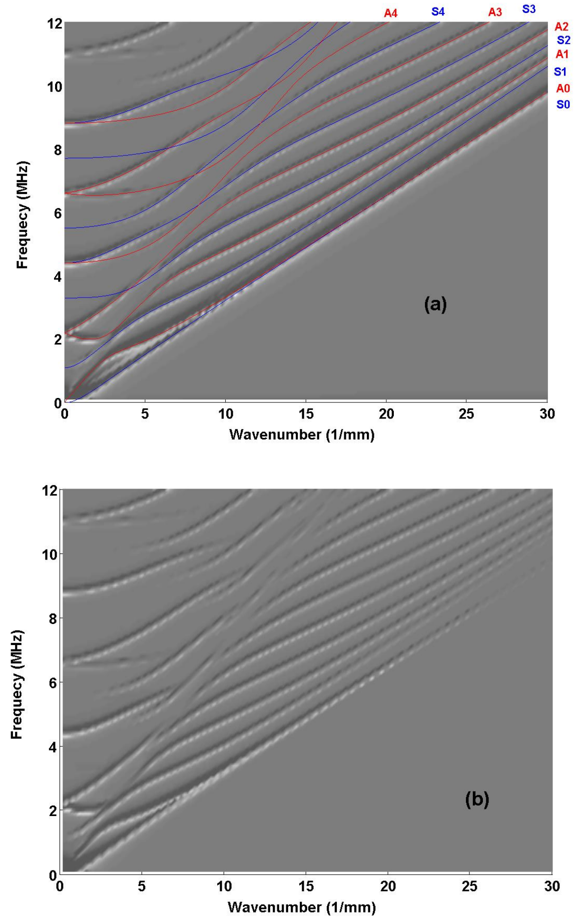

Fig.1 Dispersion spectrum for 1 *mm* aluminum plate by (a) homogeneous and (b) inhomogeneous (functionally graded) model.

To further demonstrate the theoretical model and solution, the transient displacement response has been calculated for a 5 *mm* thick aluminum plate, and then compared with that obtained by the analytical method for a homogeneous plate [7]. Various longitudinal and shear wave modes are clearly observed in Figs. 2(a) and 2(b) observed at the epicenter and non-epicenter position respectively. As shown in Fig. 2(a), the transient waveform obtained by the proposed solution (up) is close to that by the analytical method (down) regarding the arrival times and amplitudes of the direct longitudinal (L) and shear (T) waves, the reflected longitudinal (3L) and shear (3T) waves. The slight difference in the waveform is caused by the limitation of the calculation by the proposed method to a narrow range of frequency and wave number. Overpassing this numerical difficulty is in progress in our lab. Similar result for a non-epicenter observation position is shown in Fig. 2(b). The agreement further emphasizes the capability of the proposed inhomogeneous model in dealing with a homogeneous plate.

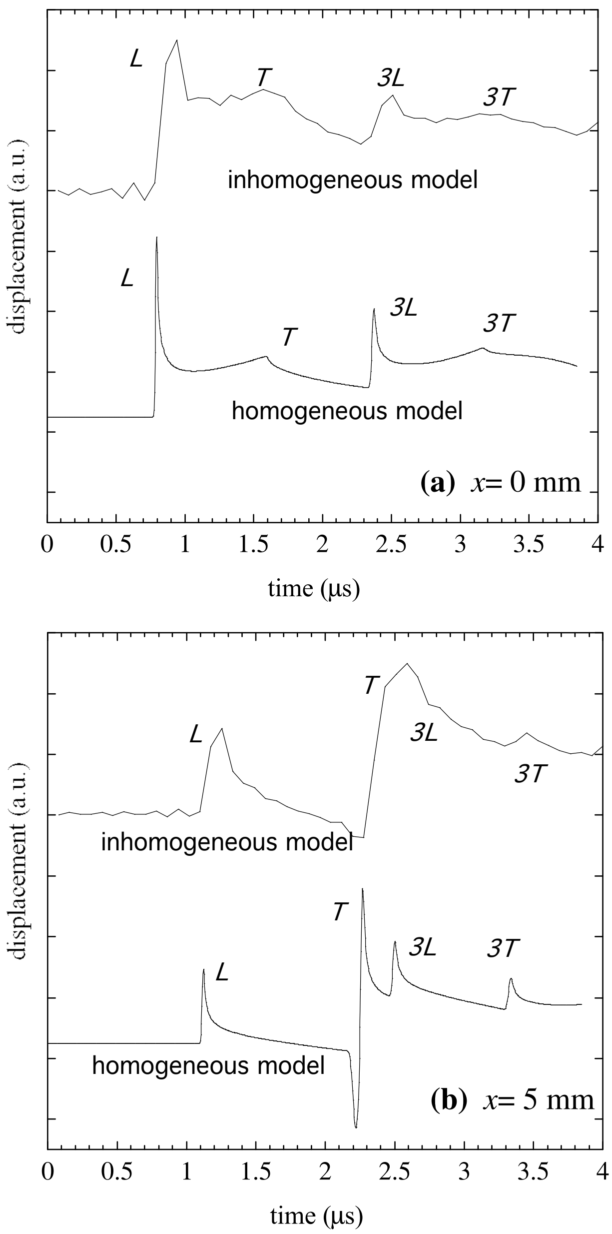

Fig.2 Laser excited transient displacement for a 5 *mm* aluminum plate calculated at (a) *epicenter x=0* and (b) *non-epicenter x=5 mm*.

4 Result on a graded plate

A functionally graded sample of thickness 6.05 *mm* and diameter 32 *mm* is now considered. Its top side is composed of 93% tungsten, the down side is made of 97% titanium, the middle is mixed with both metals in a gradual variation of density. The density of this plate is gradually increasing from the down side to the top side as show in the following table 1.

Thickness(mm)	0–1.00	1.00–1.69	1.69–2.38	2.38–3.07	3.07-3.76	3.76–4.45	4.45-5.05	5.05-6.05
Density(g/cm^3)	4.32	5.56	7.21	9.00	11.02	13.27	15.52	17.85

Table 1 The graded profile for a functionally graded plate

Laser ultrasonic experiment was carried out on this sample, and corresponding theoretical waveforms are obtained for comparison. As shown in Fig. 2(a), 31 experimental waveforms were recorded by scanning the laser line source in an interval of 0.66 *mm*, while fixing the detecting laser at the center of the opposite side. The direct arrival of longitudinal (*L*) and shear (*T*) waves are clearly observable with the arrival time corresponding to the theoretical prediction based on the profile in Table 1 (see the × mark). As shown in Fig. 2(b), 31 theoretical waveforms were calculated to simulate the laser excited acoustic field. Although the main frequency components of theoretical waveforms are relatively low, the arrival of the direct longitudinal (*L*) and shear (*T*) waves are observable, correspondingly to the theoretical prediction (see the × mark). It can be concluded that the proposed theoretical model and solution is able to predict the transient acoustic field observed in experiment. In addition, the arrivals of direct longitudinal and shear waves are delayed in comparison to that predicted by the theory for the scanning position from 8 *mm* to 10 *mm*. It is certain that there are defects in the sample resulting to the relatively low sound velocity. This provides a simple method of non destructive evaluation of the FGMs in a noncontact way.

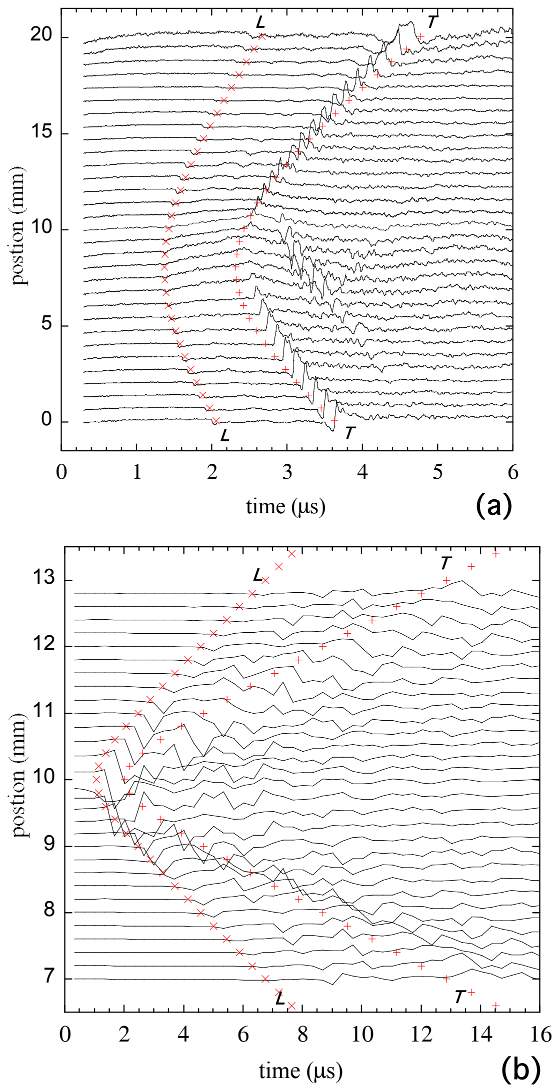

Fig.3 Laser excited transient displacment field : (a) experiment (b) theory for a functionally graded plate.

5 Conclusion

A theoretical model and solution is presented to predict the dispersion and transient displacement excited by laser in a functionally graded plate. The laser line source could be assumed as an ideal transient force, and the dispersion and transient displacement is numerically obtained and compared with that by the analytic method. The agreement on an aluminum plate demonstrates the capability of the proposed inhomogeneous model in dealing with a homogeneous plate. Laser ultrasonic measurement was carried out, and the direct arrivals of longitudinal and shear waves were observed in the experimental displacement field. The agreement between experimental and calculated theoretical waveforms provides a promise for the application to NDE of graded materials.

Acknowledgments

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