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Parametric audible sounds by phase-cancellation excitation of primary waves

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An ultrasound source with a simple configuration is considered as a theoretical model. The source with a circular aperture consists of two coaxially arranged planar projectors: i.e., one is an inner disc projector and the other is an outer ring projector. The active areas of these projectors are the same. The outer diameter of the source is 20 cm. Both the projectors are driven individually at the same frequencies of 38 and 40 kHz but different phase angles. Especially, the study focuses on two extreme cases of the usual in-phase driving and out-of-phase driving. Numerical computation using the Khokhlov - Zabolotskaya - Kuznetsov (KZK) equation demonstrates that when the driving signal is in phase the difference frequency wave of a 2-kHz wave provides quite a narrow beam a parametric acoustic array usually creates. The beam has a similar directivity when the signal is out-of-phase by 180 degrees, although the peak of the sound pressure level decreases by few decibels. Interestingly, the second harmonic pressure level of the difference frequency reduces by ten decibels or more. Needless to say, the pressure amplitudes of the primary waves are suppressed considerably near the beam axis. Experimental verification is done using an airborne ultrasound source with a 19.2-cm circular aperture.

1 Introduction

When two finite-amplitude sound beams of different but neighboring frequencies are propagated in the same direction, a parametric acoustic array is intrinsically formed in the beams. This is a well-known physical fact. Actually, nonlinear interaction of two primary waves provides a spectral component at the difference frequency. Additional components at higher frequencies such as the harmonics and the sum frequency are also generated. However, only the difference-frequency component can travel a long distance because sound absorption is generally increased with frequency, and then the waves at higher frequencies decay their amplitudes greatly compared with the difference frequency. The most remarkable property of the parametric array is its sharp directivity even for the low frequency. Additionally, side-lobes, that usually exist in a directive sound, are considerably suppressed.

The aim of the present report is to control sound fields by changing the phases of primary waves. The study begins with modeling an ultrasound source with a simple configuration. The source consists of two coaxially arranged planar projectors with circular apertures: i.e., one is an inner disc projector and the other is an outer ring projector. Incidentally, similar reports of source modeling have already been published so far. Clark utilized the Fresnel-Kirchhoff diffraction theory to model the acoustic fields produced by a variety of amplitude and/or phase apodizations of large aperture, axially symmetric ultrasound sources[1]. He found that the forms of resulting intensity maxima and their resulting suitability for surgical lesioning of tissues are carefully compared. His interests are entirely-focused on linear approach to the field analysis, although he states in the body that it is of essential necessity to model nonlinear propagation of beams for obtaining greater accuracy at high sound intensities. For a nonlinear acoustic regime, Saito and Kawagishi designed a focusing ultrasound source that consists of two coaxially and confocal transducers for avoiding the concentration of second harmonic generation at the focus and for minimizing the fundamental pressure amplitude there[2]. Unfortunately, their theoretical and experimental examination is confined to the fundamental and its second harmonic fields formed by mono-frequency excitation. In line with their ideas, one of the present authors and his colleagues have reported the possibility of controlling locally Eckart-type acoustic streaming near the focus using a concave ultrasound source with two coaxially arranged transducers[3].

The present paper extends the above-mentioned approach to the case of bi-frequency excitation, in which both the projectors are driven by the same frequencies but different phase angles. Especially, it treats two extreme cases of the usual in-phase driving and out-of-phase driving. To comprehensively explore these situations we resort to the Khokhlov - Zabolotskaya - Kuznetsov (KZK) model equation, predicting theoretically the field characteristics of the difference frequency wave. Besides, the fields of not only the second and third harmonics of the difference frequency but also the primary waves are evaluated. Some numerical examples demonstrate that several salient features are found in the beam patterns of the difference frequency wave by changing the phases. This report also pays attention to the sound pressure levels of the harmonic and primary waves. In the following, to verify the theoretically obtained results experiments are carried out in air using an ultrasound source with a 19.2-cm circular aperture.

2 Theoretical prediction

Figure 1 shows a theoretical model of a sound source. The source consists of two coaxially arranged ultrasonic projectors: i.e., one is an inner disc projector whose radius is a and the other is an outer ring projector whose radius is b. These projectors are placed on a coplanar surface, and are radiating individually two ultrasound beams of different but neighboring frequencies f_1 and f_2 $(f_1 > f_2)$:

$$\left.\begin{array}{l} p_1 = P_1 \sin(\omega_1 t + \theta) \\ p_2 = P_2 \sin(\omega_2 t + \theta) \end{array}\right\} \quad \text{(on the source)}, \qquad (1)$$

where $\omega_1 = 2\pi f_1$ and $\omega_2 = 2\pi f_2$ are the primary angular frequencies, and θ is the initial phase. Furthermore, p_1 and p_2 are the sound pressures of the primary waves on the source, P_1 and P_2 being their amplitudes. For simplicity, we assume that the areas of the two projectors are the same: i.e., $b = \sqrt{2}a$. In this case, the sound power radiated from the ring projector is equal to that radiated from the disc projector.

When a finite-amplitude ultrasound wave propagates in a fluid medium, waveform distortion occurs inevitably during propagation due to the inherent nonlinearity of the medium. For a directional sound beam, the KZK

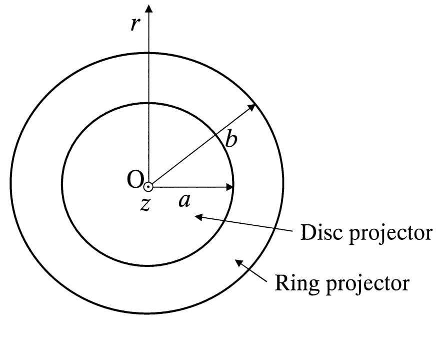

Figure 1: Theoretical model of an ultrasound source.

model equation that combines successfully nonlinearity, dissipation, and diffraction is widely used to predict nonlinear behavior of the beam theoretically. This model equation is described as[5]:

$$\frac{\partial^2 p}{\partial z \partial t'} = \frac{c_0}{2} {\nabla_\perp}^2 p + \frac{\delta}{2c_0^3} \frac{\partial^3 p}{\partial t'^3} + \frac{\beta}{2\rho_0 {c_0}^3} \frac{\partial^2 p^2}{\partial t'^2}, \qquad (2)$$

where p is the sound pressure, c_0 is the sound speed, ρ_0 is the medium density, δ is the sound diffusivity that is related to sound absorption, and β is the nonlinearity coefficient. Moreover, ${\nabla_\perp}^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is a Laplacian that operates in the $x-y$ plane perpendicular to the axis of the beam (z axis), and $t' = t - z/c_0$ is the retarded time.

Let the initial phase θ be different for the two projectors. To be more precise, θ always remains to be zero for the disc projector, while the ring projector can have phase shift θ with respect to the disc. If θ is positive, the primary waves from the ring projector lead those from the disc by θ rad. We now focus on two extreme situations: i.e., $\theta = 0$ and π. The former is the 'in-phase' excitation of the primary waves and is usually used for the formation of a parametric acoustic array. The latter is the 'out-of-phase' excitation we are especially concerned with. Taking into account of the thus stipulated initial conditions, we solve numerically the KZK equation by employing a finite difference method whose scheme has already been established methodologically and technically.

For comparison with the experiment described later, we assign source parameters as follows:

$f_1 = 38$ kHz and $f_2 = 40$ kHz.
$b = 10$ cm, a then becomes 7.07 cm.
Room temperature= 20° C, and relative humidity= 50%.

The room temperature and relative humidity determine the sound absorption coefficient of the air, that is readily predicted as a function of frequency using a relatively simple formula[4].

On-axis propagation curves of the first three harmonics of the difference frequency wave are shown in Fig. 2 with the curves of the primary waves. The source pressure levels of both the primary waves are the same to be 125 dB. As is expected, the pressure levels of the primary waves are noticeably different, being quite dependent on the phase: the levels for the out-of-phase excitation are generally higher than those for the in-phase excitation in the nearfield less than 1 m from the source. Conversely, the pressure amplitude for the out-of-phase is suppressed greatly due to the phase-cancellation effect, being lower than that for the in-phase in the farfield. For example, more than 10-dB suppression is observed at 4 m. Far away from this point, further suppression is realized effectively.

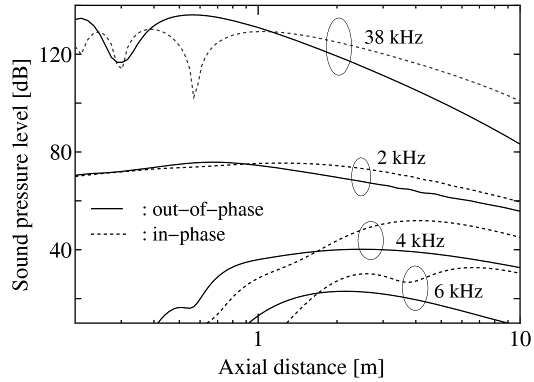

Figure 2: Propagation curves along the beam axis. The sound pressure levels on the source are 125 dB for both the primary waves. Almost the same suppression is realized for the 40-kHz primary wave when $\theta = 180°$.

In contrast with the above-mentioned features of the primary waves, the pressure levels of the difference frequency wave of 2-kHz are almost independent of the initial phases in the nearfield region less than 1 m. In the field greater than 1 m, however, the pressure levels by the out-of-phase excitation are less than those by the in-phase excitation. Typically, the pressure reduction is not great so much, being only a few decibels. Far away from the source, at 10 m, for example, the difference level seems to be decreased slightly. Additional interests are observed in the higher harmonics. The second harmonic wave of 4 kHz and the third harmonic wave of 6 kHz as well are expected to reduce their amplitudes by 20 to 30 decibels in the farfield.

Beam patterns of various frequency components at 4 m from the source are shown in Fig. 3, where the patterns of the third harmonics are not shown because of too low pressure levels. Obviously, the pressure levels of the primary waves are 10-dB or more reduced near the

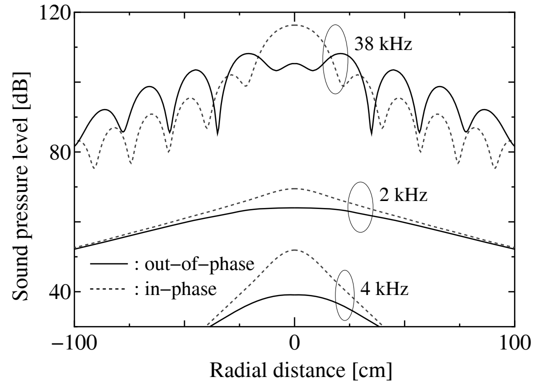

Figure 3: Beam patterns of various frequency components at 4 m from the sound source. Almost the same pattern is obtained for the 40-kHz primary wave when $\theta = 180°$.

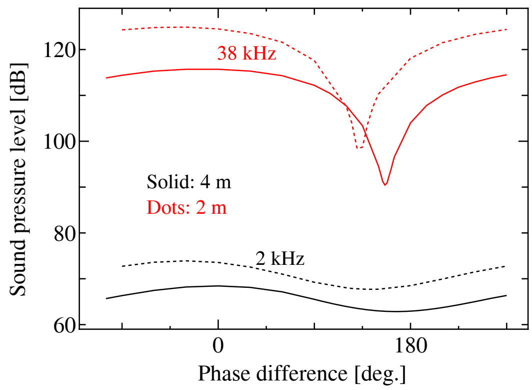

Figure 4: Theoretically predicted phase-dependence of 38-kHz and 2-kHz spectral components at 2 m and 4 m on the axis.

beam axis. Instead, the sidelobe levels increase overall by several decibels. For the difference frequency, the waves have no sidelobes within the computed range of ±100 cm, that is a prominent feature of a parametric array. The pressures around the axis for the out-of-phase excitation are indeed suppressed, being several decibels lower than those for the in-phase excitation. At 50-cm away from the axis, however, the levels are almost the same between both the excitations. Interestingly, a similar tendency is seen in the data of the second harmonic.

Now consider the case where the phase difference θ between the two projectors is changed over $-90°$ to $270°$. Figure 4 shows numerically computed SPLs at 2 m and 4 m on the axis for the 38-kHz primary and the 2-kHz difference frequency waves. It is demonstrated that the difference frequency wave decreases its pressure near the specific phase where the primary wave has the minimum pressure. To the contrary, the former wave attains the maximum pressure at the phase where the latter wave has the peak. Notably, the pressure variation of the difference frequency is less sensitive to the phase in comparison with the primary wave, independent of receiving points. Additionally, the phase that determines the minimum pressure tends to approach 180° when the receiver is away from the source.

3 Experiments and discussion

Experiment was carried out in air using an ultrasound source of 19.2 cm in diameter, as depicted in Fig. 5. The source consists of 271 small piezoelectric ceramic transducers of 10 mm in diameter. Each transducer has about 40-kHz resonant frequency and at least ±5 kHz bandwidth within 10 dB below the maximum sensitivity at 40 kHz. There are ten annular arrays of the transducers. The input terminals of all transducers of the first seven arrays from the origin are connected in parallel to be the disc projector, and all the terminals of the remaining transducers are also connected in parallel to be the ring projector. The number of the transducers is 127 in the disc projector, resulting in 17 pieces less than that in the ring. We need then an appropriate adjustment of source pressure amplitude when driving both the projectors with equal electric power. When the

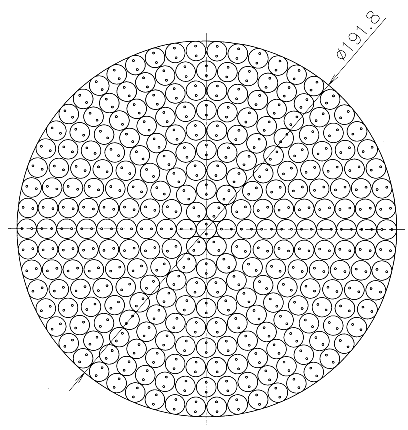

Figure 5: Configuration of an ultrasound source. Small circles denote the allocations of 271 commercially available piezoelectric ceramic transducers with an outer diameter of 10 mm and a resonance frequency of 40 kHz.

source is assumed to be driven by only the ring projector with $p_0 = 125$ dB, the pressure level can be numerically predicted 112 dB at 4 m. Likewise, the pressure of 111 dB should be predicted at the same point by driving only the disc projector. We therefore adjusted the attenuators of two power-amplifiers (see Fig. 6) so as to be able to obtain the respective pressure levels at 4 m. Actual voltages applied to the disc projector and ring projector were 21.8 V_{pp} and 19.1 V_{pp}, respectively. In our experimental conditions, the room temperature was 23° C and the relative humidity 52 %.

The experimental setup is shown in Fig. 6. Two sinusoidal signals of 38 kHz and 40 kHz are electrically mixed and are gated to generate tone-burst waves. The subsequent inverter changes the phase by just 180°. We can select either of the two excitation modes by the switch 'S'. The thus separated signals are individually power-amplified, being fed to the projectors.

Axial pressure curves of the primary waves of 38 kHz and 40 kHz are shown in Fig. 7(a). Symbols are all experimental data, and solid and dotted lines are the theoretical results obtained by solving numerically the KZK equation. On-source pressure levels are expected to be 125 dB, that correspond to $P_1 = P_2 = 50.2$ Pa in eq. (1), for both the primary waves, subject to the source pressure amplitude to be uniformly distributed. As can be seen, the agreement of theory and experiment is entirely excellent. When a pressure receiver or microphone is placed in the field less than 1 m, the sound pressure levels by the out-phase excitation are higher than those by the in-phase excitation. The levels become, however, lower drastically when the receiver is located away from that point. 10 dB or more pressure

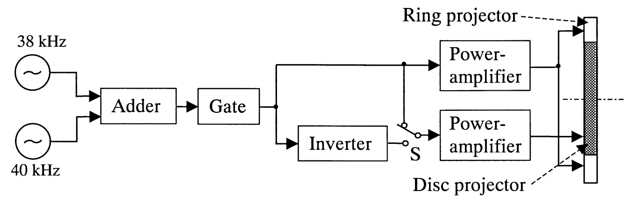

Figure 6: Experimental setup.

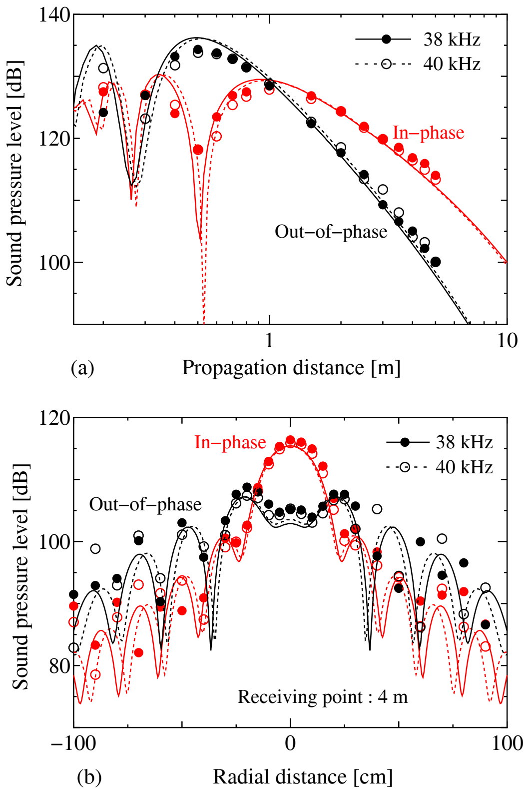

Figure 7: On-axis pressure curves of the primary waves of 38 kHz and 40 kHz (a) and their beam patterns at 4 m from the source (b). Symbols are all experimental data, and solid and dotted lines are the theoretical predictions based on the KZK equation. On-source pressures are predicted to be 125 dB for both the primary waves.

reduction is significantly achieved around 4 m from the source, where beam patterns are measured. The more the receiver is away from the source the more the level difference is enlarged.

The beam patterns of the primary waves are shown in Fig. 7(b). Like the propagation characteristics just mentioned above, the experimental data are in good agreement with the numerical predictions. Unfavorable discrepancies are found off the axis, however. A potential source of the discrepancies is probably due to the nonuniformity of pressure distribution on the source. Actually each piezoelectric ceramic transducer used has some variation in the efficiency of electroacoustic transduction. It should be then noted that the random distribution of such transducers on the source surface results in sound pressure different from the pressure predicted from the uniform distribution. The variance in pressure is directly related to the variance in the transducer's sensitivity. In the region far away from the axis where the pressure level is relatively low, variation effect is more likely to be enhanced[6]. At any rate, the out-of-phase excitation reduces the pressure level by about 10 dB within the range of ±25 cm at the expense of the pressure increase outside this range.

Comparison of experimental and theoretical results is shown in Fig. 8 for the secondary sound pressures, where the third and more higher harmonics are not presented in the figure because their levels are too low to be

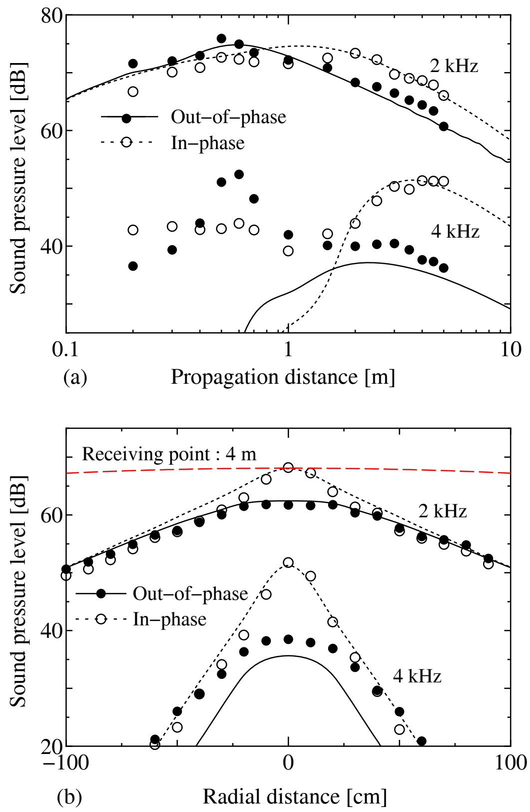

Figure 8: On-axis pressure curves of the difference frequency wave of 2 kHz and its second harmonic of 4 kHz (a) and their beam patterns at 4 m from the source (b). Symbols are all experimental data, and solid and dotted lines are the theoretical predictions based on the KZK equation. Dashed line denotes the hypothetical pattern of the 2-kHz wave is radiated linearly from the source.

comparable with acoustical noise floor of around 17 dB above 1 kHz in our experimental environment. For the 2-kHz components of the difference frequency wave, the experimental data agree well with the theoretical curves. Pressure difference between the two excitations appears markedly around the beam axis: when the source is operated in out-of-phase the pressure decreases its magnitude by about 5 decibels in comparison with the in-phase excitation, keeping almost the same levels in the region away from the beam axis by ±50 cm or more. Incidentally, a dashed line in Fig. 8(b) denotes the hypothetical pattern of the 2-kHz wave that is radiated linearly by the present source. It is evidently confirmed from this curve that the parametrically generated sound waves have quite narrow beams compared with the linearly radiated wave.

The second harmonic waves of the difference frequency (4-kHz components) are generated prominently near the source, in spite of essentially weak generation of the harmonics, as the theory demonstrates. Nonlinear vibration of the corn and/or metal diaphragm of the ceramic transducer by high voltage driving causes possibly the generation of such harmonics. In a similar fashion, high-pressure intense sound tends to induce harmonic distortion in a condenser microphone through the nonlinear vibration of the diaphragm. However, the most prevailing source for the discrepancies may be in the receiver system, that retains about 90 dB dynamic range including a bandpass filter to pick up the secondary wave components and a digital oscilloscope with a built-in FFT analyzer and an averaging function. At $z = 60$ cm, for example, the pressure level of the primary wave is 135 dB from Fig. 7(a). Measured data above 45 dB or 50 dB with allowance(margin) by 5 dB are then reliable in our measurement system. In fact, the spectral components of the second harmonic observed were comparable with signal noise floor in FFT analysis.

4 Conclusions

We have presented in this report numerical and experimental results on parametric array formation for the inphase and out-of phase excitations of the primary waves. It has been revealed that the sound pressure levels of the primary waves and the harmonics of the difference frequency wave are considerably reduced without deteriorating the acoustic properties of the parametric array by changing only the phases of the primary waves by 180°.

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