Hierarchical spline technique application for real time 3D displaying of seafloor using multibeam sonar data

Jerzy Demkowicz

Gdansk University of Technology, ul. Narutowicza 11/12, 80-952 Gdansk, Poland
demjot@eti.pg.gda.pl

Multibeam sonar records have a high resolution raster character. Unfortunately, interpolating and approximating and eventually displaying scattered 3D raster data of high volume leads to some difficulties related to a computer processing power. The paper presents some advantages of using hierarchical splines as applied to real data from multibeam EM 3002 sonar acquired during acoustic survey on Southern Baltic. The proposed approach consists of two stages: firstly, all acquired multibeam sonar raw data are interpolated with high density uniform spline interpolation. The knots and control points of interpolated network are saved for defined resolution level. In the next stage preprocessed high resolution data are combined with low resolution data sets following prior knot decimation process. Such approach allow real time 3D displaying of multibeam sonar data for different zoom levels.

Introduction

The bathymetric data features wide range of vertical resolution. Bathymetric records from multibeam sonar (MBS) posses decemeter or even higher resolution, on the other side ocean bathymetry features 1 km resolution. The applications of MBS may be in many areas obvious redundancy and ambiguity [3], but at the same time can be useful in an approximation approach. The paper presents the approximation approach using a hierarchical spline techniques. The approach allows for a flexible and appropriate resolution for different scales in the process of visualization and bottom imaging.

Spline functions

Spline functions can be expressed as linear combination:

$$s(x)=\sum_{k\in Z}c(k)\varphi^n(x-k) \qquad (1)$$

where $c(k)$ is a control point, and φ^n a base function of n degree. The linear combination is responsible for spline function smoothenes. Spline functions of integer knots can be interpreted as functions of different resolution in the context of multiscale representation [1]. The base funtion equation for $n = 0$ yields $\varphi^0(x/m)$, and is 1 for $x\in[0,m]$ and 0 in other case:

$$\varphi^0(x/m)=\sum_{k=0}^{m-1}\varphi^0(x-k)=\sum_{k\in C}h_m^0(k)\varphi^0(x-k) \qquad (2)$$

where $h_m^0(k)$ represents a filter of Z transform $H_m^0(z)=\sum_{k=0}^{m-1}z^{-k}$ so a discret impuls of length m. The $(n+1)$ convolution of the function reads:

$$H_m^n(z)=\frac{1}{m^n}\left(H_m^0(z)\right)^{n+1}=\frac{1}{m^n}\left(\sum_{m=0}^{m-1}z^{-k}\right)^{n+1} \qquad (3)$$

which represents $n+1$-th convolution of discrete puls and can be implemented as extremely fast algorithms eg. FIR filters. The coeficients of the filter yields and resemble the Pascal triangle.

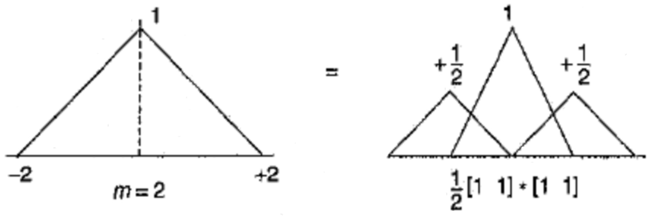

Fig.1. Level of detail process for spline function of degree 1.

This situation is shown in the Fig. 1 for spline function of order 1, as so called spline function piramyd.

Eventually, spline funtion representation for n order is represented as:

$$\varphi^n(x/m)=\sum_{k\in Z}h_m^n(k)\varphi^n(x-k) \qquad (4)$$

and can be interpreter as the spline function hierarchy.

Spline functions edge modification

The hierarchical spline functions of order 1 and 3 are presented in the Fig. 2a and Fig. 2b. The figures show the way of multiresolution approximation construction process. The new set of control points is calculated from control points from the upper scale. The scale change can be introduced as knot removement and a control point reduction. In the first, but important case, base functions are triangles (see Fig. 2a). The figure presents the main idea behind the decimation process as well. The overall process is called a generalization.

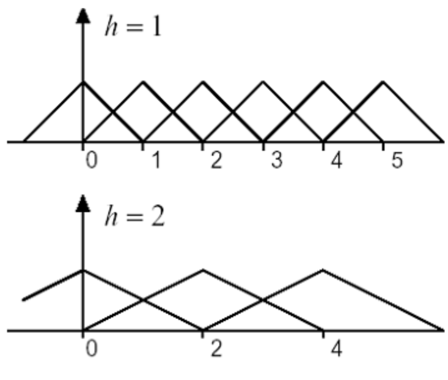

Fig.2a. Knot resolution change in the process of generalization.

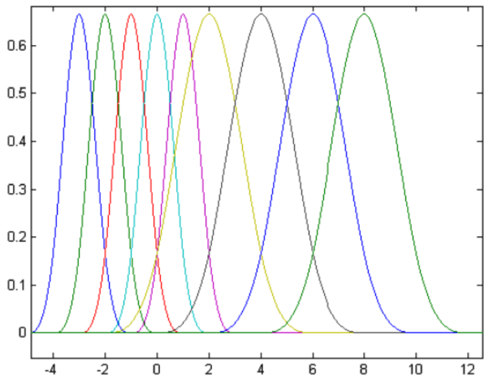

Fig.2b. Base function of order *3* with mixed resolution.

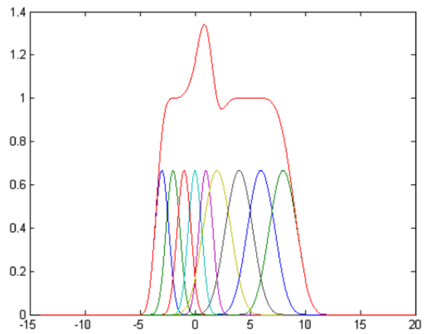

Fig.3a. The oscillation on the edge of different resolution before knot removal.

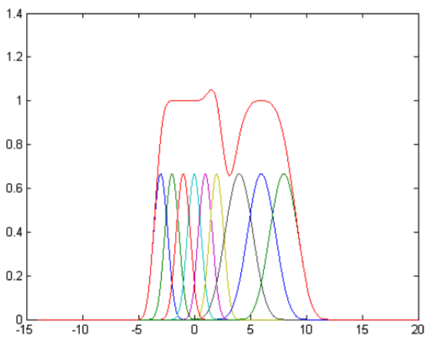

Fig.3b. The oscillation on the edge of different resolution after knot removal.

The control points decimation process means the reducing of the approximated data and the scale changing (see Fig.2a) [3]. The spline function equation (1) yields then:

$$s_h(x) = \sum_{k \in Z^p} c_h(k)\varphi(x/h - k) \qquad (5)$$

where *h* represents scale and φ represents scale function.

The Fig.3. presents a problem which emerge while different domains of different resolution are merged, i.e. oscillation on the stick edges. The Fig.3a presents the situation after knot insertion while Fig.3b, presents the situation after the knot removements. The process of the oscillation compensation consists in de Boor knot insertion. The heuristic algorithm consists of a few steps:

1. Coarse approximation of low resolution data from with spline functions
2. Knot insertion for areas of high resolution
3. Control points modification which corresponds to the high resolution area
4. Repeted knot insertion for areas of high resolution if the area is not covered by the new set of knots, other situation ends the algorithm

Hierarchical spline multiresolution analysis

The hierarchical spline approach to multiresolution representation can be analyzed using the sampling Nyquist theorem, what was introduced in the chapter 2 of the paper. The chapter underlines an important aspect of the spline functions analysis, namely their FIR filter implementation possibility. However, in the context of the paper and the 3D data representation, more convenient and general, seems to be a geometric approach.

Let V^k represents control points space of resolution *k*. There is a transformation which transform one space into another:

$$R^{[k+1]} = SV^{[k]} \qquad (6)$$

where *S* depicts the transformation. One could imagine such representation of different kind of 3D data which are represented in some places by control points of high density as shown in Fig.4. Fig.4 and Fig.5 present low and high control points representation as a regular grid in different colors. If $\vec{O}$ would represent a control points translation in newer space of higher resolution, the translation in the space reads:

$$V^{[k+1]} = R^{[k+1]} \oplus \vec{O}^{[k+1]} \qquad (7)$$

where $\oplus$ operator can be interpreted as the transposition $\vec{O}$. The result of the operation was show in the Fig. 5 as high density yellow grid overlaid the base low resolution control points grid.

The hierarchical spline representation may be applied to different kind of data e.g. from bathymetric data of electronic chart (low resolution) to bathymetric data from multibeam sonar (high density resolution) etc.

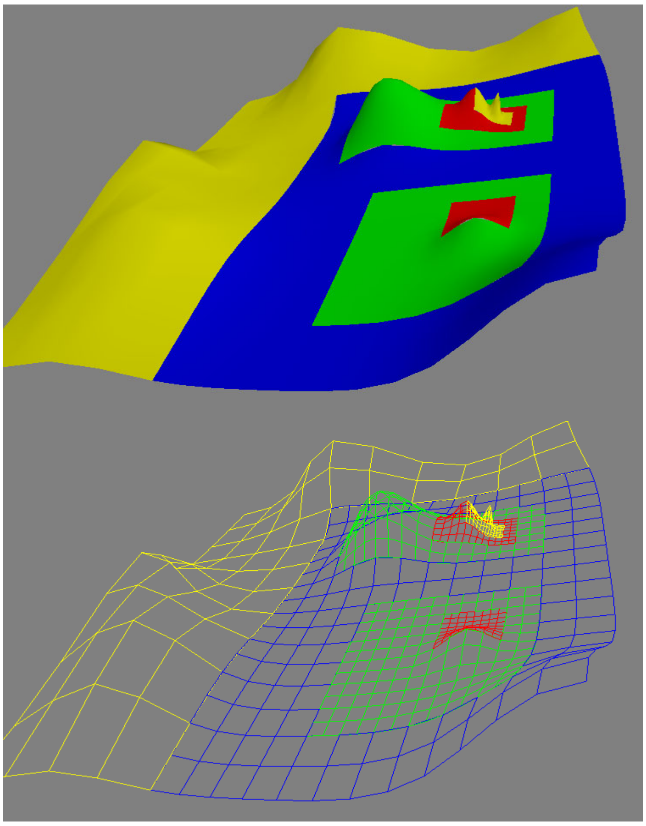

Fig.4. Multiresoution 3D data representation using hierarchical spline function techniques

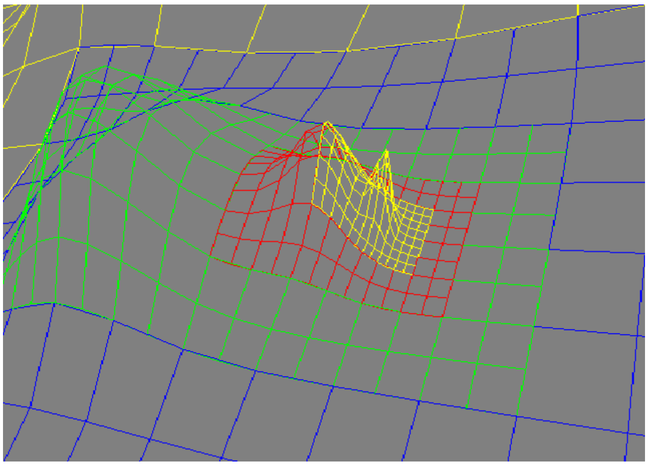

Fig.5. High and low vertical resolution data approximation using hierarchical spline function

The crucial question, which arises at this point refers to the approximation error, as the error could be used by the automatic hierarchical spline grid generation process. The error could be calculated through following mean square error:

$$ER = \sqrt{\frac{\sum_{i=0}^{n}(f(x_i) - s(x_i))^2}{\text{n - m}}} \qquad (7)$$

where $f(x_i)$ represents bathymetric measurement in the point *x* and $s(x_i)$ is the spline approximation in the point *x*. The approximation error will depends on the sampling frequency (so the Nyquist theorem) and as a consequence this is a reason the hierarchical splines are so useful - because the hierarchical splines can approximate the data taking into account their local density and their locality. Therfore the local spline approximation error can be very useful in the context of the vertical spline konts resolution estimation.

Conclusions

The problem of the efficient 3D spatial data representation is still opened. There are two main reasons for that: redundancy and the excessive amount of the data. Multibeam sonar data are typical in that context, as multibeam sonar records are an example of a high resolution quasi-raster spatial data. Interpolating and approximating and eventually displaying scattered 3D raster data of high volume lead to some difficulties related to computer processing power. The proposed approach consists of two stages: firstly, all acquired high resolution multibeam sonar raw bathymetric data are interpolated with high density uniform spline interpolation and then knots and control points of interpolated network are saved for defined resolution level and combined with low resolution data sets [3]. Some redundancy and ambiguity of the measurements is not a drawback in the context of the spline approximation, but it can be treated rather as an advantage, and in fact are indispensable [3]. This flexible approach allows for the spline hierarchical technique usage only in the areas where it is required, namely areas of high vertical resolution measurements.

At this stage of the investigation, authors have implemented algorithms for hierarchical spline representation of 3D spatial data of different resolution. The next stage will include the process automation as applied to the high volume data. This stage will use local approximation error, as a first step to the hierarchical spline knot local resolution determination.

References

F.W. Wiley at al., “Hierarchical Spline Approximation”, *CiteSeer.IST*, 2003.

QINSy Brochure, “SA Total Hydrographic”. *QINSy*, *www.qps.nl*.

J. Demkowicz, M. Moszyński, A. Stepnowski, “Application of splines and wavelets along with TIN decimation to 3D imaging of seafloor from multibeam sonar data”. *Proceedings of the Sixth European Conference on Underwater Acoustics.* ECUA 2002. Gdańsk, 24-27 June 2002.

N.K. Govil, ”Frontiers in Interpolation and Approximation”, *Chapman&Hall/CRC*, 2006

M. Unser, “Splines. A Perfect Fit for Signal and Image Processing”, *IEEE Signal Processing Magazin*, Nov. 1999