

A parametric study of interactions between acoustic signals reflected by the seafloor

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Abstract

This paper deals with the study of broadband acoustic signals reflected by the seafloor and recorded on a vertical array in shallow water areas. Only the bottom reflected paths are considered here and we study the interactions of the signals recorded on the hydrophones for different source-array ranges. Recently (see e.g. Guillon and Holland, JASA, 122, p 2974), two different aspects of the "coherence" of these signals were examined (the maximum of the cross-correlation coefficients and the phase of the cross-spectra) and it was shown that these parameters are sensitive to the geoacoustic nature of the seafloor. These results show that the interactions between signals can be used for geoacoustic inversion or for transmission loss predictions. In that regard, a detailed parametric study is needed to investigate the sensitivity of the interactions measurement to various parameters (range, water-depth, geoacoustic structure,...). This study is done by computing the signals for different geoacoustic structures. We have computed and represented the coherence versus the first grazing angle. We also propose an analytical approximation of the coherence. The results obtained show that the coherence exhibits features relative to the auto-correlation function of the source and to the ratio of penetrating energy. These results help us understanding the results obtained on experimental data and give a guide for future work on geoacoustic inversion with the coherence.

1 Introduction

The context of this study is the use of low frequency signals in shallow water areas to infer the geoacoustic properties of the seafloor. The experimental data, and therefore the numeric simulations, are obtained with the following procedure: a broadband signal (100 - 6000 Hz) is emitted by a source located just below the sea surface and recorded on a vertical array moored on the seafloor. The source is towed by a ship to change the source-receiver range and consequently the grazing angle χ (fig. 1). The original experiment was conducted by NURC in June 1997 in Mediteranean Sea near Elba Island and the data were used by C. Holland to develop a new geoacoustic inversion technique [7]. Data from two different sites were used: site 2 has a multilayered geoacoustic structure (a silty-clay fabric with intercalating sandy sediments) with an acoustic basement at 150 m below the seafloor, whereas site 3 has a more homogenious structure with 15 m of relatively coarse-grained sediments overlaying a magmatic basement.

In recent studies [6], these acoustic data were examined



Figure 1: Geometry of the problem

with a new perspective: analysis of the interactions of the signals recorded by the hydrophones on the whole array. This approach is used in sea surface analysis [5], in ambient noise imaging [8], but rarely in seafloor characterization [2]. The parameter analysed is the "coherence" along the array. Even if, strictly speaking, the term of coherence should be reserved for the frequency domain [1], we use it for studies in the time domain [2, 5]. The coherence of the signal $p_i(t)$ recorded at hydrophone *i* relative to the first (bottom) hydrophone is defined as the maximum value of the normalized cross-correlation coefficient:

$$\gamma_{1i} = \max\left[\frac{C_{1i}(\tau)}{\sqrt{C_{11}(0)C_{ii}(0)}}\right]$$
, (1)

with $C_{ab}(\tau) = p_a(t)p_b^*(t+\tau).$

In order to interpret the experimental data, a numerical model of the coherence has been implemented. The reflected field for a point source is first computed at each hydrophone for each frequency in the band of interest[3]. This computation is based on a numerical evaluation of the Sommerfeld integral:

$$p(r, z, f) = ik \int_0^{\pi/2 - i\infty} J_0(kr \cos \chi) R(\chi) e^{-k(z+H) \sin \chi} \times \cos \chi d\chi , \quad (2)$$

where χ is the grazing angle, H is the height of the source, z the height of the receiver above the seafloor, and $R(\chi)$ is the plane wave reflection coefficient of the seafloor which can be calculated for a wide variety of structures [4]. Then, the temporal signals are obtained with an inverse Fourier transform :

$$p_i(t) = \text{IFFT}\left[p(r, z, f) \times S(f)\right], \qquad (3)$$

where S(f) is the source spectrum. The coherence versus the separation of the hydrophones is then computed for the entire array using eq. (1).

The comparison between model and data made for site 2 and site 3 (fig.2) leads to two main comments [6]: 1) despite some differences, there is a general agreement between model and data, in particular, the global value of coherence, its shape, and its evolution with range are captured reasonably well by the model; and 2) the coherence has a strong dependence on the geoacoustic properties of the seafloor. This second point shows an opportunity to use this quantity for geoacoustic inversion. But, to do so, it is necessary to know the sensitivity of the coherence to the various parameters of the problem. This is the subject of the present work.

The paper is organized as follows: in section 2, the parametric study and the χ representation are presented. The latter allows us to reduce the number of parameters to be studied. In section 3, an analytical approximation of the coherence is derived and studied. The obtained results are discussed and the original results obtained on



Figure 2: Numerical and experimental coherence (from [6]) as a function of separation from bottom hydrophone for site 2 (a) and site 3 (b). Solid lines: experimental data, dashed lines: numerical coherence. red lines: long-range (grazing angle $\sim 12^{\circ}$), blue lines : short range (grazing angle $\sim 65^{\circ}$).

real data (fig. 2) are examined with the interpretations provided by the analytical approximation.

2 Synthetic model and χ representation

2.1 Synthetic model and parameters variations

To study the sensitivity of coherence to the parameters, synthetic models with multiple and simple configurations are used in this paper. The different parameters that can be varied are the range r (50 to 800 m), the total thickness of the sediment covering the basement (2, 4, 8, 16 or 32 m) and the number of layers in this stratification (1, 2, 3 or 4) (fig. 1). The variation in range is choosen to be similar to the experimental variations. This short distance measurement is required by the fact that the inversion procedure is based on the first bottom reflected path [7] and then, we must be able to separate this path from others. Consequently, this is a local inversion procedure around the moored array and the environment is assumed to be range-independent.

The water depth is fixed at 150 m. The vertical array is composed of 15 hydrophones equaly spaced from 5 m to 75 m above the seafloor. The impedance contrast at each interface is constant for a given number of layers. Then the layer impedances grow linearly from water impedance to basement impedance that is in all cases equal to 3750.10^3 Ra.

For each possible configuration, there is a simulation with an isocelerity model (c = 1500 m/s everywhere) and an isodensity one ($\rho = 1000 \text{ kg/m}^3$ everywhere) with the same acoustic impedances in both cases and without absorption. For example, for the isocelerity model of a 2 layers stratification, densities are $\rho_1 = 1500 \text{ kg/m}^3$, $\rho_2 = 2000 \text{ kg/m}^3 \rho_b = 2500 \text{ kg/m}^3$ respectively for the first layer, the second and the basement. The isocelerity model allows us to study the case of a reflexion coefficient without refraction or total reflexion phenomena. This case can be usefull for performing an equivalent ray method calculus. The number of free parameters is then 3 + 2 * N: 3 for height of the hydrophone, the height of the source, the range and 2 for

celerity and density, N being the number of layers. So, even for a very simplified synthetic model, the number of parameters to be studied (and possibly inverted) is high.

2.2 A χ representation

In [6] the representation of coherence is done as a function of the separation from bottom hydrophone which is then the reference for each range simulation or measurement. This representation has two drawbacks: the results depend on two parameters (the range r and the separation along the array) and two results at two different ranges are obtained with two different grazing angles χ which makes the interpretation more complicated since the reflection coefficient is a function of the grazing angle.

To overcome this two drawbacks, we propose to compute and to represent the coherence as a function of the grazing angle on the first interface χ (fig. 1), the reference hydrophone (eq. 1) being the one with the highest grazing angle. With this approach, range, height of the source and the receiver are three parameters transformed in one but there are three problems.

First, the experimental results from previous studies and presented in figure 2 were not obtained with this representation and, consequently, their interpretation leaded through this approach should be done carefully.

Second, for a given grazing angle χ , the coherence is obtained through the cross-correlation coefficient between two hydrophones that are located at two different distances. So, the signal emitted by the source should be perfectly reproducible.

And third, there is an infinity of pairs of range r and



Figure 3: Geometry of the problem for a fixed χ

height z + H that give a fixed grazing angle χ (fig. 3). For two different ranges that give the same angle χ on the first interface, the grazing angles on the buried interfaces will be different and then the coherence could be different whereas the grazing angle is the same. This is the most crucial problem from the three mentionned. To evaluate it, we have computed the coherence with a fixed grazing angle at 4 different ranges and a higher grazing angle hydrophone as a reference (fig. 3) for a one-layer stratification.

The results for the isocelerity and isodensity seafloors are presented in figure 4. The maximum difference is



Figure 4: Simulation of coherence with the same grazing angle at four different distances r and for a one layer media. blue lines: isocelerity model, red lines: isodensity model

about 0.1 for a grazing angle of 45°. This result illustrates that for both low and high (close to 90°)grazing angles and for a given χ , the coherence is almost insensitive to the range. Consequently, for numerical studies presented here, this representation is validated and allows us to make parametric studies with less free parameters. For studies based on experimental data, this χ representation should be made carefully. The first point to check is to know how the source is reproductible. Then, the validity of the representation depends on the complexity of the seafloor. For a particular configuration, computations such as ones presented in figure 4 must be performed to check if the coherence is insensitive to range for a given grazing angle χ .

3 Approximation of the coherence

3.1 Analytical expression

In order to interpret the obtained results and to have a better understanding of the basic phenomena, we propose a simplified analytical expression for the coherence. To obtain it, we have accepted three hypothesis.

First, we assume that the reflected signals can be accurately described as a sum of the local reflections from each interface, i.e. that is multiple reflections are neglected (fig. 5(b)). Second, we assume that the maximum of the coherence between two signals is obtained when the echoes with the highest amplitudes are in phase (fig. 5(a)). Then, echoes from other interfaces with a lower amplitude become out of phase as a function of the grazing angle. And third, we neglect the refraction effects. So, we can have a simplified analytical expression for the time delays between two echoes. With these three hypothesis, we can write the coherence as a sum of auto-correlation functions C_{ss} of emitted signal, multiplied by an amplitude factor dependent on the geometric dispersion and reflection coefficient for each echo:

$$\gamma_{1i} \approx \frac{\sum_{n=1}^{N+1} A_{n1} A_{ni} \times C_{ss}(\Delta t(\chi_n^1, \chi_n^i))}{\sqrt{\sum_{n=1}^{N+1} A_{n1}^2 Css(0)} \times \sum_{n=1}^{N+1} A_{ni}^2(\chi_n^i) Css(0)},$$
(4)



Figure 5: Illustration of the approximation. (a): signals aligned with the maximum amplitude pulse. (b): approximation of the ray travel without refraction

with,

$$A_{ni} = \frac{R_n(\chi_n^i)}{D_{ni}} \times \prod_{p=1}^{n-1} \left(1 - R_p^2(\chi_n^i) \right)$$
(5)

where R_n is the local reflexion coefficient of the interface n (function of the grazing angle χ), D_{ni} is the ray travel length, and N is the total number of layers.

The value of the auto-correlation C_{ss} is taken at zero for the maximum amplitude echo, and at $\Delta t = T_{n1} - T_{ni}$ for the others. T_{ni} is the time delay between the maximum amplitude echo and the considered one with index n corresponding to the reflection over the n^{th} interface and i being the index of the signal. The coherence is computed with signal 1 as a reference which is the signal recorded at the maximum grazing angle. The T_{ni} are calculated under the hypothesis of straigth rays:

$$T_{ni} \approx \frac{H + z_i}{c_w \sin(\chi_n^1)} + \sum_{j=1}^{n-1} \frac{2h_j}{c_j \sin(\chi_n^i)} - \frac{H + z_i}{c_w \sin(\chi_n^i)} \quad (6)$$

where h_j is the j^{st} layer thickness, z_i is the height of the hydrophone i, c_w is the water sound speed and c_j is the j^{st} layer celerity. One can see that $T_{1i} = 0$.

3.2 Analytical results

Figure 6 and 7 compare results for the coherence obtained using the simulation based on the Sommerfield's integral and owr approximation. On figure 6, simulations are without refraction in the layered media (isocelerity model) and there is a general agreement between simulation and approximation. The general form of the coherence is, for high grazing angles (above 60°), a lobe that has the general form of the auto-correlation of the emitted signal and for low grazing angles, a constant. One can see that the width of the lobe is sensitive to the total thickness stratification and the constant *Cte* is the ratio of energy of the highest echo over the sum of energies from all echoes:

$$Cte = \frac{A_{11}A_{1i}}{\sum_{n=1}^{N+1} A_{n1}A_{ni}} \,. \tag{7}$$

Peaks on the simulation occur when a reflection from an interface (viewed from the reference signal) intersects a reflection over an other interface (viewed from another signal with lower grazing angle). This phenomenon is not taken into account in the approximation but it is possible.

When refraction is taken into account through the isodensity model, the comparison between simulations and approximations shows significant differences (fig. 7).

These differences are mainly due to the frequency dependence of the reflection coefficient below the critical angle and the refraction phenomena that are not taken into account by the approximation. Only the lobe near the reference grazing angle is well described.

One bad consequence of these results is that two different seafloors can lead to the same coherence. Indeed, for a given number of layers with a given thickness, the constant (eq. 7) can be obtained with many different celerities and densities. This problem may lead to illposed inverse problem and should be studied in details in the future.

3.3 Interpretation of experimental results

As mentionned in section 2.2., the experimental results presented in figure 2 were not obtained with the χ representation. Despite this difference, some features of these experimental results can be interpretated with the above comments:

- 1. for the short range measurements, the lobe described above is thin for the site 2 corresponding to a large stratification without knowledge of layer repartition, and large for the site 3 corresponding to a thin stratification.
- 2. for the long range measurements, the form of the coherence is due to the total reflexion phenomenon. At the site 3, coherence is high because the grazing angle is sub-critical for all hydrophones, but for the site 2 only the bottom reference hydrophone is under the critical angle that might explain the fast decrease of coherence.

Simulations with refraction in figure 7 show that peaks occuring when a reflection from an interface interferes with another one, are difficult to see. This simulation is closer to the reality than without refraction. In that way, we can consider go back to coherence representation with one reference hydrophone for each range measurement and then, interpret the evolution of the lobe width or the value of the constant if visible.

4 Conclusions

In this paper, the coherence of signals reflected by the seafloor and recorded on a vertical array has been analysed. The coherence of two kind of seafloors (isocelerity or isodensity) is computed using a numerical model, represented versus grazing angle, and analysed. It appears that the coherence as a function of grazing angles can be described as a constant depending on the energy ratio and on the sum of auto-correlation function of the source. This constant and this lobe width are functions of the thickness and the stratification of the sediment layer. The experimental results obtained previously [6] are correctly explained with this interpretation.

Before turning to the inverse problem (i.e. recovering the geoacoustic properties from coherence measurements), some additional work should be done. The maximum amplitude echo can change from an interface to another as the grazing angle changes that can lead to jumps of the coherence curve. This phenomenon is out of the scope of this paper and should be investigated in the future. The peaks of the coherence curve can also give information on the number of layers.

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Figure 6: Numerical and approximated coherence function of the grazing angle for various configurations of the isocelerity stratification (without refraction). Blue lines: simulation, red lines: approximation. Number of layers is changing with lines and thickness is changing with rows.



Figure 7: Numerical and approximated coherence function of the grazing angle for various configurations and an isodensity stratification (with refraction). Blue lines: simulation, red lines: approximation, green line: critical angle. Number of layers is changing with lines and thickness is changing with rows.