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Analysis of the effects of the oscillations of a rigid sphere inside a cylindrical cavity containing a standing acoustic wave

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Under certain driving conditions of a single-axis acoustic levitation device, a suspended sample leaves its stability state and starts to oscillate-vertically around the initial equilibrium position. A published theory on such instabilities predicts the occurrence of time delays between the response of the cavity of the device and the motion of the sample inside it. In this paper, an experimental investigation on similar time delay effects is described. A solid sphere was moved in a controlled way inside a closed cylindrical cavity by means of a rod connecting the object to the outside of the system. A standing wave was generated inside the cavity by using a second speaker. In this way, oscillations of the sphere were produced and the response of the sound field to such movement was studied. The effect of the frequency of the oscillations of the sphere on the time delay between the sound pressure and the movement of that object is reported. In addition, the relations between the obtained results and the published theory on oscillational instabilities is discussed.

1 Introduction

Any acoustic wave that collides with an object exerts over it an acoustic force; such force may be increased in magnitude by the usage of standing (instead of travelling) waves. Acoustic forces are useful to sustain the position, displacement, and inert state of a small object in the medium where it is located. It is possible to suspend an object against the gravity pull; this phenomenon gets the name of acoustic levitation.

The research on acoustic levitation began with experiments performed in gravity-free environments where small acoustic forces are enough to control the position of samples. Nowadays, acoustic levitation is a technique of container-less processing of samples and substances such as those that easily react or can be contaminated by contact; it also can be used for material properties measurement such as fluid viscosity. However, some problems have to be studied and solved before acoustic levitation may become a mainstream technique.

This paper is focused on single-axis acoustic levitation devices whose medium is enclosed in cylindrical cavities and whose standing waves are driven along the cylinder axis. Under certain driving conditions, samples suspended statically at the equilibrium points inside the standing wave start to oscillate along the cylinder axis and around the equilibrium position. Current theory on such instabilities [1] is based upon the existence of a time delay between the motion of the sample and the response of the cavity, such delay is predicted to be linear to the sample velocity.

The main objective of our research was to experimentally study and measure such delay. Although conditions can be attained for the instabilities to appear in the acoustic levitation device, our approach simplifies the problem to gain control on the system variables; this proved fundamental for the research outcome.

An experiment was designed with the purpose of controlling the relevant system variables, particularly the frequency and amplitud of the sample oscillations, and the frequency and amplitude of the sound field excited in the cavity. This experiment consisted on a spherical sample that was put to oscillate in the same fashion as the instabilities occur. Thus we were able to study the system response to such movements and measure the time delays, which are reported on Sec. 6 and 7.

2 Theoretical background

Inside a cavity, acoustic levitation is attained by exciting standing waves whose wavelength is bigger than the sample dimensions. To simplify the system, we worked only with longitudinal waves along the cylinder axis. Transverse resonant modes were not used since they introduced forces on the sample that had to be accounted for and, therefore, increased the complexity of the problem.

Inside the standing wave in a gravity-free environment, the sample tends to levitate at the position of a sound pressure node, which is the equilibrium position. If the sample happens to be nearer to or at the pressure antinode, it is expelled and forced to go to the equilibrium position.

In gravity environments, due to the weight of the samples, there is a shift of the equilibrium position. In addition, the amplitude of the standing wave has to be maximized for the resultant acoustic force to be able to compensate for the sample weight. Thus, resonance frequencies have to be used since these produce the highest sound pressure amplitud.

However, the presence of a sample in the standing wave modifies the acoustic pressure field inside the cavity. This produces a shift, $\Delta\omega/\omega_0$, in the resonance frequency of the cavity that is a function of the sample position. The shift is predicted by the following equation for spherical samples

$$\begin{aligned}\frac{\Delta\omega}{\omega_0} \quad = \quad & \frac{V_s}{V_c}\left\{\left[-\frac{1}{4}+\frac{5}{4}\cos(2kz)\right]\right.\\ & \left.-\left[\frac{7}{360}+\frac{169}{360}\cos(2kz)\right](kR)^2\right\}. \quad (1)\end{aligned}$$

This equation has been derived to the second order in kR by means of the Boltzmann-Ehrenfest principle of adiabatic invariance by Santillan *et al.* [2], where $\Delta\omega = \omega_f - \omega_0$, ω_f is the angular resonance frequency of the cavity with the sample, ω_0 the resonance frequency of the empty cavity, V_s the sample volume, V_c the cavity volume, z the position of the sample along the cylinder axis, R the sample radius and k the wavenumber. The behaviour of this shift is shown for an arbitrary system at the mode $n = 2$ in Fig. 1, where $\Delta\omega/\omega_0$ is plotted against the normalized position (z/L) of the sample inside the cavity. It is clear that the resonance frequency increases if the sample is near the pressure anti-nodes ($z/L = 0$, 0.5 and 1), from where it is expelled. The resonance frequency decreases if the sample is near the

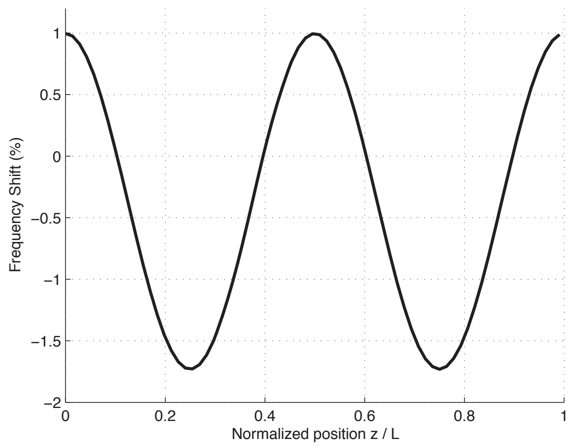

Figure 1: Resonance frequency shift of a cylindrical cavity as a function of the sample normalized position (z/L) along the axis of the cavity.

pressure nodes ($z/L = 0.25$ and 0.75), towards where it is forced to displace.

When the resonance frequency changes, the whole frequency response curve of the cavity shifts. Let consider an acoustic levitator device a at fixed temperature. If we excite the driver that generates the standing wave in the cavity with a voltage at a fixed frequency and a constant amplitud, the perturbation of the position of the levitated sample will produce a shift of the frequency resonance and thus both the amplitude of the standing wave and the acoustic force will change.This could easily render the system unstable for there may occur an undesired change in the sample position.Further experiments have shown that instabilities occur when the cavity is driven at a frequency above the resonance of the levitating mode of the empty cavity.

Since the system is causal, there is a time gap between the sample perturbation and the response of the acoustic field to such perturbation. Current theory on the instabilities by Rudnick *et al.* [1] is based upon the time delays to predict a linear sample velocity-dependent term in the acoustic force that acts on the sample. In the following sections we describe the linear dependence of the system response to the sample velocity that we experimentally produced and measured.

3 Experimental setup

The central idea of the experiment was to produce a change on the system and measure the time it takes for the system to react to that change. The change consisted on the movement of the sample, and the reaction was the variation on the amplitude of the standing wave due to its relationship with the sample position inside the cavity (as explained on Sec. 2).

The purpose of the planned experimental setup was to achieve control over the main system variables, particularly the amplitude, form, and frequency of the sample oscillations. With such control, we put the sample to oscillate along the cylinder axis in a sinusoidal way with controlled amplitude and frequency. This oscillation was in a very similar fashion as those oscillational instabilities that appear on acoustically levitated samples. Hence we could study the response of the cavity to such movements. It is important to remark that both phenomena, the controlled oscillations and the instabilities, are not the same though.

The experimental setup is shown in Fig. 2. A closed cylindrical cavity with plane reflecting walls perpendicular to the cylinder axis was used. Following the experimental setup of previously reported experiments [3], the cylinder had a diameter $\phi = 5.08$ cm and a length $L = 7.06$ cm. The resonance frequency of the mode $n = 2$ of the empty cavity was measured to be 4883.5 Hz at 23.0° C. Since the resonance frequency is a linear function of the temperature, we measured the shift of the resonance frequency of the system to be 8.53 Hz per degree Celsius. The cavity quality factor for the mode $n = 2$ was also measured to be $Q = 243.75$.

A driver from a horn loudspeaker was used to excite the standing wave inside the cavity. However, such driver introduced a problem for it had a conical output and the volume from its inner diaphragm to its outside area was important when compared to the cavity volume. If the driver were attached directly to the cavity through the reflecting wall, its volume would add to that of the cavity. To avoid this situation a wave guide that went from the aperture of the driver to the cavity was used. This wave guide had both a conical shape that functioned as an impedance coupler and an aperture on the center of the reflecting wall of just 0.30 cm that worked as a point source that excited the standing wave.

Since the aperture of the wave guide behaves not as

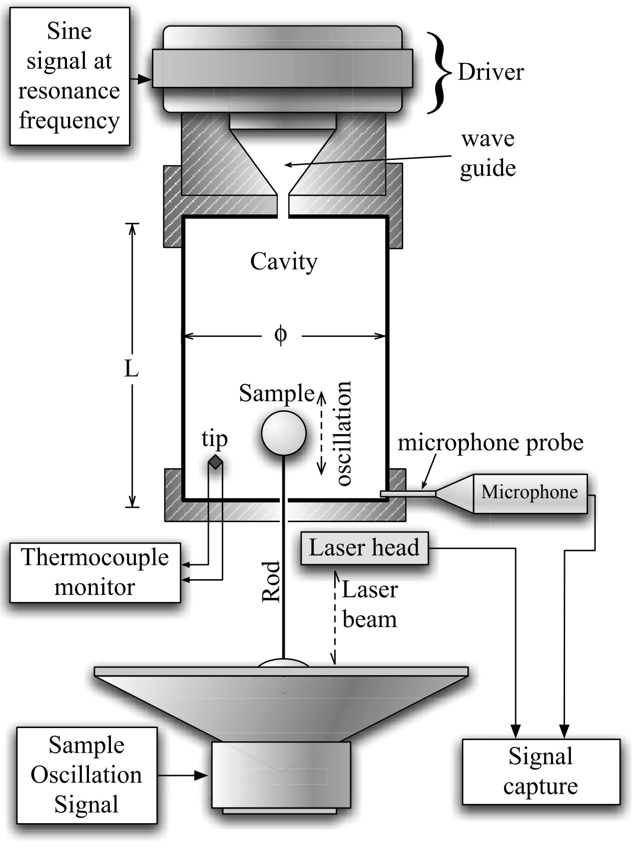

Figure 2: Experimental setup

a planar but as a point source, the normal axial mode $n = 1$ does not behave well very near the aperture. For this reason the second plane-wave normal mode was used. We located the sample in the area next to the reflecting wall opposite to that with the wave guide. In this region the standing wave behaves well enough for our requirements. Since we obey the condition $kR \ll 1$, where R is the sample radius, normal modes higher than $n = 2$ require a smaller sample. This would worsen the measurement resolution since the amplitude of the cavity response is a function of the quotient V_s/V_c. This is predicted by every equation for the resonance frequency shift for a cylindrical cavity [1-6]. The effects on the system are more noticeable with bigger samples.

A rigid glass sphere was used as the sample. With a diameter $\phi = 1.73$ cm, it is the biggest available sample that did not trespass the sample size limit, which is experimentally explored and reported in Ref. [3]. The sample was positioned inside the cavity as seen in Sec. 5, it was attached to the tip of a thin and rigid rod that went by a hole through the center of the cavity reflecting wall along the axis of the cylinder. The opposite end of the rod was attached to a low-frequency (sub-woofer) loudspeaker, next to the dust cap, which is the most rigid point of the diaphragm, to prevent distortion on the oscillations.

An amplified sine signal with controlled amplitude and frequency was feed into the loudspeaker. The diaphragm of the loudspeaker, the rod and the sample inside the cavity oscillated accordingly along the cylinder axis.

The laser head of a Laser Doppler Interferometer (LDI) was pointed into a reflecting surface on the loudspeaker diaphragm; in this way we obtained a signal proportional to the position of the sample inside the cavity. With this signal, it was possible to prove the sine-like behaviour of the sample oscillation. In the lowest frequency tested (0.5 Hz), the THD was less than 0.25 % and decreased at higher frequencies.

The experiments were performed at room temperature, which was not fixed; therefore the frequency of the standing wave excited by the driver had to be adjusted to that of the resonance frequency of the empty cavity at the current temperature. The temperature was measured with a thermocouple whose tip was located inside the cylindrical cavity. The frequency-adjusted sine signal was feed into the driver and the standing wave was produced.

A probe microphone was used to measure the standing wave inside the cavity. The probe microphone went through the cavity walls and then located in the nearest attainable position to the pressure maximum of the standing wave, which is the reflecting wall.

4 Processing of the measurement signals

Averaging of several measurement cycles were performed to decrease the noise levels on both signals, of which one example is shown in Fig. 3. The signal from the LDI indicated the instantaneous position of the sample around the equilibrium point (upper graphic). The signal from the microphone is the standing wave that behaves as a carrier wave for the response of the system, which is the envelope (lower graphic). Thus, we located the time position of the apex of both, the sample position and envelope signals, and then we calculated the time difference between those. The resulting time delay is then converted into a phase delay relative to the period τ of the sample oscillation.

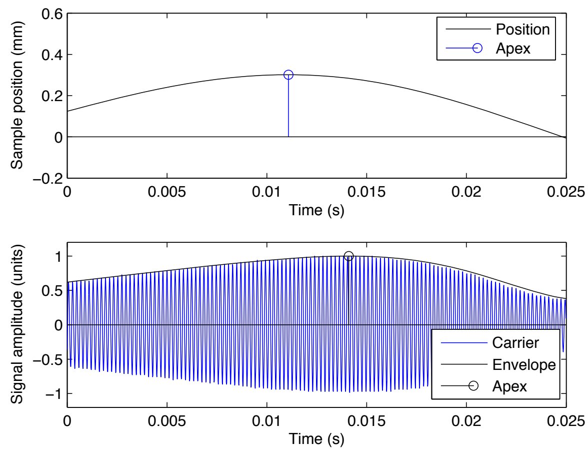

Figure 3: Example of the signal analysis. The time difference between both apex is measured.

In Fig. 3 the time delay between the instant in which the sample reaches the highest position of its path inside the cavity (the apex of the upper graphic) and the instant in which the cavity reacts to such event (the apex of the lower graphic) is shown. A series of measurements performed under several different oscillation amplitudes and frequencies show the clear behavior of this reaction delay, as we discuss in the next sections.

5 Measurement range inside the standing wave

During instabilities the sample oscillates around the equilibrium position. A first approach was to measure the response of the cavity with the sample oscillating in the same fashion; however, the measurement resulted in an oscillating response with twice the frequency of the sample, this decreased the accuracy of the delay measurement (and increased the complexity of the analysis).

A different region in the cavity for the movement of the sample was chosen. The limits of the displacement of the sample during the oscillations were the ones that go from the node ($z/L = 0.25$) to the antinode ($z/L = 0.5$,) with z the position along the cylinder axis. The response of the system with a static sample was measured within this range and is shown in Fig. 4. Such response is the amplitude of the standing wave for each position of the sample and is the one to which the system tends when the oscillational frequency decreases to 0 Hz. Also in Fig. 4, the sample oscillation range used for the measurements is shown delimited by circles. In this figure, the cylinder axis is horizontal and the sample oscillations are centered within this range. As seen on the following sections, the biggest peak-peak oscillation amplitude used for the measurements was 5 mm. This is just below the limit of the sub-woofer driver. Since there would be a crossing of the sample through either the node or antinode, oscillations with amplitudes above $L/4$ can not be used for the problem afore mentioned would reappear.

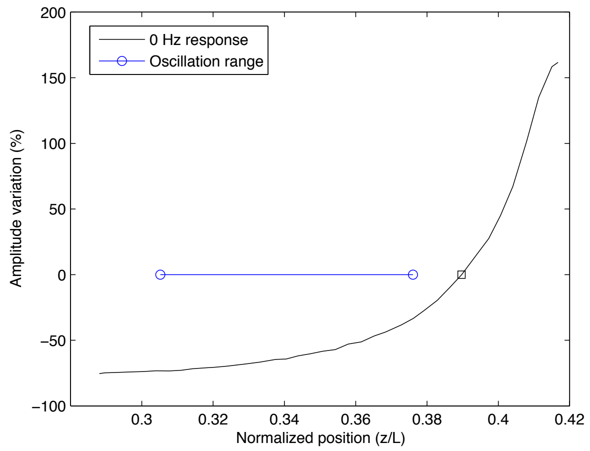

Figure 4: Response of the system with the sample at a fixed position along the axis of the cavity, and the range of movement of the sample inside the standing wave for the measurements.

6 Phase delay as a function of the oscillation frequency

The phase delay as a function of the sample oscillation frequency was measured. It consist on the data shown in Fig. 5. The amplitude of the sample oscillation was adjusted to 5 mm peak-peak and the frequency went from 1 to 20 Hz. This is about the same range of frequencies in which the instabilities occur. It is not possible to go below in frequency due to the response of the audio amplifier used, and higher oscillation frequencies yielded a cavity response that lost the signal envelope profile required for the signal analysis.

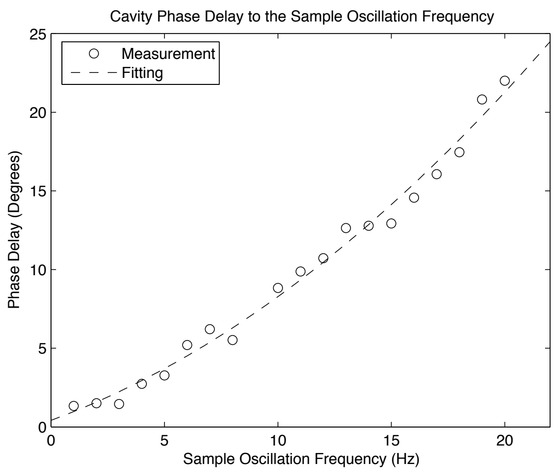

Figure 5: Phase delay as a function of the sample oscillation frequency

These data were fitted into a second-degree polynomial

$$y = 0.0257x^2 + 0.5285x + 0.4211 \qquad (2)$$

shown in Fig. 5, which when converted into time delay instead of phase delay, becomes a linear function of the frequency. For low frequencies the coefficient with most weight is the linear one, thus the whole behaviour of the phase delay as a function of the oscillation frequency can be simplified to a linear function with a slope $m = 0.5285$.

7 Phase delay as a function of the oscillation amplitude

A measurement of the phase delay as a function of the sample oscillation amplitude at a constant frequency of 20 Hz yields the data in figure 6. This frequency was chosen since it is the one with the highest cavity response obtained in the previous measurement, and this yields the highest accuracy attainable. These data were collected by measuring the phase delay while changing the amplitude of the oscillation of the sample from 0.5 mm peak-peak to 5 mm peak-peak.

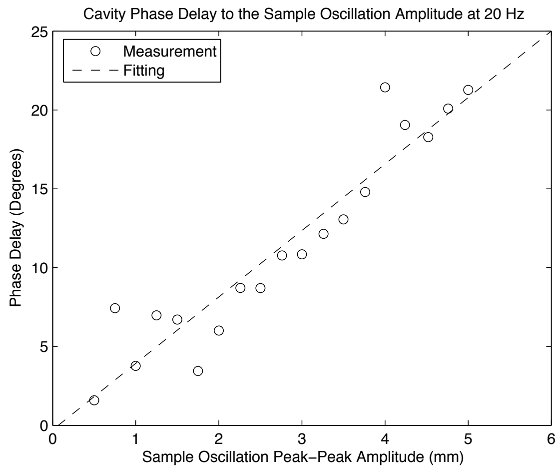

Figure 6: Phase delay as a function of the sample oscillation amplitude

For small amplitudes of the sample oscillation the effect of the cavity is small, thus barely measurable; this accounted for the scatter of the data since the accuracy decreases. Despite of this issue, the behaviour of the phase delay is clear; it increases proportionally to the oscillation amplitude, and hence to the sample velocity. When the oscillation amplitude increases with a fixed frequency, the sample has to travel a longer path in the same time, so the velocity of the sample increases.

Applying a least-squares (linear) fitting, we got a linear function with positive slope $m = 4.21$, shown in the graph as a dotted line.

8 Discussion

The current theory that describes the oscillational instabilities for acoustically levitated samples [1] is entirely based upon the fact that there is a time delay in the reaction of an acoustic mode to the changing position of a moving object, which gives rise to a linear velocity dependent term in the acoustic force on that object. The nature and details of such function are not provided though.

Thus, further research is required before a direct and analytical link between the linear velocity-dependent acoustic force term of the theory and the result of this paper could be established. However, from the results presented in this work, the existence of a linear function of the cavity delay to the sample velocity in system with an oscillational instability-like behavior is confirmed.

9 Conclusions

In this paper we experimentally studied the effects of the velocity of movement of an spherical sample on the time delay of the cavity response to such movement. It is a new approach to the study of the oscillational instabilities of acoustically levitated samples, whose sample oscillations have a similar behaviour than those we performed.

After careful experimental planning, it was possible to measure a time (and phase) delay between the movement of a sample inside a cavity and the response of the cavity excited at the resonance frequency of its second pure plane-wave mode. The results of such measurements conclude that there is, indeed as theory predicts, a phase delay of the cavity response to the sample velocity, which is a result of both the sample oscillation amplitude and frequency.

The extent of this results is just for the studied frequency and amplitude range. At higher frequencies the cavity response deviates from the expected behaviour, thus we could not obtain useful information; however, this constrain belongs to the tinkered experiment, not to the system itself. Further research is suggested in this direction. Also these results could be extended to other sample shapes since resonance frequency shift measurements with disks, cylinders and cubes yields different behaviour than that for a sphere [5, 7].

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