Convolution-scattering model for staircase echoes at the temple of Kukulkan

David Lubman

DL Acoustics, 14301 Middletown Lane, Westminster, CA 92683, USA
dlubman@dlacoustics.com

Chirped echoes from staircases at the temple of Kukulkan at Chichen Itza, Mexico have stimulated interest in archaeoacoustics since first reported by the author in 1998. In 2002, the author demonstrated chirped echoes and other acoustical effects at a Chichen Itza tour following the First Pan-American/Iberian Meeting on Acoustics in Cancun, Mexico. Among those present was Nico Declercq. In 2004, Declercq et al claimed credit for the first scientific explanation for the chirp. That claim overlooks this author's earlier explanation at the Cancun Meeting. This paper suggests advantages of Lubman's convolution-scattering model to Declercq's diffraction model. Lubman models the clap-echo process as a time-invariant linear system. His result explicitly shows echo dependence on the incident sound spectrum. It is computationally far more efficient, provides instant auralization, and achieves higher near-field accuracy by avoiding the unnecessary assumptions of plane wave impingement on infinite corrugated periodic surfaces. It also yields more richly detailed echo sonograms. Echoes are calculated by convolving incident sound (e.g., handclaps) with staircase impulse responses. Impulse responses are modelled as scattering from staircase steps. Lubman's solution allows ethnologists, ethnomusicologists and others to conveniently simulate and auralize staircase echoes for *any* sound stimulus, whether handclaps, voices, or ethnic sound instruments.

1 Introduction

Chirped echoes from staircases at the temple of Kukulkan at Chichen Itza, Mexico have stimulated much interest since first reported at scientific meetings by this writer in 1998 - 1999 [1, 2, 3]. The temple – a world heritage site - has become an icon of Mexico.

Handclaps made while standing before the temple produce downward "chirped echoes". Hearing these echoes is, for most people, a remarkable listening experience. One might ask why? Echoes are everyday experiences for most people. What makes this echo so remarkable?

Fig.1 Temple of Kukulkan at Chichen Itza, Mexico. Photo shows one of its four 91-step staircases responsible for a remarkable echo - the subject of this paper.

The answer may be that the echo has an unexpected feature that defies our everyday listening experience.

Listeners expect echoes to sound like delayed replicas of their stimuli (a handclap, in this case). In other words, we expect the echo of a clap to sound like a clap. The chirped echo defies that expectation because its sound (auditory percept) is so unlike the stimulus.

Many listeners react with wordless astonishment. It is wordless because that expectation is either instinctive or learned experientially and wordlessly.

It is amusing to observe tour groups brought to Kukulkan by guides, who clap their hands repeatedly to demonstrate the chirped echo before moving them on to the next attraction. Often, astonished individuals abandon their tour groups and stay behind to play with the echo.

Some listeners go so far as to impute magical or spiritual meanings to the echo. If true that imputation suggests two competing explanations. The chirped echo signal may causally stimulate imply that the

The imputation of magical or spiritual meaning to the echo may not be so far fetched. Chichen Itza was a ceremonial city of the ancient Maya. Normally, only priests and caretakers were present there, except for special occasions. On those occasions thousands of Maya faithful probably filled the huge plaza to witness sacrifice and such spectacles as the spring equinox shadow that appears on the north-west balustrade of a chirping staircase. That ancient spectacle has become famous again, drawing thousands of tourists and pilgrims since its rediscovery in modern times.

The zigzag shadow is often described as the "descending feathered serpent". Some believe it represents the god Kukulkan's annual descent from heaven. The ancient highland Maya would also have heard the chirped echo at those times. It might have been stimulated by a priest, or by a synchronized chorus of hand clappers or percussion instruments. (synchronized clapping choruses make the chirp louder.)

The chirped echo would have been especially meaningful to the ancient Maya, because it sounds uncannily like the primary call of the quetzal – a cloud forest bird widely venerated in Mesoamerica, but most famously by the ancient Maya who traded in its long and colourful tail feathers. Some ancient Mexicans believe the quetzal was a messenger of the gods.

Not only a sound but also a sight is emulated by the pyramid. The descending serpent shadow may emulate the diving flight of the quetzal. That flight begins exactly at the spring equinox in the cloud forest. Thus, the ancient Maya would have had a special reason to hear the chirped echo as a culturally specific, magical, or spiritual sound.

1.1 Objectives

This paper describes the chirped echo and compares it to the chirp of the quetzal

This paper outlines a mathematical explanation for the echo, first given at an acoustical meeting in Cancun, Mexico [1]. The explanation is interesting for the physical insight it offers. It also prescribes a practical method for determining the echo response to any sound, whether live, recorded or mathematically described, such as handclaps and ethnic sound instruments.

The author's formulation is compared with one by Declercq et al that employs diffraction theory [5]. Practical advantages of the author's formulation are described.

2 Description of the chirped echo

A sonogram of the chirped echo is shown in Fig. 2. The sonogram depicts the chirp as graph of sound spectrum vs. time. Time, the abscissa, is marked in tenths of a second. The left ordinate is linear frequency, running from about 400 Hz to 3.1 KHz. Relative intensity in decibels [dB] is color coded as shown on the right axis, where red is the highest and black is the lowest intensity.

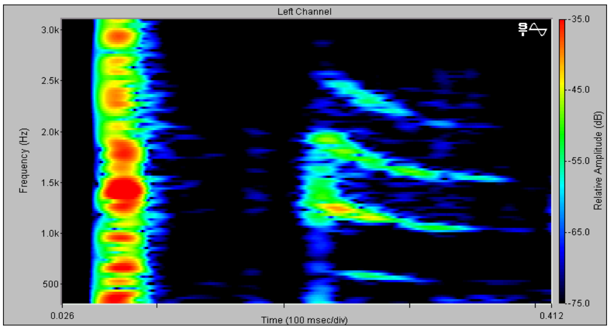

Fig. 2 Sonogram of chirped echo from the N. staircase of the temple of Kukulkan.

The colored vertical bar on the left of Fig 3 is the handclap stimulus. The chirped echo is comprised of the four curved lines on the right. It persists for less than 0.2 sec. The curved lines are harmonically related tones. Their downward curvatures tell that the chirp frequency falls with time.

3 Echo and quetzal chirp compared

The chirped echo seems even more astonishing when its sound and sonogram are compared with the quetzal [4]. A quetzal chirp sonogram is shown in Fig 3. Is the striking similarity with the chirped echo coincidental? Or did the ancient Maya intentionally design the echo to sound like the bird? If intentional, the chirped echo, created long before the Edison phonograph, is the earliest known sound recording,.

4 Physical and mathematical models

In 2004, Declercq et al attempted to explain the chirped echo with diffraction theory [5]. However, as pointed out by the authors, the explanation was incomplete. The authors recognized that echoes depend strongly on the "type" of incident sound, but could not determine that dependence.

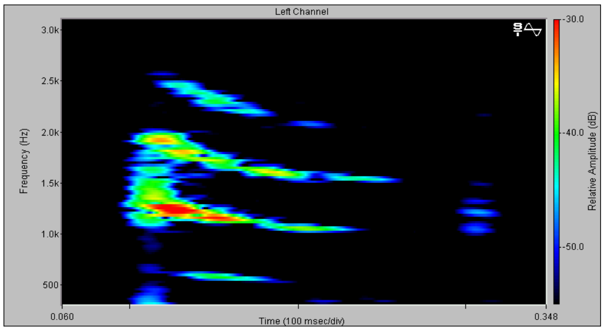

Fig. 3 Sonogram of a quetzal chirp bears a close resemblance to the chirped echo.

The solution to that problem was presented by this author at the First Pan-American/Iberian Meeting on Acoustics at Cancun, Mexico [2]. That paper is among those cited in their own [5]).

That presentation explicitly showed the echo's dependence on handclap spectrum (more generally, the time-frequency structure of impinging sound.)

It included a model of the chirped echo, and an echo simulation and auralization That physical and mathematical model is outlined in Figs. 4 and 5. An observer's handclap produces an acoustical disturbance, f(t) that propagates to and interacts with the staircase. Altered by the staircase, the disturbance returns to the observer's ears as a chirped echo, g(t).

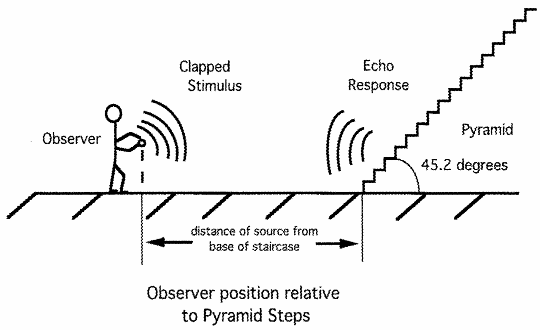

Fig. 4: Key physical elements of the chirped echo system reduced to two-dimensions

A theory of the chirped echo must explain how an arbitrary stimulus such as a handclap f(t) is transformed into another sound, g(t), such as a chirped echo.

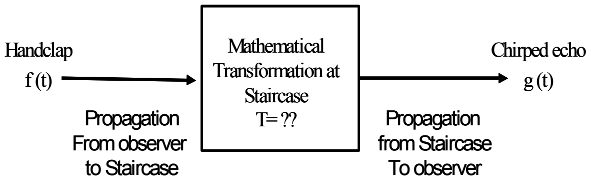

Figure 5: Schematic of the chirped echo system as a mathematical problem

4.1 Convolution-Scattering theory

The problem shown in Figs. 4 and 5 comprise a time-invariant linear system, as shown in Fig. 6. The solution is found simply by invoking the convolution theorem shown in Eq.(1). Here, $h(\tau)$ is the staircase impulse response.

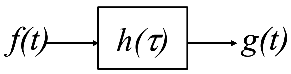

Fig. 6. Input-output relations for a linear system

$$g(t) = \int f(\tau) \cdot h(t - \tau) \, d\tau \qquad (1)$$

The convolution integral implies that the chirped echo g(t) is determined by the stimulus f(t) and the impulse response of the staircase, $h(\tau)$.

The staircase impulse response can be modelled as scattering from the staircase. For this purpose it is convenient to assume that the scattering centers are located at vertexes of stair risers and treads, as shown by the red dots in Fig. 7. In this two dimensional reduction each stair is treated as a point scatterer. Thus, the staircase impulse response is the time-ordered sum of scattered returns from the 91 stairs.

It follows from the convolution integral that the chirped echo can be defined in the frequency domain as well by Fourier transformations – this is another useful formulation.

$$G(j\omega) = F(j\omega).\ H(j\omega) \qquad (2)$$

where,

$$G(j\omega) = F\,[g(t)] \qquad (3)$$

$$H(j\omega) = F[h(t)] \qquad (4)$$

and,

F [y(t)] is the Fourier transform of y(t) (5)

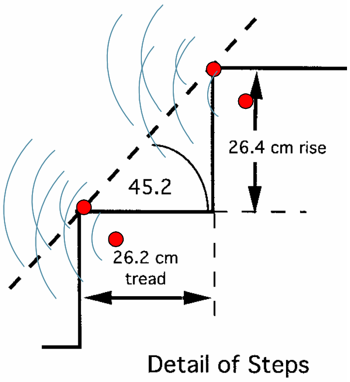

Fig. 7 Staircase modelled as an array of 91 point scatterers centred at intersections of risers and treads. Measured average dimensions are shown.

The stimulus (handclap) power spectrum is

$$|F(j\omega).\ F^*(j\omega)| = |F(j\omega)|^2 \qquad (6)$$

where, * designates complex conjugate.

The echo power spectrum is

$$|G(\omega)|^2 = |F(j\omega)|^2 .\ |H(j\omega)|^2 \qquad (7)$$

In words, the echo power spectrum is the product of the stimulus power spectrum (handclap) and the squared magnitude of the Fourier transformed impulse response.

The echo power spectrum is easily measured with standard instrumentation and measurement technique.

5 Convolution Results

Fig. 8 is the sonogram of a computer simulated chirped echo first shown at the 2002 Cancun meeting. The calculated echo was auralized as a *.wav.

This particular result was obtained by convolving a 1 ms sample of band-limited random noise with a mathematical model of the impulse response. The impulse response consisted of 91 values of time delay and 91 values of pressure amplitude. [The echo sound pressure varies as 1/2r, where 2r is the round trip distance between the clapper and the staircase scatterer]. Calculations for this example assumed the clapper stood 11 meters from the staircase.

Calculations took less than 1 second on an ordinary 1999 Dell desktop computer and the result - a *.wav file - was auralized immediately.

Several notable differences between Figs. 8 and 9 are observed.

The simulation of Fig. 8 is richer in harmonics than the actual echo recorded at the temple.

In Fig 9, the sonogram of a chirped echo recorded at the same distance is shown for comparison.

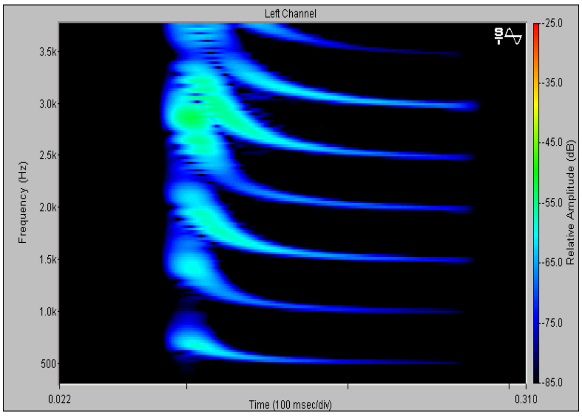

Fig. 8 Chirped echo simulated by convolution on a simple desktop computer.

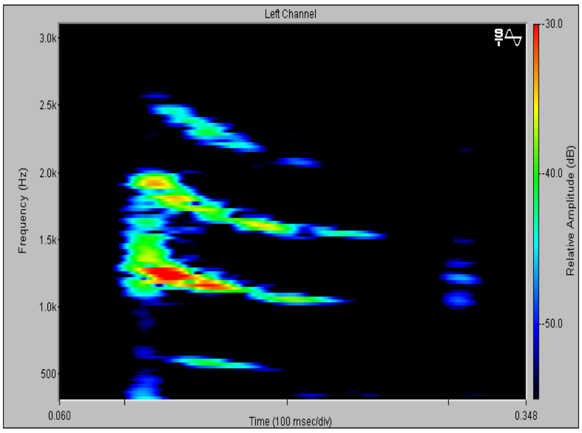

Fig. 9. Actual chirped echo from a handclap at the temple of Kukulkan's north staircase.

One reason is that no allowance was made for the handclap spectrum. Each handclap has a unique power spectrum. Some guides at Chichen Itza have learned to clap effectively to produce a strong echo.

It is simple to incorporate handclap spectra by suitable modelling f(t).

Another reason is staircase deterioration. In ancient times, staircases were periodically replastered with smooth and non-porous limestone plaster. Over time, plaster eroded and weathered making the stairs irregular and porous. Those features reduce the staircase scattering strength, especially at higher frequencies.

It is simple to incorporate the scattering strength of the staircase by modifying the staircase impulse response according to measurements of the staircase scattering strength.

It seems likely that the long stone staircases of every Mesoamerican pyramid chirped strongly when freshly plastered, and were even louder than they are today. If staircase tread lengths and (to a lesser extent) riser height dimensions are similar the chirped echo will sound like the quetzal. Today however, the chirped echoes are inaudible or barely audible at most Mesoamerican pyramids. However, the chirp can be made audible by increasing the stimulation strength. That can be done at the site by recruiting synchronized clapping choruses. A synchronized chorus of 20 tourists, for example, will increase the echo strength by 10 log (20) = 13 dB.

With the convolution-scattering method, the chirp of now - silent pyramid staircases can be simulated off-site from their physical dimensions.

6 Comparison of diffraction and convolution – scattering models

The diffraction solution of Declercq et al [5] is complex yet needlessly restrictive. For example, it models the staircase as a corrugated surface of infinite extent. In contrast, the convolution – scattering model puts no restriction on staircase extent.

Moreover, Declercq et al's solution is valid only in the far field because it requires plane wave incidence on the staircase. Assuming plane waves leads to large errors at short ranges where the wave front curvature is significant. In contrast, the convolution - scattering solution gracefully accepts wave front curvature and is thus correct in both the near and far field.

Declercq et al's reported that their computed sonograms required huge amounts of computer time even with Ghent University's highest speed computer. Yet their graphic results seem far less impressive than the simulated echo sonogram shown here. The simulation of Fig 9 took less than a second on an ordinary desktop computer.

In their abstract, Declercq et al raised the "critical question" of what physical effects cause the chirped echo? The answer was obscured by their diffraction formulation. It was answered earlier, easily and transparently in ref [2] using the convolution – scattering model.

7 Conclusions

The chirped echo phenomenon is amenable to many useful treatments, including diffraction, convolution scattering, and others not discussed here.

Among the merits of the convolution-scattering model are:

- Stimuli f(t) are unrestricted. They can be analytical, sound recordings, or live sounds
- Response echoes g(t) are efficiently calculated with ordinary desktop computers
- Calculated echoes are auralized (heard) instantly.
- Echo sonograms are readily obtained from g(t)
- It is hoped that ethnographers and musicologists will find this tool useful in their attempts to reconstruct or imagine the sounds of rituals in the plaza.
- A hypothesis for intentional design of the chirped echo is suggested. It provides a motivation for intentional design from the history and mythology of the Maya and the natural sounds of their ancient environment.

This paper is an example of an archaeological acoustic investigation. Previously, echoes and unusual sounds around ancient monuments were ignored by archaeologists, if not by natives, tourists, and acousticians. With the publication of the monograph Archaeoacoustics [6] archaeologists are now encouraged to consider the sounds and soundscapes around ancient monuments as true archaeological artefacts that may merit scholarly study.

Since many unusual sounds can arise by chance, the question of intent is central to archaeological acoustic studies. That said, ancient builders lived in a much quieter world and were good listeners. If such speculations as these are validated, it would mean that some ancients were capable of superb acoustical engineering beyond the previous imaginings of modern workers.

Archaeoacousticians can ask if at certain times and places ancient builders created sounds and soundscapes that were meaningful in their particular cultures.

Archaeologists may employ archaeoacoustics to learn more about disappeared cultures by recreating the sounds of their natural and built environments.

Acknowledgments

The author gratefully acknowledges the patient and generous support of the Paul S. Veneklasen Research Foundation for preparation of this manuscript.

The author also acknowledges the contributions of many vocational and avocational Mesoamerican archaeologists. Some of them participate in Aztlan – a Mesoamerican interest list server. From their books, papers, lectures, and internet postings this author has obtained useful and stimulating knowledge and understandings.

References

[1] Physical explanation and mythological hypothesis for the chirped echoes resounding from the pyramid staircase at Chichen Itza, V Mexican Congress on Acoustics, Querrtero, QRO., Mexico, Sept 1998.

[2]Acoustical features of two Mayan monuments at Chichen Itza: Accident or design? J. Acoust. Soc. Am. **112** (5), 2285 (A) (2002).

[3] Maya acoustics: Of rainbows and resplendent quetzals, J. Acoust. Soc. Am., **26** (4), 2228, Nov, ’99.

[4] Sounds and sonograms of bird and echo at http://www.ocasa.org/membersites.htm

[5] A theoretical study of special acoustic effects caused by the staircase of the El Castillo pyramid … Declercq et al (J. Acoust. Soc. Am. **116**, (6), December 2004, pp 3328-3334)

[6] Archaeoacoustics Monograph published by the McDonald Inst. For Archaeological Research, University of Cambridge, U. K. Chris Scare & Graeme Lawson, Ed. (2006)