External sound radiation of vibrating trombone bells

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The issue of the influence of bell vibrations on the sound of brass instruments is still debated. For such instruments, external sound field is the superposition of the sound field resulting from acoustic oscillations of the air column and the sound field resulting from direct radiation of the bell and walls. The aim of this paper is to quantify the bell contribution with respect to the air column one, and to investigate the conditions for which the former may become audible. For this purpose, the structural modes of a trombone bell are identified using an experimental modal analysis. For each mode shape, the radiation efficiency is computed using a model of the radiated sound based on a distribution of equivalent monopoles. A critical frequency is defined for the bell and allows us to determine at which condition a given structural mode is radiating.

1 Introduction

The contribution of the vibrating walls to the overall sound produced by brass wind instrument is a controversial issue [1, 2, 3, 4, 5, 6, 7, 8, 9]. Although it has been shown that the input impedance could be disturbed by the vibrations of the walls [1, 2], the direct radiation from the walls to the external field could also have an influence. The external sound field results from the superposition of the contribution of the acoustic oscillations of the air column, which is the most important part, and the contribution of the vibrating bell and walls. The vibrating bell contribution could thus take part to the sound field. The questions of its relative strength with respect to the air column contribution, and more importantly the physical processes involved are still not answered. In this paper, the study is focused on eigenmodes of vibration of a trombone bell. These modes are investigated experimentally and numerically. Their ability to possibly radiate sound efficiently enough to take an audible part in the overall sound in then investigated.

2 Structural modes of a trombone bell, measurements and finite elements analysis

The study of the sound radiation from a vibrating body imply a study of its dynamic behavior. In the literature, the study of the vibration of brass instrument bells has been investigated experimentally using optical interferometry techniques. Some vibration patterns due to the eigenmodes of a trumpet bell were observed by Smith [6] and more recently by Moore [7], using such techniques. The vibration patterns at resonances can be classified considering their nodal properties. As it is the case for a simple cylinder, the various modes can be described using a longitudinal index, corresponding to the number of nodal circles along the bell, and a circumferential index, corresponding to the number of nodal meridians around the bell.

Some measurements on a trombone bell which can be removed from the instrument itself have been carried out using a different technique. Using a shaker as excitator, the vibrations have been recorded using a scanning laser vibrometer on various points of the bell. The set of scanned points allows to build vibration patterns on the surface of the bell. Due to the curvature of the structure, only a part of the surface was scanned. Frequency response functions were recorded for all the scanned points, and the vibration patterns at the resonances were analysed. An example of an obtained vibration pattern at a resonance is shown on Fig1, clearly showing nodal circles and meridians. The results of this analysis

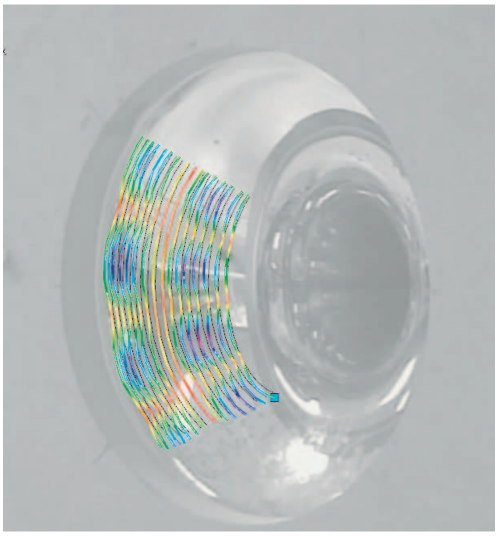


Figure 1: Typical result for a vibration pattern at a resonance, obtained by laser scanning vibrometer measurements

is given in Table 1, giving the resonance frequencies as a function of the circumferential and longitudinal index. Blanks in Table 1 correspond to vibrational patterns

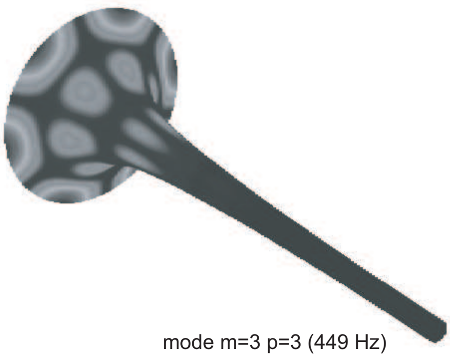
Table 1: Bell modal frequencies as a function of m, circumferential index, and p, longitudinal index.

	p=3	p=4	p=5	p=6
m = 1	297 Hz	735 Hz		1456 Hz
m = 2	259 Hz	577 Hz	1403 Hz	2091 Hz
m = 3	561 Hz	676 Hz	2140 Hz	2474 Hz
m = 4	716 Hz	979 Hz	2298 Hz	2957 Hz
m = 5	1240 Hz			
m = 6	1383 Hz	1630 Hz		3035 Hz

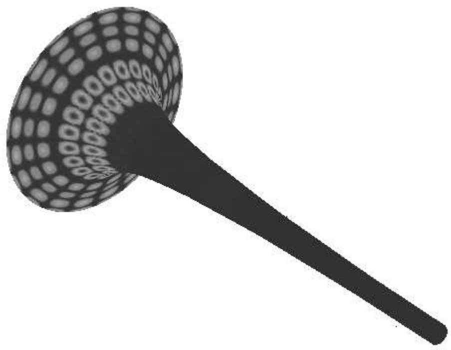
that were not obviously identified. Moreover the longitudinal index is uncertain because of the fact that only a part of the bell was scanned.

Because of the complex shape of trombone bells the experimental investigation is difficult. Finite element computations have thus also been carried out [10]. Using COMSOL, a finite elements software, the eigenmodes of the structure have been computed. The model of the bell was built using a measured bell bore and thickness, and estimated brass material parameters (Young's modulus $E = 110$ GPa, density $\rho = 8700$ kg/m^3). The bell is clamped at its entrance and free to vibrate at its end. The obtained eigenmodes showed similar properties as the measured one, regarding the structure of nodal lines. Three hundred modes were obtained between 0 and 6150 Hz. Most of the modes were pairs of modes with same indexes m and p but with close eigenfrequencies and rotated modal shapes of an angle $\pi/(2m)$. This is due to the asymmetry of the mesh used for the computation, which leads to a breaking of modal degeneracy. Three examples of eigenmodes are displayed on Fig2, they are classified considering their modal indexes m and p.

mode m=2 p=3 (233 Hz)



mode m=3 p=3 (449 Hz)



mode m=10 p=6 (6102 Hz)

Figure 2: Eigenmodes of the bell obtained by finite elements computation

3 Radiation efficiency of the vibrating bell

In order to evaluate the ability of such eigenmodes to radiate sound, radiation efficiencies computation have been carried out.

3.1 Computation method

For a given eigenmode of the trombone bell, each vibrating cell is modeled by a monopole which is positioned in the center of the region and whose strength is proportional to its size and to its maximum of amplitude of displacement. In order to take into account the phase opposition between two adjacent vibrating cells, the strength of two adjacent monopoles are opposite. For example, the $(2,3)$ mode is modeled using 12 monopoles, and $(10,6)$ using 120 monopoles, one for each vibrating lobe.

Sound pressure due to this distribution of monopoles is evaluated in far field on a sphere which is centered on the barycenter of the monopoles. The pressure field on a point of the sphere can thus be written as :

$$p(r) = \sum_{q=1}^{N} p_q(r) = \sum_{q=1}^{N} \frac{j\omega\rho Q_q}{4\pi} \frac{e^{jkr_q}}{r_q}, \qquad (1)$$

where N is the number of monopoles, ω is the angular frequency, k the acoustic wave number, ρ the air density, Q_q the strength of the monopole of index q, and r_q the distance between the position of the monopole of index q and the point on the sphere. The radiated sound power P is then computed using the expression [11] :

$$P = \frac{1}{2} \int\int_S \frac{p(r)^2}{\rho c} dS. \qquad (2)$$

This integral is evaluated numerically by calculating the pressure on a large number of points on the sphere.

To compute the radiation efficiency, the sound power radiated is normalised by the sum of the radiated power due each monopole taken individually [11] : P_{ref}. We have :

$$P_{ref} = \sum_{q=1}^{N} \frac{\omega^2 \rho Q_q^2}{8\pi c}. \qquad (3)$$

The radiation efficiency of the considered eigenmode of the bell is thus :

$$\sigma = \frac{P}{P_{ref}}. \qquad (4)$$

Examples of radiation efficiencies for three modes : $(2,3)$, $(3,3)$ and $(10,6)$ are given on Fig3.

3.2 Interpretation of the radiation efficiencies

On Fig3, it is shown that the radiation efficiency has a "high-pass filter" behavior. When the driving frequency is below the cut off frequency, the vibration pattern is inefficiently radiating sound. Each mode shape has a critical frequency, above which the value of radiation efficiency is close to 1. If the eigenfrequency of the mode is above the critical frequency, it may be able to radiate

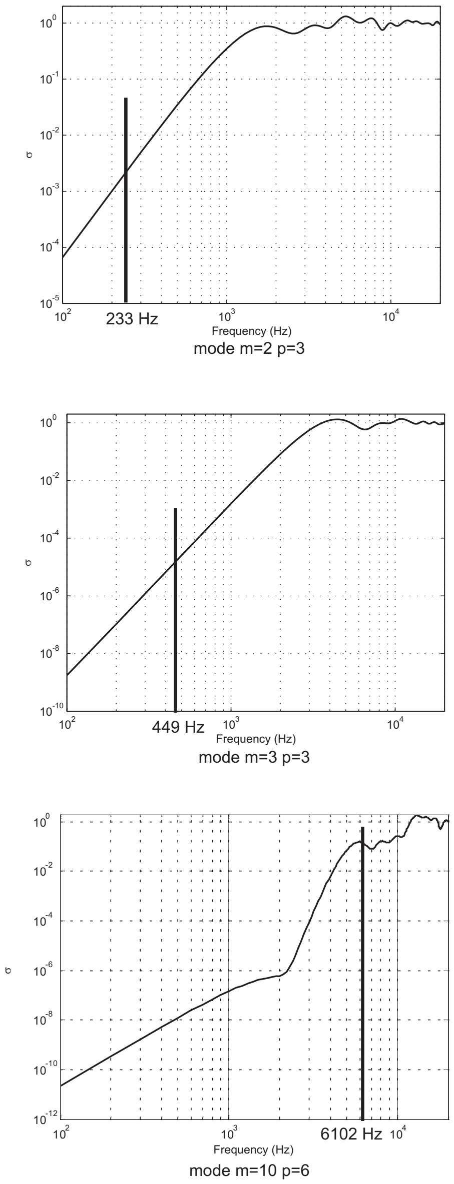


Figure 3: Radiation efficiency versus frequency for 3 modes of the bell (m=2,p=3), (m=3,p=3) and (m=10,p=6). Each vertical line indicates the eigenfrequency of each mode.

power if it is excited. On the contrary, even if it is excited, the radiated sound power would be weak. An eigenmode of the bell which is excited would resonate but depending if its eigenfrequency is above or below the critical frequency it could radiate efficiently or not. Modes that do have an eigenfrequency above the critical frequency are candidates to participate to the overall sound of a trombone when it is played, because even if they are slightly excited, they would radiate sound power.
In the case of the trombone bell, it is shown on figure 3 that for the low frequency modes such as $(2,3)$ or $(3,3)$ the eigenfrequency, indicated with a thick vertical line, is not located in the efficient part of the curve. This imply that these modes might be excited when the trombone is played but are not likely to radiate a lot of sound power. On the contrary, more high frequency modes like $(10,6)$ are candidates to radiate sound power when excited, as the eigenfrequency is close to the efficient part of the curve.

4 Conclusion

It is shown that the eigenmodes of a trombone bell could be classified using two indexes : a longitudinal index p, corresponding to the number of nodal circles along the bell, and a circumferential index m, corresponding to the number of nodal meridians around the bell. These eigenmodes when excited could participate to the overall sound of the trombone if their radiation efficiency is high enough. It is shown that using a model of equivalent monopoles for each mode of the bell, it is possible to compute the radiation efficiency of each mode. This radiation efficiency can be used to discriminate between modes that would not radiate efficiently even if excited and modes that are efficient.

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