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Hybrid Inversion technique for predicting geometrical parameters of Porous Materials

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Abstract

In prediction of acoustical behavior of porous materials, five geometrical parameters play a very important role, but some of these geometrical properties are very difficult to measure directly. So many authors have suggested different inversion strategies for getting these properties from directly measured acoustical properties of the material using standing wave tube. These approaches can be divided in two different categories: analytical (based on the limit behavior of the bulk properties) and minimization based methods (which make use of searching algorithms to determine the best solution that minimizes a cost function calculated by means of a prediction model). Recent studies have shown the reliability of the analytical methods for the determination of the airflow resistivity and the minimization based approach by using genetic algorithms for getting the other physical parameters. Consequently, a hybrid inversion technique can be proposed for the complete calculation of the geometrical quantities and here it is presented in detail. Moreover the paper compares the results from the hybrid approach with the experimentally measured parameters and the values of the five parameters obtained by using genetic algorithms. Finally, the paper presents the effect of both inversion techniques on acoustical properties using Johnson-Allard-Champoux model.

Introduction

In recent years study of acoustic porous materials has become very important in the development of new acoustic materials as well as in the design of sound absorbing packages in the transport Industry. The prediction of acoustic materials is governed by five physical and three mechanical parameters. The acoustical behavior of porous materials is governed by five physical (e.g. Porosity, flow resistivity, tortuosity, VCL and TCL) as well as three mechanical parameters (e.g. Young's modulus, Poisson ratio and loss factor). Out of these five physical parameters, porosity and high flow resistivity can be measured directly by available direct methods. But measurement of physical parameters like tortuosity, viscous and thermal characteristic lengths is very difficult. So as an alternative, many authors have proposed different inversion strategies for getting these properties from directly measured both characteristic and surface properties of the material using standing wave tube.

These approaches can be divided in two different categories: analytical and minimization based methods. These inverse characterization schemes are based on the equivalent fluid model (e.g. Johnson-Allard-Champoux-Model) [1] in which the solid frame is assumed to be rigid, i.e. motionless. The inverse characterization of the parameters is performed over a wide frequency range [50-4200 Hz]. The test specimen is backed by the rigid end termination of the measurement plane wave tube. In the following sections, a description of the equivalent fluid model is firstly presented. Secondly, the inverse problem strategies are briefly discussed. Thirdly, inverse characterization results on several porous samples are given. Finally the paper presents the effect of both inversion techniques on acoustical properties using Johnson-Allard-Champoux model.

The Equivalent Fluid: Johnson-Champoux-Allard Model

Open cell Poroelastic materials are very well described by Biot theory [1]. At the same time, in many situations when a material sample is excited by acoustical waves, the frame of this material behaves approximately as acoustically rigid (motionless) over a wide range of frequencies. In this case, the porous material can be replaced on a macroscopic scale by an equivalent fluid of effective density $\rho(\omega)$ and effective bulk modulus $K(\omega)$. The motionless frame condition can occur either because of high density or elasticity modulus, or because of particular boundary conditions imposed during the test. In the widely used equivalent fluid model of Johnson-Champoux-Allard, these effective quantities depend on five macroscopic parameters: the flow resistivity (σ), the porosity (ϕ), the tortuosity (α_∞), and the viscous (Λ) as well as thermal (Λ') characteristic lengths. The dynamic density $\rho(\omega)$ and complex compressibility $K(\omega)$ for Johnson Model are given by following equations.

$$\rho(\omega) = \rho_0 \alpha_\infty \left[1 + \frac{\sigma\phi}{j\omega\rho_0\alpha_\infty} \sqrt{\frac{4j\alpha_\infty{}^2\eta\omega}{\sigma^2\Lambda^2\phi^2}} \right] \quad (1)$$

$$K(\omega) = \gamma P_0 \left[\gamma - (\gamma-1) \Big/ 1 + \frac{8\eta}{j\Lambda' N_{pr}\omega\rho_0} \sqrt{1 + j\rho_0 \frac{\omega N_{pr}\Lambda'}{16\eta}} \right]^{-1} \quad (2)$$

where ρ_0 is density of fluid, P_0 is atmospheric pressure, γ is specific heat ratio N_{pr} is Prandtl number, η is coefficient of viscosity of air and ω is circular frequency.

For a porous sample of thickness d, backed by rigid wall, its specific acoustic surface impedance is

$$Z_s = -j \frac{Z_c}{\rho_0 \cdot c_0} \cot(k_c\, d) / \phi \quad (3)$$

where Z_c and k_c are the characteristic impedance and the complex wave number of the porous specimen respectively. They are related to the effective properties of the porous medium by Eq.(4) and Eq.(5).

$$Z_c = \left(\rho(\omega)\, K(\omega)\right)^{1/2} \quad (4)$$

$$k_c = j\omega.\left[\rho(\omega) / K(\omega)\right]^{1/2} \quad (5)$$

Inversion problem strategy

In this section, the inversion techniques for low frequency limit analytical method and non linear curve optimization technique are discussed in detail.

1.1.1 Analytical Method

It is based on the acoustical model from which analytical expressions linking the material parameters to acoustical measurements are derived. The methods using this approach are also qualified as indirect methods. It uses the low frequency limit behavior of the bulk acoustic properties like effective bulk modulus and effective density. The static resistivity is determined from dynamic resistivity given as [2]

$$\sigma = -\lim_{\omega \to 0}\left[\operatorname{Im}\left(\rho(\omega)\right).\omega\right] \quad (6)$$

The imaginary part of the low frequency limit of the dynamic resistivity is the static resistivity.

For other parameters, the analytical method could lead to incorrect values mainly because non availability of sufficient high frequency measurement range and due to the difficulty in finding an adequate range for the linear interpolation [3].

1.1.2 Optimization Based Method: Genetic Algorithm

Genetic algorithm is based on the Darwin's theory of Evolution. It is used to solve the optimization problem with constraints and bounds on the solution. It repeatedly modifies a population of individual points using rules modelled on gene combinations in biological reproduction. At each step, the genetic algorithm selects individuals at random from the current population to be "parents" and uses them produce the "children" for the next generation. Over successive generations, the genetic algorithm improves the chances of finding a global solution. In the final analysis, normalized surface impedance is used as cost function [3]. The cost function minimized is defined as Eq.(7)

$$Z_s = \sum \left| Z_{s,Mea} - Z_{s,Model} \right| \quad (7)$$

The bounds implemented on the physical parameters are given in the table 1

Bounds	σ	ϕ	α_∞	Λ	Λ'
Lower Bounds	1000	0.1	1	10	10
Upper Bounds	200000	1	10	2000	2000

Table 1 Bounds on Physical Parameters

Also non-linear bound was implemented on characteristics lengths such that

$$\Lambda \le \Lambda'$$

This condition is true for almost all porous materials.

The optimization problem with constraints was implemented in Matlab®.

2 Experimental Methods

For Experimental measurements, 4 different types of porous as well fibrous materials like Melamine foam, Cellular Rubber, Polyurethane foam and Glass wool were selected with density in between 10 and 78 *kg/m*^{3} and thickness in between 14 and 40 *mm*. The diameter of all samples was 45 *mm*. The open porosity was directly measured by a method based on Boyle's law [4] which uses isothermal compression of air volume within and external to the tested material. The static flow resistivity was measured by flow resistivity test rig based on standard ISO-9053 [5]. Finally, the tortuosity was determined by a method based on determination of the high frequency limit for the complex phase velocities within the air and the material [6]. While the characteristic lengths were inverted using Genetic algorithm with directly measured porosity, flow resistivity, tortuosity as additional input. The directly measured physical material parameters are tabulated in the table 1.

In proposed hybrid inverse characterization, only flow resistivity is calculated by analytical method, while other four physical parameters are calculated from genetic inverse with fixed value of flow resistivity from Analytical inversion. Afterwards the complex acoustical parameters (i.e. characteristic impedance and complex wave number) were determined by means of a transfer matrix approach by using a three microphone technique [7]. The surface acoustic properties (i.e. surface impedance and the normal incidence sound absorption coefficient) were measured according to the ISO 10534-2 [8]. Finally the measurement test rigs are shown in Fig. 1

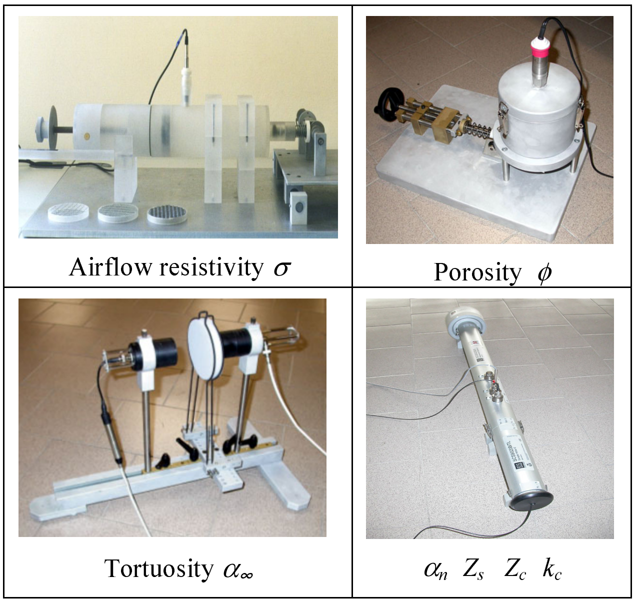


Figure 1 Experimental Test Rigs

3 Results and Discussions

In this section the inverse parameters are compared with those measured from experimental measurements. The comparison is tabulated in tables 2-5. From these results; it is clear that there is good comparison between flow resistivity calculated from analytical method at low frequency limit and directly measured values.
In some cases, flow resistivity calculated from genetic algorithm is higher than expected. It is because of genetic optimization will try to find out the solution for five intrinsic physical parameters by minimizing error between measured data and theoretical model and so the solution could be only mathematical but not the physical one, even the inverted parameters from genetic optimization provides better results for acoustical properties. For all materials, flow resistivity calculated from Genetic algorithm is comparable with directly measured value of flow resistivity; still the relative error remains high as compared to Analytical inversion as shown in Table 6 and so the analytical inversion seems to be good solution. For other physical parameters, there is good comparison between directly measured values and values from Genetic as well as Hybrid inversion techniques.

Melamine Foam 8.4 Kg/m^3-20mm			
	Exp.	*Genetic*	*Hybrid*
Flow Resistivity (Ns/m^4)	10550	12340	11697
Porosity [-]	0.99	0.92	0.92
Tortuosity [-]	1.01	1	1.03
VCL (μm)	81*	92	100
TCL (μm)	255*	178	188

Table 2 Melamine Foam

PU-Foam 25 Kg/m^3-20mm			
	Exp.	*Genetic*	*Hybrid*
Flow Resistivity (Ns/m^4)	12901	39628	12470
Porosity [-]	0.98	0.96	0.98
Tortuosity [-]	1.41	1.87	1.82
VCL (μm)	21*	32	30
TCL (μm)	33*	32	41

Table 3 Polyurethane Foam

Cellular Rubber 64 Kg/m^3-24mm			
	Exp.	*Genetic*	*Hybrid*
Flow Resistivity (Ns/m^4)	123501	116866	96149
Porosity [-]	0.83	0.87	0.87
Tortuosity [-]	2.64	1.27	1.26
VCL (μm)	23*	10	10
TCL (μm)	23*	10	10

Table 4 Cellular Rubber

The relative error for all physical parameters is calculated and the average relative error for all materials is given in the table 6

Glass Wool 17 Kg/m^3-20mm			
	Exp.	*Genetic*	*Hybrid*
Flow Resistivity (Ns/m^4)	14186	24906	14258
Porosity [-]	0.99	0.96	0.97
Tortuosity [-]	1	1	1
VCL (μm)	59*	73	59
TCL (μm)	193*	140	181

Table 5 Glass Wool

* inverted using Genetic algorithm

Rel. Error %	σ	ϕ	α_∞	Λ	Λ'
Genetic	76	4	21	37	29
Hybrid	9	3	21	31	28

Table 6 Average relative error for all material samples

Finally the effect on sound absorption of all these measured and inverse parameters from both techniques is presented. The values of simulated sound absorption coefficients are compared with the measured values. The results for two materials are shown in Fig. 2 and Fig 3

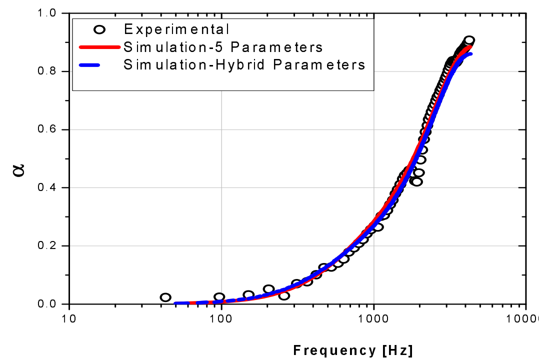


Fig: 2 Comparison of Sound Absorption Coefficient for Melamine foam

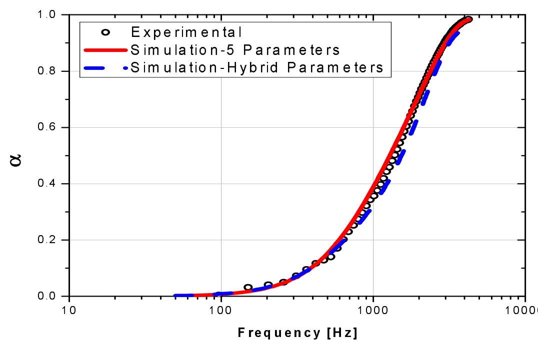
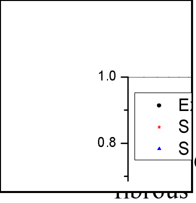


Fig: 3 Comparison of Sound Absorption Coefficient for Glass Wool

From Fig 2 and 3 it is clear that there is satisfactory agreement between experimental sound absorption and the same determined by simulation from inverted parameters using Johnson-Champoux-Allard model.



... 4 and 5 surface impedance of tested materials is ...ed with simulated surface impedance of porous and ... materials. From these graphs, it is observed that reliable results for acoustic properties can be obtained by Hybrid technique with only flow resistivity inverted from analytical method instead of inverting all five parameters from Genetic optimization Technique.

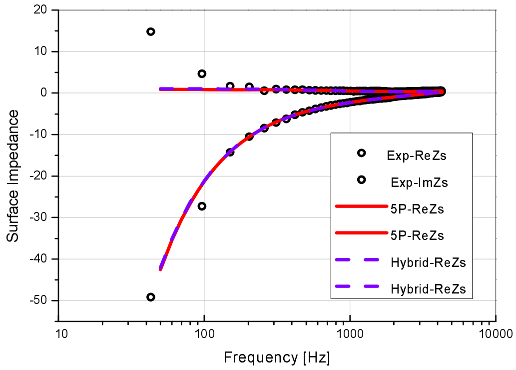


Fig: 4 Comparison of Surface Impedance for Melamine Foam

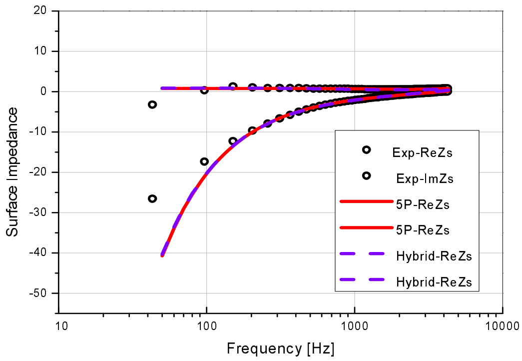


Fig: 5 Comparison of Surface Impedance for Glass Wool

Conclusion

In this article, an experimental and inverse investigation by Genetic and hybrid techniques is presented for poroelastic materials. Analytical and genetic algorithm techniques are implemented for calculation of flow resistivity and other four physical parameters. Material samples were experimentally characterized by measuring characteristic and surface acoustical properties by three microphone tube. The flow resistivity calculated from both techniques is compared with the directly measured value and it is found that flow resistivity calculated from hybrid inversion is more reliable. The effect of all parameters calculated from hybrid technique is also studied on Sound absorption curves and it is observed that the results are comparable with the measured acoustic properties. Finally from this study, it is concluded that proposed hybrid inversion technique seems to provide reliable results for physical parameters as well acoustical properties.

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