experimental evaluation of the wavenumber in stacked screen regenerators

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The experimental method to evaluate the wave number of an acoustic wave propagating in porous media is proposed. The method was applied to four types of stacked screen regenerators. The value of the experimentally obtained wave number was found to be similar to that of the theoretically obtained wave number in a circular channel. However, it was found that it depended on the types of regenerators. Based on the dependence on the regenerator types of the evaluated wave number, the effective radius in the stacked screen regenerators was addressed.

1 Introduction

Thermoacoustic engine and cooler have no or a few moving parts, and thus they have high reliability. Moreover the recently-developed thermoacoustic engine and cooler employ the thermodynamic cycle similar to the Stirling cycle and thus, they have potential to achieve high efficiency as the same as conventional Stirling engine and cooler.[1, 2, 3]

In thermoacoustic devices, energy conversion takes place in the porous media. The porous media is generally called as 'stack' or 'regenerator'. As a stack, parallel placed plates, an array of narrow channels, and an pin array are usually used. On the other hand, as a regenerator, metallic spheres packed in a cylinder and the stacked mesh screens are used. In the thermoacoustic devices based on the Stirling cycle, the regenerator is usually adopted as an energy conversion component. This is because the excellent thermal contacts between gas and solid wall are favorable for the execution of the Stirling cycle and a regenerator has a manufactured advantage to realize the excellent thermal contacts compared with a stack.

In order to design thermoacoustic engine and cooler, one must characterize a stack and regenerator. They can be characterized by wave number.[4, 5, 6] The wave number of a stack can be analytically calculated because of the simple geometry of a stack. It was confirmed that the analytically calculated wave number of a stack agree well with the experimentally obtained one.[7] On the other hand, it is difficult to analytically calculate the wave number in a regenerator. This is because flow channels in a regenerator are very complex.

In this study, the experimental method to evaluate the wave number in porous media is proposed, and the proposed method is applied to stacked-screen regenerators. The experimental results show that a stacked-screen regenerator can be modeled as an array of circular channels.

2 Theory

This section describes the theory for the experimental evaluation of the wave number in a stacked screen regenerator. In the theory, the transfer matrix method[8] is used and the "capillary-tube-based" theory[9] that models a stacked screen regenerator as an array of pores having uniform cross section is used.

When the temeprature takes some constant value along a flow channel, the momentum and continuity equations[6] are written as

$$\frac{dP}{dx} = -\frac{1}{A}\frac{i\omega\rho_m}{1-\chi_\upsilon}U \tag{1}$$

$$\frac{dU}{dx} = -\frac{i\omega A\left[1+(\gamma-1)\chi_\alpha\right]}{\gamma P_m}P, \tag{2}$$

where P is oscillatory pressure, U is volume velocity, ω is angular frequency, A is the cross-section of the flow channel, and ρ_m, P_m, T_m, γ, and σ are the mean density, the mean pressure, the mean temperature, the ratio of specific heats, and Prandtl number of a working gas, respectively. The functions χ_α and χ_ν in Eqs. (1) and (2) are the thermoacoustic functions.[4, 6, 5]

The thermoacoustic functions in a flow channel having a particular geometry can be analytically determined. In order to express the thermoacoustic functions, the two parameters, τ_α and τ_ν, are used.[5] τ_α is thermal relaxation time defined as

$$\tau_\alpha = r^2/(2\alpha), \tag{3}$$

and τ_ν is viscous relaxation time defined as

$$\tau_\nu = r^2/(2\nu), \tag{4}$$

where r is the characteristic length in the channel, α is thermal diffusivity, and ν is kinematic viscosity. For the case of circular channels, the radius of the circular can be used as r and the thermoacoustic functions χ_α and χ_ν are expressed as

$$\chi_\alpha = \frac{2J_1(Y_\alpha)}{Y_\alpha J_0(Y_\alpha)}, \quad \chi_\nu = \frac{2J_1(Y_\nu)}{Y_\nu J_0(Y_\nu)}, \tag{5}$$

where

$$Y_\alpha = (i-1)\sqrt{\omega\tau_\alpha}, \quad Y_\nu = (i-1)\sqrt{\omega\tau_\nu}. \tag{6}$$

J_1 and J_0 are the first and zeroth-order Bessel functions, respectively.

When the flow channel has a uniform cross section, Eqs. (1) and (2) can be solved analytically. The solution of them can be written in a matrix form as

$$\begin{pmatrix} P_0 \\ U_0 \end{pmatrix} = M(x_1, x_0)\begin{pmatrix} P_1 \\ U_1 \end{pmatrix} \tag{7}$$

$$M(x_1,x_0) \equiv \begin{pmatrix} \cos(k(x_1-x_0)) & \frac{-i\omega\rho_m \sin(k(x_1-x_0))}{Ak(1-\chi_\upsilon)} \\ \frac{Ak(1-\chi_\upsilon)\sin(k(x_1-x_0))}{i\omega\rho_m} & \cos(k(x_1-x_0)) \end{pmatrix}.$$

Here, P_j and U_j represents oscillatory pressure and volume velocity at x_j, respectively, and the complex wave number k is given as

$$k = \frac{\omega}{a}\sqrt{\frac{1+(\gamma-1)\chi_\alpha}{1-\chi_\nu}}, \tag{8}$$

where a is adiabatic sound speed.

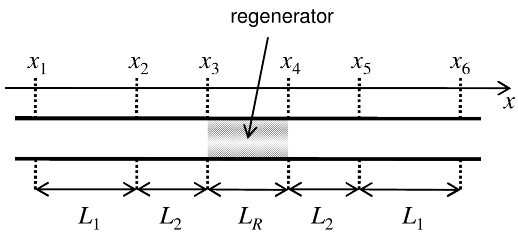

Figure 1: Coordinate system for the experimental evaluation of the wave number in the stacked-screen regenerator.

We consider the case that a regenerator of L_R in length is sandwiched between circular tubes as shown in Fig. 1. χ_ν and k in the circular tubes can be achieved from Eqs. (5) and (8), and they are denoted as $\chi_{\nu,T}$ and k_T, respectively. The axial coordinate x and the lengths L_1, L_2, and L_R are set as shown in Fig. 1.

By using Eq. (7), the pressure and volume velocity at $x = x_1$, (P_1, U_1), can be related to those at x_2, (P_2, U_2);

$$\begin{pmatrix} A_a & B_a \\ C_a & A_a \end{pmatrix} \begin{pmatrix} P_1 \\ U_1 \end{pmatrix} = \begin{pmatrix} P_2 \\ U_2 \end{pmatrix}, \qquad (9)$$

where

$$A_a = \cos k_T L_1 \qquad (10)$$
$$B_a = \frac{-i\omega\rho_m \sin k_T L_1}{k_T(1-\chi_{\nu T})} \qquad (11)$$
$$C_a = \frac{k_T(1-\chi_{\nu T}) \sin k_T L_1}{i\omega\rho_m}. \qquad (12)$$

Here, $L_1 = x_2 - x_1 (= x_6 - x_5)$. As the same as the relation between (P_1, U_1) and (P_2, U_2), pressure P_2 and volume velocity U_2 at $x = x_2$ can be related to pressure P_3 and volume velocity U_3 at one end of the regenerator, $x = x_3$, as

$$\begin{pmatrix} A_b & B_b \\ C_b & A_b \end{pmatrix} \begin{pmatrix} P_2 \\ U_2 \end{pmatrix} = \begin{pmatrix} P_3 \\ U_3 \end{pmatrix}, \qquad (13)$$

where

$$A_b = \cos k_T L_2 \qquad (14)$$
$$B_b = \frac{-i\omega\rho_m \sin k_T L_2}{k_T(1-\chi_{\nu T})} \qquad (15)$$
$$C_b = \frac{k_T(1-\chi_{\nu T}) \sin k_T L_2}{i\omega\rho_m}. \qquad (16)$$

Here, $L_2 = x_3 - x_2 (= x_5 - x_4)$. Note that the loss[10] introduced at the connecting point between the regenerator and the tube is neglected. Equation (9) yields

$$U_1 = (P_2 - A_a P_1)/B_a \qquad (17)$$
$$U_2 = C_a P_1 + A_a U_1. \qquad (18)$$

By using Eqs. (13), (17), and (18),

$$P_3 = A_b P_2 + B_b \left(C_a P_1 + \frac{A_a}{B_a}(P_2 - A_a P_1) \right) \qquad (19)$$
$$U_3 = C_b P_2 + A_b \left(C_a P_1 + \frac{A_a}{B_a}(P_2 - A_a P_1) \right) \qquad (20)$$

are obtained. By the same proceeding of achieving Eqs. (19) and (20), two equations can be obtained:

$$P_4 = A_b P_5 + B_b \left(C_a P_6 + \frac{A_a}{B_a}(P_5 - A_a P_6) \right) \qquad (21)$$
$$U_4 = -C_b P_5 - A_b \left(C_a P_6 + \frac{A_a}{B_a}(P_5 - A_a P_6) \right), \qquad (22)$$

where P_4 and U_4 are pressure and volume velocity at the other end of the regenerator, $x = x_4$.

Although the thermoacoustic function $\chi_{\nu,exp}$ and the wave number k_{exp} in the regenerator are still unknown, the relation between (P_3, U_3) and (P_4, U_4) can be written as

$$\begin{pmatrix} A_R & B_R \\ C_R & A_R \end{pmatrix} \begin{pmatrix} P_3 \\ U_3 \end{pmatrix} = \begin{pmatrix} P_4 \\ U_4 \end{pmatrix}, \qquad (23)$$

where

$$A_R = \cos k_{exp} L_R \qquad (24)$$
$$B_R = \frac{-i\omega\rho_m \sin k_{exp} L_R}{k_{exp}(1-\chi_{\nu,exp})} \qquad (25)$$
$$C_R = \frac{k_{exp}(1-\chi_{\nu,exp}) \sin k_{exp} L_R}{i\omega\rho_m}. \qquad (26)$$

This is because the regenerator is modeled as an array of pores having uniform cross section. Since the determinant of the matrix in Eq. (23) is one, i.e., $A_R^2 - B_R C_R = 1$,

$$\begin{pmatrix} A_R & -B_R \\ -C_R & A_R \end{pmatrix} \begin{pmatrix} P_4 \\ U_4 \end{pmatrix} = \begin{pmatrix} P_3 \\ U_3 \end{pmatrix}. \qquad (27)$$

Eq. (23) and Eq. (27) yield

$$A_R = \frac{P_3 U_3 + P_4 U_4}{P_3 U_4 + P_4 U_3} \qquad (28)$$
$$B_R = \frac{P_4^2 - P_3^2}{P_4 U_3 + P_3 U_4}. \qquad (29)$$

By using Eqs. (24) and (28),

$$k_{exp} = \left(\arccos \left(\frac{P_3 U_3 + P_4 U_4}{P_3 U_4 + P_4 U_3} \right) \right) / L_R \qquad (30)$$

can be obtained. The signs of the real and imaginary parts of k_{exp} are determined physically. Equation (30) indicates the following; when a stacked screen regenerator is sandwiched by the circular tubes in which χ_ν and k are analytically calculated, the pressure measurements at four positions allow us to evaluate the wave number in the regenerator. This is because (P_3, U_3) and (P_4, U_4) in Eq. (30) can be expressed by A_a, B_a, C_a, A_b, B_b, C_b, and the values of pressure at four positions, P_1, P_2, P_5, P_6. (see Eqs. (19)-(22.)

3 Experimental setup and procedure

The constructed experimental setup is shown in Fig. 2. The setup was composed of a loud speaker, branching resonator, and looped tube. The looped tube was used

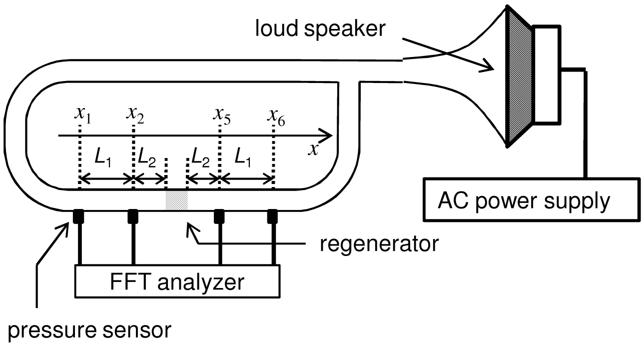

Figure 2: Experimental setup to determine the wave number and thermoacoustic function in the regenerator.

so as to make a phase difference between measured pressures large enough to be measured. A stacked screen regenerator of L_R=20 mm in length was inserted into the looped tube. Four pressure sensors were mounted on the wall of the looped tube. The distance between the positions of the mounted pressure sensors, L_1, (see Fig. 2) was set to 0.30 m and the distance of the mounted pressure sensor from the regenerator, L_2, (see Fig. 2) was 0.10 m. As a working gas, dry atmospheric air was used.

Sine signal was continuously supplied to the loud speaker by using an alternating-current power supply, so that pressure in the looped tube was oscillated. The pressure oscillation was measured with the pressure sensors at four positions x_1, x_2, x_5, and x_6 (see Fig. 2). The measured pressure signals were input to an FFT analyzer, and the amplitude and phase of P_1, P_2, P_5, and P_6 were achieved. By substituting the measured P_1, P_2, P_5, and P_6 into Eqs. (19) − (22), the pressure and volume velocity at both ends of the regenerator were calculated. By using the calculated pressure and volume velocity and Eq. (30), the wave number was evaluated. The measurements were performed as quickly as possible to avoid making the temperature gradient along a regenerator due to the thermoacoustic effect[3]. Moreover the measurements were done under the condition that the pressure amplitude is small enough to avoid nonlinear effects.

4 Experimental results

The experimental evaluation of k_{exp} in stacked screen regenerators was performed. The four types of regenerators were used. The geometrical properties of the regenerators are listed in Table 1. Note that f in this table will be described latter. The measurements were executed under some frequencies between 40 to 490 Hz, and they were done four times at a given frequency for each regenerator.

The experimentally obtained k_{exp} is shown as a function of $\omega\tau_\nu$ in Fig. 3, where k_{exp} is divided by $k_0 = a/\omega$. In order to calculate $\omega\tau_\nu$ in the regenerators, we defined half of the hydraulic diameter in a regenerator as r. In Fig. 3, the theoretically obtained wave number k_{theo} in a circular channel is also shown as a reference.

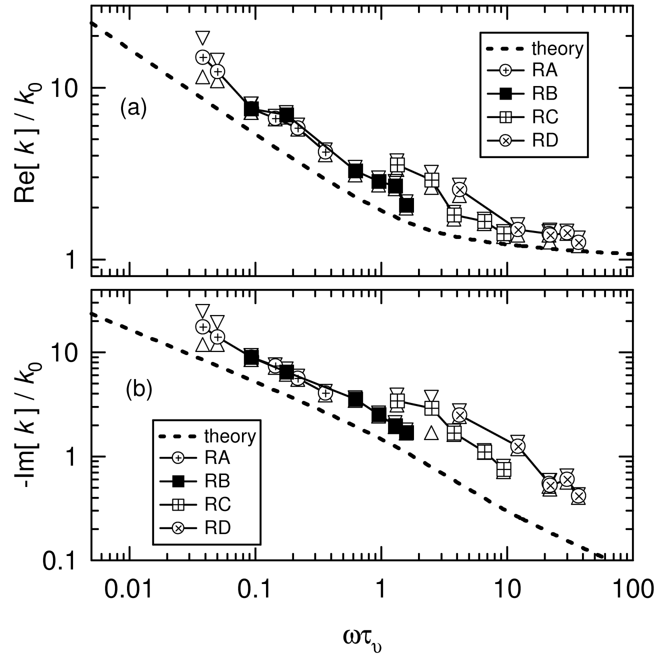

Figure 3: The experimetal results of the wave number k_{exp} in the stacked screen regenerators. The wave number k_{exp} is shown as a function of $\omega\tau_\nu$

Table 1: Geometrical properties of the stacked screen regenerators. OPF and HD mean "Open Frontal Area" and "Hydraulic Diameter", respectively.

Type	RA	RB	RC	RD
Mesh No.	#200	#80	#40	#24
Wire dia. (mm)	0.04	0.10	0.14	0.22
OPF (%)	47	47	61	63
Porosity (%)	76	73	83	86
HD $2r$ (mm)	0.13	0.26	0.65	1.23
f	2.6	2.4	4.8	6.8

As shown in Fig. 3, the $\omega\tau_\nu$ dependence of k_{exp} in the stacked screen regenerators is similar to that of k_{theo}. However, Re[k_{exp}] and -Im[k_{exp}] were always larger than Re[k_{theo}] and -Im[k_{theo}], respectively, and at a given value of $\omega\tau_\nu$, the value of k_{exp}/k_0 depends on the type of the regenerators. Therefore, it can be said that the wave number in the stacked screen regenerators depends not only $\omega\tau_\nu$ but also the types of the regenerators.

In order to evaluate an effective radius r_{eff} in a stacked screen regenerator, a new function depending on the type of regenerators, f, is introduced and r_{eff} is denoted as

$$r_{eff} = r/\sqrt{f}, \tag{31}$$

i.e.,

$$\omega\tau'_\nu = \omega r_{eff}^2/(2\nu) = \omega\tau_\nu/f. \tag{32}$$

By using $\omega\tau'_\nu$, we conform the experimentally obtained k_{exp} to k_{theo} of a circular channel case. For each type of the regenerators listed in Table 1, f was determined so that $k_{exp}(\omega\tau'_\nu)$ is fitted to $k_{theo}(\omega\tau'_\nu)$ of a circular channel as close as possible. For the theoretical case of a circular channel, f is one, i.e. $\omega\tau'_\nu = \omega\tau_\nu$.

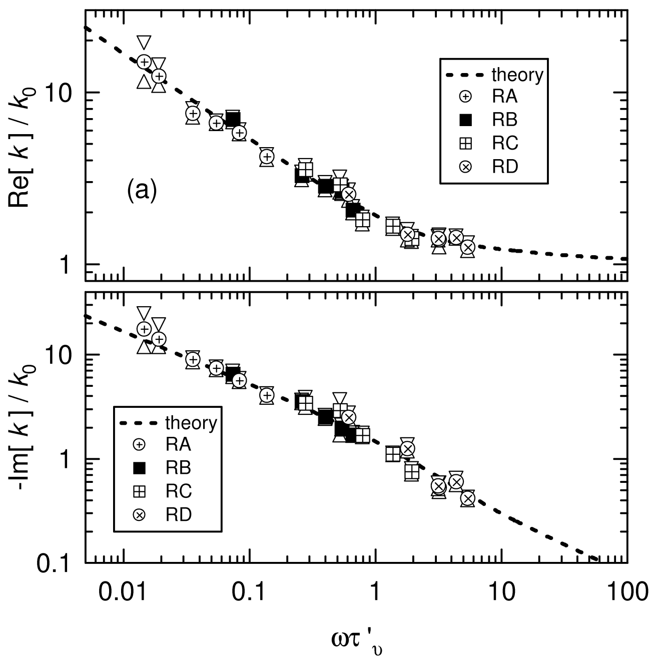

Figure 4: The experimental results of the wave number k_{exp} in the stacked screen regenerators. The wave number k_{exp} is shown as a function of the modified $\omega\tau'_\nu (= \omega\tau_\nu/f)$

The obtained values of f are shown in Table 1 and $k_{exp}(\omega\tau'_\nu)$ is shown in Fig. 4. This figure shows that the real and imaginary parts of the experimentally obtained wave number k_{exp} agree well with those of k_{theo} by using f, respectively. Based on the agreement between the experimentally and theoretically obtained k, I consider that stacked screen regenerators having complex flow channels can be modeled as an array of circular channels whose radius is r_{eff} calculated from Eq. (31).

5 Summary

The experimental method to evaluate the wave number in porous material, in which geometry of flow channels is unknown, is shown. The method was applied to the stacked screen regenerators. The experimental results show that the regenerators can be modeled as an array of circular channels by using the experimentally obtained function f depending on the type of the regenerator.

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