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Particles Filter applied in the real-time bearings-only tracking problem of a sonar target

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The development of a passive sonar tracking system in real-time needs algorithms each time more accurate and that can be able to follow rapid changes in the signal characteristics. In a bearings-only tracking problem, the bearing model is a nonlinear function of the target states. Nowadays, methods that deal with nonlinear and non-Gaussian estimation are receiving great attention. Particles filter is one of these methods. Particles filters are Sequential Monte Carlo methods that represent the required probability density function as a set of random samples. This present paper describes the particles filter application in the bearings-only tracking problem. Simulated and real data of targets are displayed in energy versus bearing graphics obtained in an operational naval environment. These data were used with the Sampling Importance Resampling algorithm (SIR). The particles filter is formulated in Cartesian coordinates and then the result is transformed to modified polar coordinates. Results are compared with Extended Kalman Filter (EKF) and the effectiveness and limitations of SIR algorithm for tracking targets using a bearing/time record are examined.

1 Introduction

The problem of tracking a target using a history of noisy bearing measurements requires nonlinear estimation methods due to the fact that bearings are nonlinear function of the parameters of location. This problem is also referred as bearings-only tracking (BOT) or target motion analysis (TMA) [1, 2]. From the many solutions proposed to this problem, the more used are based on pseudo-linear formulations and on the Kalman filtering [3]. These methods are based on the assumption that the noise is gaussian, which is not often the case in the underwater environment, causing their performance to degrade rapidly.

With the advent of modern computers, traditional statistical methods have taken a new direction. Several methods that use intensive computation have been presented in the literature for the treatment of non-gaussianity and nonlinearity, and can be used, too, for the treatment of problems of recursive nature and that require solutions in real time. The particles filter (PF), whose principal idea is the use of Monte Carlo simulation to implement bayesian recursive filters, is one of these methods. The advantage of this method is its ability to estimate any statistical needs with few assumptions and can be applied to automated processes in many different situations [4, 5, 6, 7].

The objective of this paper is to present a study of the particles filter in the underwater target tracking problem using bearings only. The work is organized as follows: Section 2 shows the formulation of the target tracking problem, Section 3 shows the methodology of particles filter, Section 4, its application to data obtained in simulated and real operational naval environment, and Section 5, the conclusions and future works.

2 Problem Formulation

Geometric configuration for the target motion estimation problem is shown in Figure 1, where (x_p, y_p) and (x_A, y_A) are the coordinates of the platform and the target respectively, B is the bearing of the target to the platform, v_{xA} and v_{yA} are the components of the velocity of the target, α_p is the angle between the north and the heading of the platform and r is the distance between the platform and the target. Consider that platform and target are in the XY plane and that the target moves in a constant velocity and on a fixed heading. The platform has its trajectory consisting of some "legs", i.e., segments with the same heading and velocity.

The components of the target position when the target moves in constant velocity (constant heading and velocity) during a time interval and in the same plane, can be given by the following equations:

$$
\begin{aligned}
x_A(t_i) &= x_A(t_o) + \Delta t_i v_{xA} \\
y_A(t_i) &= y_A(t_o) + \Delta t_i v_{yA}
\end{aligned}
$$

where $\Delta t_i = t_i - t_o$. The equation that associates positions of platform and target to obtain the real bearings of the target to the platform, B, is:

$$tgB(t_i) = \frac{x_A(t_i) - x_P(t_i)}{y_A(t_i) - y_P(t_i)}$$

The noisy bearing measurement of the target is:

$$\beta(t_i) = B(t_i) + w(t_i)$$

where $w(t_i)$ is the noise.

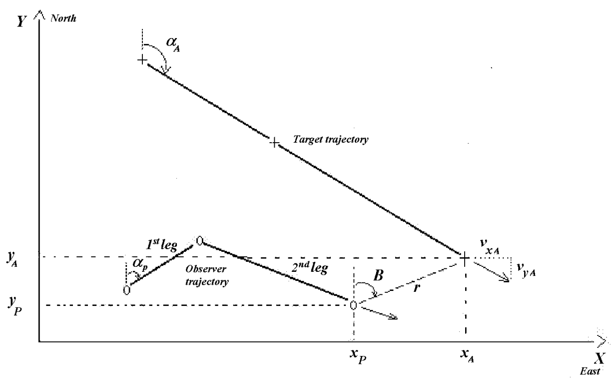

Fig. 1 –General scenario

The aim of the target estimation problem is to calculate the target trajectory using noise corrupted bearing measurements from a single observer. In real naval applications, target's course, speed and range are the desired estimated parameters. Apparently this problem seems to be simple but the bearings-only problem is not easy to solve because the problem is inherently nonlinear.

3 Methodology

3.1 Bayesian Approach

In a general way, the tracking problem can be defined considering that the evolution of the state sequence, $\{x_t \in \Re^n\}$, of a target can be represented by

$$x_t = f_t(x_{t-1}, u_{t-1}) \quad (1)$$

where $f_t : \Re^n X \Re^m \to \Re^n$ is the transition system evolution function and $u_t \in \Re^r$ is a sequence of zero mean white noise, independent of past and current states and with known probability density function (pdf). The measures, which are obtained over time, are related to the state vector equation through a measurement equation

$$\beta_t = h_t(x_t, w_t) \quad (2)$$

where $h_t : \Re^n X \Re^r \to \Re^p$ is the function of the measures and $w_t \in \Re^r$ is another sequence of white noise with zero mean, independent of the past states, and of the system noise and with known pdf. It is considered that the pdf of the initial state and the functions f_t and h_t are known.

In a bayesian approach, the target tracking problem calculates the degree of belief of the current state given the measures obtained at the moment [9]. The pdf of the current states, given all the information possible, can be obtained recursively in two stages: prediction and update. On the prediction stage, the system model is used to obtain the prior pdf of the state in the next time step by

$$p(x_t|\beta_{1:t-1}) = \int p(x_t|x_{t-1})p(x_{t-1}|\beta_{1:t-1})dx_{t-1} \quad (3)$$

considering that the pdf of that time is known. The probabilistic model of the development of the state, $p(x_t|x_{t-1})$, which is a Markov model of the first order, is defined by the system equation and the known statistics of u_{t-1} i. e., $p(x_t|x_{t-1}) = \int p(x_t|x_{t-1}, u_{t-1})p(u_{t-1}|x_{t-1})du_{t-1}$

Supposing $p(u_{t-1}|x_{t-1}) = p(u_{t-1})$, we have

$$p(x_t|x_{t-1}) = \int \delta(x_t - f_{t-1}(x_{t-1}, u_{t-1}))p(u_{t-1})du_{t-1}$$

where $\delta(\bullet)$ is the Dirac function.

The update stage involves the measurement model in the following way: given the measurement, the prior density can be updated by the Bayes rules

$$p(x_t|\beta_1,\ldots,\beta_t) \equiv p(x_t|\beta_{1:t}) = \frac{p(\beta_t|x_t)p(x_t|\beta_{1:t-1})}{p(\beta_t|\beta_{1:t-1})} \quad (4)$$

where

$$p(\beta_t|\beta_{1:t-1}) = \int p(\beta_t|x_t)p(x_t|\beta_{1:t-1})dx_t \quad (5)$$

depends on the conditional pdf of data, i.e., the probability function, defined for the model of the measurements and the known statistics of w_t,

$$p(\beta_t|x_t) = \int \delta(\beta_t - h_t(x_t, w_t))p(w_t)dw_t$$

In the update of the equation (4), the measure β_t is used to modify the forecast of the prior density from the previous instant to get the desired posterior density of the current state. The equations Eq.(3) and Eq.(5) constitute the formal solution for the bayesian problem of recursive estimation [8]. The integral of the equation (5) does not have a closed analytical form. For many problems the solution of this integral is very difficult. To avoid this obstacle, methods as the Kalman filters consider that the models of the system and of the measurements are linear and/or gaussian and that the noise process is also gaussian, with known variances and additives. However, in some problems these considerations are not valid.

3.2 Particle Filters

Methods that use intensive computation are based on the Monte Carlo simulation (MC) to estimate the statistics of interest, recalculating its value by using artificial samples that are obtained by modifying the real samples. The main idea of these methods is to represent the probability density function of the state vector by a set of weighted random samples that are updated and propagated by the algorithm. The method provides an accurate representation and equivalent pdf when the number of samples is large. The method has the advantage of being applied to any type of state or measurements. The fact of the stage of updating the algorithm (bayesian rule) being implemented as a bootstrap weighted means the method is considered also a bootstrap filter [5, 10]. The bootstrap method idea is to use resampling to obtain additional artificial samples and then to extract information from these samples to enhance inference. These samples are concentrated in areas of high density of probability.

There are some types of particles filters: based on MCMC, acceptance-rejection, and Importance Sampling. In this work, we will present the Sequential Importance Sampling algorithm - SIS, which is the basis of some particles filters developed until the moment, and the Sampling Importance Resampling algorithm – SIR.

3.2.1 SIS Algorithm

The principal idea of this algorithm is to represent the posterior density for a set of weighted random samples (q_t) and to estimate the parameters of interest based in these samples and weights. Let $\{x_{0:t}^i, q_t^i : i = 1,\ldots,N_S\}$ be a sample measurement set characterizing the posterior density $p(x_{0:t} \mid \beta_{1:t})$ where $\{x_{0:t}^i, i = 0,\ldots,N_S\}$ is a set of auxiliary points with weights $\{q_t^i, i = 1,\ldots,N_S\}$ and $x_{0:t} = \{x_j, j = 0,\ldots,t\}$ is the set of all states until time t. The weights are normalized, $\sum_i q_t^i = 1$. The posterior density in time t can be approximated by $p(x_{0:t}|\beta_{1:t}) \approx \sum_{i=1}^{N_S} q_t^i \delta(x_{0:t} - x_{0:t}^i)$.

So we have an approximation of the discrete weighted true posterior $p(x_{0:t}|\beta_{1:t})$. In this case, the weights are chosen by using the importance sampling. The importance sampling technique consists in modifying the density $p(x)$, $p_m(x)$, in which the events related to the desired parameter occur more frequently [5, 6]. Consider $x^i \sim p_m(x)$, $i = 1,\ldots,N_S$ generated samples of the modified density. An approximation of the weighted density is given by $p(x) \approx \sum q^i \delta(x - x^i)$ where $q^i \propto \frac{p(x^i)}{p_m(x^i)}$ is the normalized weight of the particles.

The weights using sample $x_{0:t}^i$ obtained by modified density $p_m(x_{0:t}|\beta_{1:t})$ are $q_t^i = \frac{p(x_{0:t}^i|\beta_{1:t})}{p_m(x_{0:t}^i|\beta_{1:t})}$.

In each iteration of the sequential case we will have samples constituting an approximation for $p_m(x_{0:t-1}|\beta_{1:t-1})$

and want to approximate $p(x_{0:t} \mid \beta_{1:t})$ with a new samples set. If the modified density is chosen such that

$$p_m(x_{o:t} \mid \beta_{1:t}) = p_m(x_t \mid x_{0:t-1}, \beta_{1:t-1}) p_m(x_{0:t-1} \mid \beta_{1:t-1})$$

then the samples $p_m(x_{o:t} \mid \beta_{1:t})$ can be obtained increasing each existing sample $x_{0:t-1}^j \sim p_m(x_{0:t-1} \mid \beta_{1:t-1})$ with a new state $x_t^j \sim p_m(x_t \mid x_{0:t-1}, \beta_{1:t-1})$. The weight equation is the following one: $q_t^j = q_{t-1}^i \dfrac{p(\beta_t \mid x_t^i) p(x_t^i \mid x_{t-1}^i)}{p_m(x_t^i \mid x_{0:t-1}^i, \beta_{1:t})}$

Considering $p_m(x_t \mid x_{0:t-1}, \beta_{1:t}) = p_m(x_t \mid x_{t-1}, \beta_t)$ we have modified the density dependent only on x_{t-1} and β_t. So, the modified weights are $q_t^j \propto q_{t-1}^i \dfrac{p(\beta_t \mid x_t^i) p(x_t^i \mid x_{t-1}^i)}{p_m(x_t^i \mid x_{0:t-1}^i, \beta_{1:t})}$ and the filtered posterior density $p(x_t \mid \beta_{1:t})$ can be approximate by $p(x_t \mid \beta_{1:t}) \approx \sum_t q_t^i \delta(x_t - x_t^i)$.

A problem that frequently occurs is the degeneration, i.e., the accentuated reduction of the weights of all the particles. A severe degeneration occurs when size of effective sample is less than or equal to N_S, i.e., $N_{ef} \le N_S$ and N_{ef} is small. The effect of this degeneration can be reduced if N_S is too large, which is often impractical in certain situations. To avoid this problem you can use two alternatives: a) a good choice of the modified density and b) the use of resampling. With the advance in computation, resampling has become a major cooperator for solving such problems. The basic idea of resampling is to eliminate particles that have small weights and concentrate on larger weight particles [7].

3.2.2 SIR algorithm

The SIR algorithm utilizes the resampling in each time interval. The algorithm stages are described as following:

Initialization

(i) Generate N_S samples $x_o^i \sim p(x_o)$, $i = 1, \ldots, N_S$.

(ii) Calculate weights $q_o^i = p(x_o^i)$.

(iii) Normalize the weights $q_o^i = q_0^i / \sum_{j=1}^{N_S} q_0^j$.

Update

(iv) Generate N_S samples $u_t^i \sim p(u_t)$

(v) Update samples by Eq. 1 where x_{t-1}^i and u_{t-1} are obtained in stages (i) and (iv) respectively

(vi) Update the weights, $q_t^i = q_{t-1}^i p(\beta_t \mid x_t^i)$, where $p(\beta \mid x_t^i)$ is deduced by Eq.2.

(vii) Normalize the weights, $q_t^i = q_t^i / \sum_{j=1}^{N_S} q_t^j$

Resample

(viii) Make $c_1 = 0$

(ix) Generate $N_S - 1$ points c_i, $i = 2, \ldots, N_S$ ...: $c_i = c_{i-1} + 1/N_S$, $i = 2, \ldots, N_S$

(x) Initialize i=1.

(xi) Generate initial point: $u_1 \sim U(0, 1/N_S)$.

(xii) Generate a set of N_S samples in the following format

- For each $j = 1, \ldots, N_S$.
- Make $u_j = u_1 + (j+1)/N_S$
- If $u_j > c_i$, make $i = i + 1$
- Make $x_t^{j^*} = x_t^i$, $q_t^j = 1/N_S$, $i^{j^*} = i$.

4 Applications and Results

In this section the application of the SIR particles filter is presented to the target tracking problem using simulated and real sonar data. The data are bearing measurements of a target obtained with a passive sonar system. It is considered that the target moves in the XY plane and adopts the following state and measurement models:

$$x_t = Fx_{t-1} + Gu_{t-1} \text{ and } \beta_t = B_t + w_t \qquad t = 1, \ldots, N_S \quad (6)$$

where

$$F = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad G = \begin{bmatrix} T^2/2 & 0 \\ T & 0 \\ 0 & T^2/2 \\ 0 & T \end{bmatrix}$$

and T is the sampling interval. The system and measurement noise are zero mean gaussian white noise process with covariance matrix and variance equals $\mathbf{Q}$ and R, respectively. The initial target state is assumed gaussian distributed with known mean $\bar{x}$ and covariance $\mathbf{P}$.

There are several different formulations and coordinates systems to model the measurement process and the state dynamics. Frequently, Cartesian coordinates are used to formulate bearings-only estimation problems. Also, modified polar coordinates are used in bearings-only problem because the unobservable component (range) is not coupled with the observable components [1]. The modified polar state vector consist of the following components: bearing, bearing rate, range rate divided by range and the reciprocal of range. In this paper, we consider that the data, in modified polar coordinates, are transformed to cartesian coordinates to implement the target tracking algorithm and then the results are transformed back to modified polar coordinate.

The transformation of modified polar into cartesian coordinates [1] is given by

$$\begin{bmatrix} r_{xt} \\ v_{xt} \\ r_{yt} \\ v_{yt} \end{bmatrix} = \frac{1}{1/r_t} \begin{bmatrix} sin\beta_t \\ (\dot{r}_t / r_t) sin\beta_t + \dot{\beta}_t \cos\beta_t \\ \cos\beta_t \\ (\dot{r}_t / r_t)\cos\beta_t - \dot{\beta}_t sin\beta_t \end{bmatrix}$$

where v_{xt}, v_{yt}, r_{xt} and r_{yt} are the relative velocity and position of target in x and y coordinates in time t, $\dot{\beta}_t$, $\dot{r}_t / r_t$, β_t and $1/r_t$ are the bearing rate, range rate divided by range, bearing and reciprocal of range of target in time t. The transformation of cartesian into modified polar coordinates is given by

$$\begin{bmatrix} \dot{\beta}_t \\ \dot{r}_t / r_t \\ \beta_t \\ 1/r_t \end{bmatrix} = \begin{bmatrix} (v_{xt} r_{yt} - v_{yt} r_{xt}) / (r_{xt}^2 + r_{yt}^2) \\ (v_{xt} r_{xt} + v_{yt} r_{yt}) / (r_{xt}^2 + r_{yt}^2) \\ \tan^{-1}(r_{xt} / r_{yt}) \\ 1/\sqrt{r_{xt}^2 + r_{yt}^2} \end{bmatrix}$$

Applications of EKF and PF are utilized to obtain an estimate of the bearing, course, speed and range of the target.

We consider that when the observer begins tracking the target it is at position $(0,0)$. The performance of the target motion estimation depends on the initial target state estimate. The choice of an initial target state is an important task. The initial range is set with tactical and underwater considerations [11]. The observer motion can help the quality of track performance. Besides of a good initial target state estimate, the initial error covariance matrix must also be determined. It is important to reflect as best as possible the errors in the target state estimate.

4.1 Simulated Data

The simulated data used were obtained from Gordon scenario [5]. The initial position of the observer is at the origin and its heading direction is north. The initial trajectory of the target is on a smooth curve. The bearings utilized to estimate target trajectory are the measurements obtained through the model $\beta_t = \tan^{-1}(r_{xt} / r_{yt})$, where r_x and r_y are relative target positions. These bearings were applied in the algorithms SIR and EKF. The initial state was set $\bar{x} = [-0.1 \quad 0 \quad 1.4 \quad 0.05]$. SIR algorithm was applied with $N = 4000$. The others parameters were set $R = 2.5x10^{-3}$, $Q = diag(10^{-6})$. Figure 2 presents errors of EKF and SIR estimates of the modified polar coordinates. These results demonstrate that the EKF errors is highest than SIR. Figure 3 presents SIR and EKF estimates of bearing, course, speed and range. The SIR algorithm performance is better than EKF.

4.2 Real Sonar Data

In this case, the data were obtained from real bearings of a submarine passive sonar in an operational naval environment. Figure 2 shows the energy that arrives at the sonar sensors in all directions. The top graph shows the energy of 360 degrees bearings on the current time and the graph below shows the energy versus bearings over time in an interval of colors in which most dark represents highest levels of energy. The target that we would like to track is on a trajectory with constant heading and speed. It is observed that there are other sources of noise besides the desired target. The energy from these sources contributes to mask the energy of the desired signal of the target.

Two data set are presented in this work to apply the algorithms: without observer's maneuvers and with one observer's maneuver. Raw bearings have been preprocessed before the SIR and EKF algorithm. In the first set, the target is traveling at constant speed of 5.5 m/s and course of 10° with 3000 meters of distance to the observer in the beginning of the tracking. The observer is with 2.5 m/s and course of 0°. Figure 4 presents the true bearings obtained from sonar sensors. For this set, we have 150 bearings taken at *T=2s*. The algorithm was implemented using SIR with $N = 20000$. The parameters of initialization are the same for both algorithm. The initial value of the state vector is $[-200 \quad 1 \quad -3500 \quad 3]$. The measurement noise variance is $R = 2x10^{-2}$ and the system noise covariance is $Q = diag(10^{-2})$. Results of bearings, course, speed and range are presented in Figure 5. Even though the implementation of EKF is faster than SIR, the results demonstrate that the SIR performance is better than EKF.

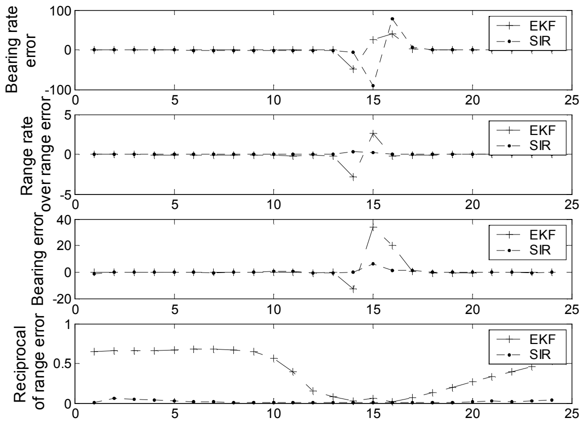

Fig. 2 – Simulated data - SIR and EKF estimates errors of modified polar coordinates.

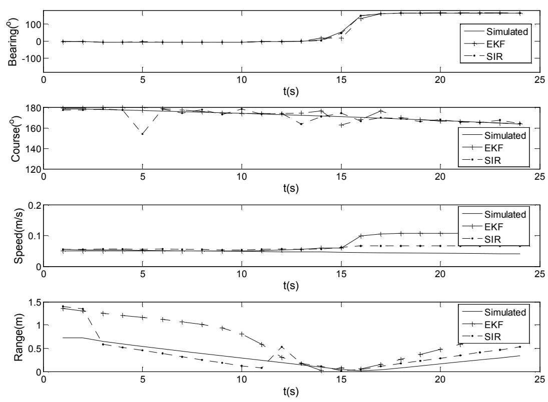

Fig. 3 – Simulated data: SIR and EKF estimates of bearing, course, speed and range.

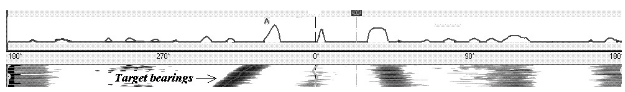

Fig. 4 – Bearing/Time records: first data set.

In the second set, the target is travelling at a constant speed of 5.5 m/s and course of 10° with 14000 meters of distance to the observer in the beginning of the tracking. The observer starts with course of 110° and change to 63° and velocity of 2.5 m/s. Figure 6 presents the true bearings obtained from sonar and Figure 7 shows the bearings. For this set, we have 450 bearings. The algorithm was implemented using SIR with $N = 20000$. The initial value of the state vector is $[-2000 \quad 1 \quad -10500 \quad 4]$. Results of bearings, course, speed and range are presented in Figure 6. For this data set, EKF was unable to respond to the target's trajectory variation as the SIR algorithm. The performance of SIR algorithm is again better than EKF.

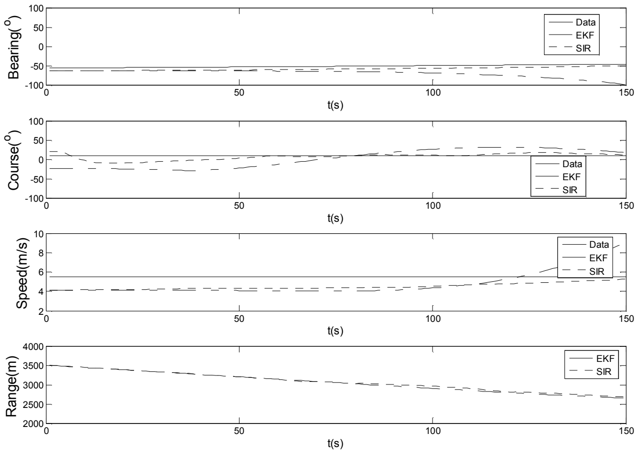

Fig. 5 – First data set – SIR and EKF estimates.

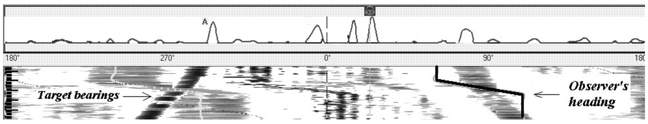

Fig.6 - Bearing/Time record – second data set.

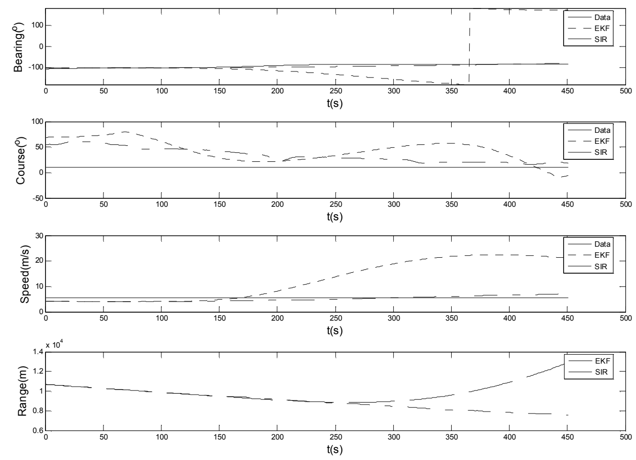

Fig.7 - Second data set – SIR and EKF estimates.

5 Conclusion

This paper presents the application of the particles filter in the problem of tracking underwater targets using real bearings of passive sonar in the presence of noise. All bearing measurements were obtained by a single moving observer which makes the problem more complex.

SIR algorithm was implemented and applied to one simulated and two real scenarios. The results had shown the viability of the PF in the target tracking problem and that it outperformed the EKF filter for the same problem.

Future works must consider maneuvers of the target comparing the results with the EKF. Others PF algorithms should be applied in order to compare its computational cost with that of the implementation of SIR. The use of other distributions for the system noise instead of the gaussian distribution used in this work and the investigation of the initialization of the PF algorithm should also be addressed in future works.

Acknowledgments

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