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Effects of enclosure design on the directivity synthesis by spherical loudspeaker arrays

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Spherical loudspeaker arrays have been used to generate non-uniform directivity patterns. It is known that the poor radiation efficiency of spherical sources and the loudspeaker electroacoustic behavior impose constraints on the directivity synthesis at low frequencies, which are aggravated as the source volume is made smaller. In this work, the effects of the enclosure design on the loudspeaker signal powers are analyzed. Two different approaches have been reported in literature, although quantitative comparisons have not been provided. In the first approach, the drivers share the same enclosure volume and in the second, they have their own independent sealed cavities. Here, an analytical model that takes into account the interior and exterior acoustic coupling is used in order to evaluate the voltages that must feed the array drivers. It is shown that the signal powers can be reduced at low frequencies by letting the drivers share the same enclosure volume. However, this leads to controllability problems, since some natural frequencies of the enclosure are in the operation range of the spherical array. If controllability at natural frequencies is neglected, a simple lumped parameter model of the enclosure presents good agreement with the continuous model, indicating that heavy calculations may be unnecessary.

1 Introduction

A spherical loudspeaker array is an electroacoustic device with several drivers mounted on a sphere-like structure. Convex regular polyhedra (Platonic solids) are most commonly used due to their high symmetry. Unlike omnidirectional sources, each driver is fed independently in order to achieve non-uniform directivity patterns.

These arrays have been developed mainly to provide electroacoustic music composers with spatialization tools and to reproduce the directivity of musical instruments, but other applications can be contemplated, as the measurement of room impulse responses for auralization purposes [1] and the information diffusion in privileged adjustable directions. Some remarkable works can be found in [2, 3, 4, 5]. Although analysis and synthesis methods could be found in these works, there is still a lack of quantitative results concerning especially the voltages that must be applied to the drivers in order to achieve a given directivity pattern.

It is known that synthesized patterns by compact loudspeaker arrays become less accurate as frequency increases. This can be dealt with by reducing the size of the loudspeaker array [2]. However, the poor radiation efficiency of spherical sources and the loudspeaker electroacoustic behavior impose severe constraints on the directivity synthesis at low frequencies, which are aggravated as the source volume is made smaller. The low frequency performance of a spherical source can be improved by working on the enclosure design. Two different approaches have been reported in literature [1, 4, 6], although quantitative comparisons have not been provided. In the first approach, the drivers share the same enclosure volume and in the second, they have their own independent sealed cavities.

Spherical harmonic functions constitute a natural basis for representation of sound source directivities, since they emerge from the solution of the Helmholtz equation in spherical coordinates. Therefore, the control strategy generally adopted is to provide the spherical array with some preprogrammed basic directivities corresponding to spherical harmonic patterns. Then, different directivities can be achieved simply by changing the gains associated with the basic directivities, so that it is not necessary to redesign the filters when a different target pattern is desired.

In this work, the voltages that must feed the drivers of a spherical source in order to produce spherical harmonic patterns in the far-field are evaluated analytically. A continuous model and a lumped parameter model are used for this task. The driver signals corresponding to the two different enclosure designs mentioned above are compared. The sound fields radiated by the array are evaluated analytically, according to the multipole source model presented in [4].

2 Analysis and synthesis of radiation patterns

2.1 Spherical array model

A spherical loudspeaker array can be modeled as a set of vibrating spherical caps mounted on a rigid sphere. This model is briefly described here. The reader will find more details in [4].

Sound propagation of spherical sources is suitably described in the frequency domain by the Helmholtz equation in spherical coordinates. Since this equation is linear, the sound field produced by each cap can be evaluated separately and the total field obtained by superposition.

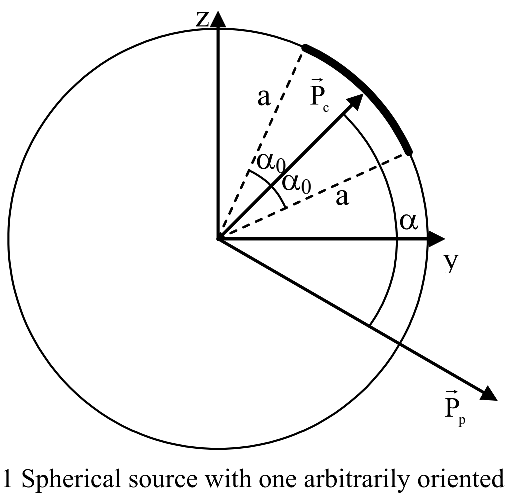

Fig.1 Spherical source with one arbitrarily oriented cap.

Figure 1 illustrates a spherical source with only one arbitrarily oriented cap. $\vec{P}_c(x, y, z)$ is the position of the center of the cap (note that it defines a symmetry axis),

$\vec{P}_p(x,y,z)$ is the position of the point in which the sound pressure is evaluated, α is its angular coordinate, α_0 is the angle that defines the cap size and a is the sphere radius.

Now, let the cap oscillate with a constant radial velocity over its surface. Since this problem has a symmetry axis, the following boundary value problem governs the sound radiation:

$$\text{(I)}\quad \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial p(r,\alpha)}{\partial r}\right)+\frac{1}{r^2\sin\alpha}\frac{\partial}{\partial\alpha}\left(\sin\alpha\frac{\partial p(r,\alpha)}{\partial\alpha}\right)+k^2p(r,\alpha)=0\ , r\geq a, 0\leq\alpha\leq\pi$$

$$\frac{\partial p(a,\alpha)}{\partial r}=j\omega\rho_0\begin{cases}U_0 & ,0\leq\alpha\leq\alpha_0\\ 0 & ,\alpha_0<\alpha\leq\pi\end{cases}$$

where p is the complex sound pressure amplitude, k is the wave number, r is the radial coordinate, $j=\sqrt{-1}$, ω is the angular frequency, ρ_0 is the equilibrium density of the medium and U_0 is the amplitude of the radial velocity of the cap.

Problem (I) can be solved analytically by the method of separation of variables, so that the sound radiated in a free-field by the cap is (this solution was obtained by assuming a temporal dependence given by $e^{-j\omega t}$)

$$p(ka,r/a,\alpha_0,\alpha)=\sum_{n=0}^{\infty}A_n(ka,\alpha_0)\,h_n^{(1)}(kr)\,P_n(\cos\alpha) \qquad (1)$$

where $h_n^{(1)}$ is the spherical Hankel function of the first kind and order n, P_n is the Legendre polynomial of degree n and A_n is given by

$$A_n(ka,\alpha_0)=j\rho_0c_0\frac{U_n(ka,\alpha_0)}{\left.\frac{dh_n^{(1)}(kr)}{d(kr)}\right|_{kr=ka}} \qquad (2)$$

where c_0 is the sound speed and U_n is

$$U_n(ka,\alpha_0)=\frac{1}{2}U_0(ka)\,\left(P_{n-1}(\cos\alpha_0)-P_{n+1}(\cos\alpha_0)\right) \qquad (3)$$

Note that U_0 can be made frequency dependent and observe that k, a and α_0 are constants in problem (I), but the authors have preferred to write them explicitly in Eqs.(1) – (3) in order to emphasize the main non-dimensional parameters involved in the spherical cap design: ka, r/a and α_0.

Similarly, the interior sound propagation is governed by the following boundary value problem:

$$\text{(II)}\quad \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial p(r,\alpha)}{\partial r}\right)+\frac{1}{r^2\sin\alpha}\frac{\partial}{\partial\alpha}\left(\sin\alpha\frac{\partial p(r,\alpha)}{\partial\alpha}\right)+k^2p(r,\alpha)=0\ , r\leq a, 0\leq\alpha\leq\pi$$

$$\frac{\partial p(a,\alpha)}{\partial r}=j\omega\rho_0\begin{cases}U_0 & ,0\leq\alpha\leq\alpha_0\\ 0 & ,\alpha_0<\alpha\leq\pi\end{cases}$$

Problem (II) can also be solved analytically by the method of separation of variables, so that the inner field is

$$p(ka,r/a,\alpha_0,\alpha)=\sum_{n=0}^{\infty}D_n(ka,\alpha_0)\,j_n(kr)\,P_n(\cos\alpha) \qquad (4)$$

where j_n is the spherical Bessel function of order n and D_n is

$$D_n(ka,\alpha_0)=j\rho_0c_0\frac{U_n(ka,\alpha_0)}{\left.\frac{dj_n(kr)}{d(kr)}\right|_{kr=ka}} \qquad (5)$$

2.2 Optimization

The functions considered in the present work are defined on a spherical surface. These functions are sampled so that $\theta_t = t\Delta\theta$ and $\varphi_m = m\Delta\varphi$; where θ is the elevation angle, φ is the azimuth angle, t = 0, 1, 2 … T-1 and m = 0, 1, 2 … M-1. Then, s = TM is the number of samples, $\Delta\theta = 180^0/(T-1)$ and $\Delta\varphi = 360^0/M$. The meshed spherical surface motivates the use of the following inner product:

$$\langle u,v\rangle = v^H W u \qquad (6)$$

where u, $v \in \mathbb{C}^s$ and $W\in\mathbb{R}_+^{s\,x\,s}$. W is diagonal and it contains non-dimensional area weight factors that are determined by surface integration over appropriate sections of the sphere. Thus, the diagonal terms of W, w_i, are given by

$$w_i=\begin{cases}\dfrac{1}{4\pi}\displaystyle\int_{\varphi_m-\Delta\varphi/2}^{\varphi_m+\Delta\varphi/2}\int_0^{\Delta\theta/2}\sin(\theta)d\theta\, d\varphi=\frac{1}{M}\sin^2\left(\frac{\Delta\theta}{4}\right), & \text{if } t=0 \text{ or } t=T-1\\[2ex] \dfrac{1}{4\pi}\displaystyle\int_{\varphi_m-\Delta\varphi/2}^{\varphi_m+\Delta\varphi/2}\int_{\theta_t-\Delta\theta/2}^{\theta_t+\Delta\theta/2}\sin(\theta)d\theta\, d\varphi=\frac{1}{M}\sin(\theta_t)\sin\left(\frac{\Delta\theta}{2}\right), & \text{if } 1\leq t\leq T-2\end{cases} \qquad (7)$$

where i = mT + t + 1.

In this work, the spherical array capability in reproducing spherical harmonic functions is studied. These functions constitute a natural basis for directivity representation since they emerge from the solution of the Helmholtz equation in spherical coordinates. A spherical harmonic function of degree n and order m is given by the following equation [7]:

$$Y_n^m(\theta,\varphi)=(-1)^m\sqrt{\frac{(2n+1)}{4\pi}\frac{(n-m)!}{(n+m)!}}e^{jm\varphi}P_n^m(\cos\theta) \qquad (8)$$

where $-n \leq m \leq n$, $m \geq 0$ and P_n^m is an associated Legendre function of the first kind of degree n and order m. Spherical harmonics so defined are orthonormal functions.

It is known that function spaces spanned by spherical harmonics of the same degree are linear subspaces that are invariant with respect to rigid rotation through spatial angles [8]. For example, if a given pattern is in the subspace generated only by harmonics of degree 3, any rotation of this pattern also possesses a spherical harmonic expansion consisting only of harmonics of degree 3. Then, if $b_n \in \mathbb{C}^s$ contains samples of a function in the subspace generated by spherical harmonics of degree n, it can be expressed as $b_n = B_n c_n$, where $c_n \in \mathbb{C}^{2n+1}$ contains coefficients and $B_n \in \mathbb{C}^{s\,x\,2n+1}$ is a matrix whose columns contain spherical harmonics of degree n and orders from –n to n.

Now, let $A(ka, r/a, \alpha_0) \in \mathbb{C}^{s\,x\,L}$ have the directivities of the L drivers of the spherical array as columns that are obtained by letting $U_0 = 1$ in Eq.(3), and $x(ka, r/a, \alpha_0) \in \mathbb{C}^L$ contain driver velocities, i.e., U_0 values for each driver. So, due to the superposition principle, the following optimization problem can be formulated, which must be solved for each ka for a given array geometry:

$$\text{(II)}\qquad \min_x \| Ax - B_n c_n \|_2$$

This is a well-known convex optimization problem (least-squares) whose solution is given by [9]

$$x^* = (A^H W A)^{-1} A^H W B_n c_n \quad (9)$$

Now, let $c_n^H c_n = 1$ and $X_n^*(ka, r/a, \alpha_0) \in \mathbb{C}^{L \times 2n+1}$ have the optimal cap velocities associated with the 2n+1 columns of B_n. Then, maximum and minimum singular values of $W^{1/2}(AX_n^* - B_n)$ provide, respectively, upper and lower error bounds associated with the subspace spanned by spherical harmonics of degree n. The directivity patterns associated with such bounds can be determined by examining the right-singular vectors obtained in the singular value decomposition [3].

3 Electrodynamical loudspeaker model

In order to evaluate the electrical tensions that must feed the drivers so that they achieve optimum velocities given by Eq.(9), an electroacoustic model of the source is used.

Let many drivers share the same enclosure. Application of second Newton's law and Fourier transform for the n-th driver yields to

$$Z_{mn} V_n + \sum_{l=1}^{L} \left(\phi_{nl}^{(ext)} - \phi_{nl}^{(int)}\right) S_n S_l V_l = F_{en} \quad (10)$$

where Z_{mn} is the mechanical impedance of the n-th driver, V_i is the diaphragm velocity of the i-th driver, L is the number of drivers, S_i is the effective area of the i-th driver and F_{en} is the Lorenz force. $\phi_{nl}^{(int)}$ and $\phi_{nl}^{(ext)}$ contain, respectively, interior and exterior acoustic coupling between drivers and they are given by

$$\phi_{nl}^{(ext)} = \frac{P_{nl}^{(ext)}}{S_l V_l} \quad (11)$$

and

$$\phi_{nl}^{(int)} = \frac{P_{nl}^{(int)}}{S_l V_l} \quad (12)$$

where $P_{nl}^{(ext)}$ and $P_{nl}^{(int)}$ are, respectively, the external and internal effective pressure acting on the n-th driver due to the movement of the l-th driver.

F_{en} can be evaluated by Eq.(13).

$$F_{en} = \frac{b_n l_n}{R_{en}} e_{gn} - Z_{men} V_n \quad (13)$$

where b_n is the magnetic flux density in the n-th driver air gap, l_n is the length of the n-th voice-coil conductor in magnetic field, R_{en} is the electrical resistance of the n-th voice-coil, e_{gn} is the tension of the n-th electrical source and $Z_{men} = (b_n l_n)^2 \div R_{en}$ is the mechanical equivalent impedance of R_{en}.

Finally, substitution of Eq. (13) in (10) yields to

$$(Z_{mn} + Z_{men}) V_n + \sum_{l=1}^{L} \left(\phi_{nl}^{(ext)} - \phi_{nl}^{(int)}\right) S_n S_l V_l = \frac{b_n l_n}{R_{en}} e_{gn} \quad (14)$$

Then, voltages that must feed the drivers so that the spherical array could reproduce a given pattern are obtained by letting $V = x^*$.

$\phi_{nl}^{(ext)}$ and $\phi_{nl}^{(int)}$ can be obtained by letting kr = ka in Eqs.(1) and (4), respectively, so that the sound pressure can be evaluated at the inner and outer surface of the sphere. On the other hand, at low frequencies, a simple lumped parameter model can be used to calculate the internal coupling between drivers by letting [10]

$$\phi_{nl}^{(int)} = \frac{1}{j\omega C_{Bnl}} \quad (15)$$

where C_{Bnl} is the acoustic compliance of the enclosure. If V_B is the total enclosure volume, then C_{Bnl} is given by [10]

$$C_{Bnl} = \frac{V_B}{\rho_0 c_0^2} \quad (16)$$

In addition, if the external acoustic coupling is ignored, $\phi_{nl}^{(ext)}$ can be approximated by the radiation impedance of a piston mounted on an infinite baffle, which is [11]

$$\phi_{nl}^{(ext)} = \frac{\rho_0 c_0}{\pi a_{pn}^2} \left[1 - \frac{J_1(2ka_{pn})}{ka_{pn}} + j\frac{H_1(2ka_{pn})}{ka_{pn}}\right] \delta_{nl} \quad (17)$$

where a_{pn} is the effective radius of the n-th driver diaphragm, J_1 is the Bessel function of first kind and order 1, H_1 is the Struve function of order 1 and δ_{nl} is the Kronecker delta.

If each driver has its own sealed enclosure, Eq. (14) still holds, but the enclosure acoustic stiffness ($1/C_{Bnl}$) must be evaluated according to Eq.(18) instead of Eq.(16).

$$\frac{1}{C_{Bnl}} = \frac{\rho_0 c_0^2}{V_{Bn}} \delta_{nl} \quad (18)$$

where V_{Bn} is the volume of the n-th enclosure.

4 Results

Here, some simulation results are presented in order to illustrate the effects of the enclosure design on the voltage that must feed the drivers of an icosahedral array in order to achieve some directivity patterns. Two different approaches were considered. In the first one, the drivers share the same enclosure volume and in the second, they have their own independent sealed cavities.

The following values were used in the simulations: L = 20 (icosahedral source), s = 79x40 = 3160 mesh points, c_0 = 343m/s, ρ_0 = 1.21kg/m^3, $\alpha_0 = 20^0$ and r/a = 20; where r is the radius of the sphere on which the sound pressure is evaluated. The Legendre polynomial series presented in Eq.(1) was truncated to 10 terms in order to evaluate the radiated fields of the spherical source at r. However, 30 terms were retained in Eqs.(1) and (4) in order to compute the sound pressure on the spherical caps.

All drivers were supposed to be equal with the following characteristics: 3" drivers, resonance frequency 80Hz, mechanical quality factor 8.0, electrical quality factor 0.73, total moving mass 0.0043kg, R_{en} = 7.6Ω and S_n = 0.0031m^2. Since α_0 = 20°, S_n = 0.0031m^2 and r/a = 20, the radius of the spherical source and evaluation surface are a = 0.0918 m and r = 1.84 m, respectively. Eq.(8) was multiplied by a constant in order to provide the target directivity patterns with a constant sound pressure level of

80dB (ref. 20μPa) at a distance r from the array center in the main radiation lobe. The signals were evaluated from ka = 0.05 to ka = 6, i.e., from f = 29.7 Hz to f = 3.6 kHz.

Figure 1 shows the root mean square error (RMSE) that arise in reproducing functions in spherical harmonic subspaces up to degree 3 by an icosahedral source. Theses curves were obtained as described in the end of session 2.2. Only one curve was plotted for n = 0, 1 and 2 for clarity, since computations have shown that upper and lower bounds for each one of these subspaces are not distinguishable. It can be verified that synthesized basic patterns become less accurate as ka and n increase.

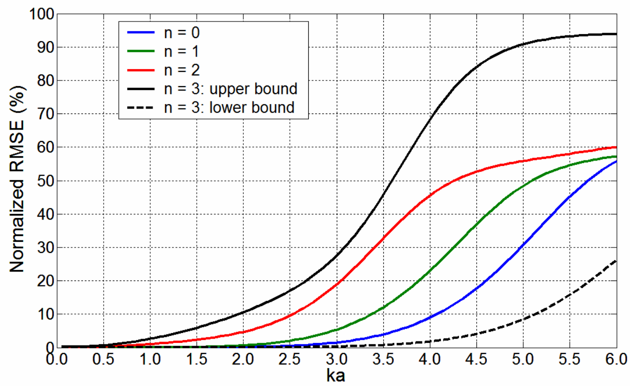

Fig.1 Normalized RMSE arisen in reproducing functions in the subspace spanned by spherical harmonics of degree n by an icosahedral source.

Figures 2 to 5 show voltage magnitude that must feed the drivers in order to synthesize spherical harmonic functions. A comparison between drivers sharing the same enclosure ("common enclosure" curves) and drivers with its own sealed enclosure ("own enclosure" curve) is provided. For the common enclosure design, two different curves are presented according to the method used to evaluate $\phi_{nl}^{(ext)}$ and $\phi_{nl}^{(int)}$. They were calculated by using Eqs.(1) and (4) (continuous model) and by using Eqs.(15) and (17) (lumped model). The vertical red lines indicate the lowest natural frequencies of the spherical cavity.

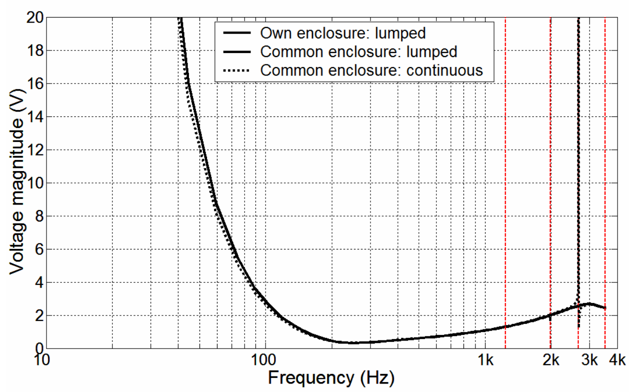

Fig.2 Voltage magnitude that must feed the drivers in order to synthesize the spherical harmonic of degree 0 and order 0 (monopole)

If the drivers share the same enclosure, Figs.2-5 show that there are no remarkable differences between the voltages evaluated by the lumped and the continuous model, except for frequencies corresponding to the natural frequencies of the spherical cavity, in which controllability problems can be identified by the continuous model. The lowest 4 natural frequencies of a rigid spherical cavity correspond to the following ka values: 2.0816, 3.3421, 4.4934 and 5.9404. Then, it is suggested that driver voltages can be evaluated by using such a simple lumped parameter model for the enclosure and by neglecting external interaction between the drivers, so that heavy calculations involved in the continuous modeling may be unnecessary.

Due to the fact that all drivers are equal, the symmetry of the icosahedron and the uniformity of the directivity pattern, all drivers must be feed by the same voltage in order to produce an omnidirectional sound field. In addition, voltages evaluated by the lumped model remain unchanged whether the drivers have their own sealed cavities or share the same enclosure, as can be verified in Fig.2.

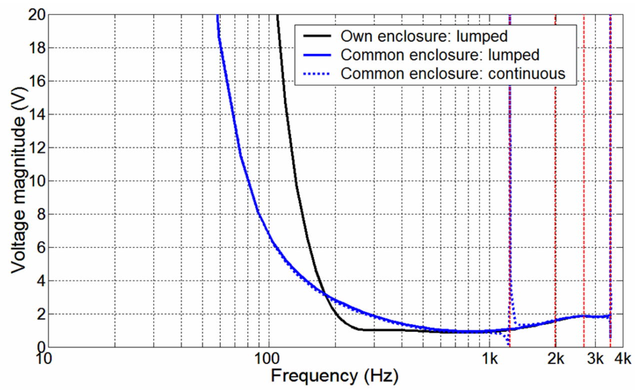

Fig.3 Voltage magnitude that must feed the most solicited drivers in order to synthesize the spherical harmonic of degree 1 and order 0 (dipole).

Figures 3 to 5 indicate that there is a single frequency value in the low-frequency range at which the "common enclosure" and "own enclosure" approaches lead to the same voltage magnitude. Lower voltages are achieved by the "common enclosure" design at frequencies lower than this value. Otherwise, "own enclosure" design presents some improvements. The transition frequency can be adjusted by changing the driver characteristics. However, for the icosahedral source considered in this work, only dipole synthesis can take advantage of the improvements provided by the common enclosure approach, since unachievable voltages are necessary to produce spherical harmonics other than monopole and dipoles at low frequencies.

High voltages must be applied to the drivers at low frequencies due to their electroacoustic behavior and the poor radiation efficiency of spherical sources at low ka values, which implies high driver velocities. One can deal with these constraints by using larger drivers, i.e., by increasing the source radius. This provides lower driver velocities and high enclosure compliance. However, for a given frequency, the error associated with directivity synthesis is increased as can be verified in Fig.1, so that the operation range of the array is reduced. Then, a spherical source design is a compromise between high and low frequency accuracy.

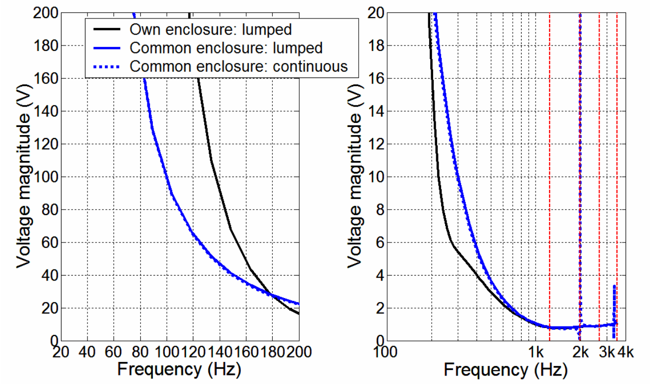

Fig.4 Voltage magnitude that must feed the most solicited drivers in order to synthesize the spherical harmonic of degree 2 and order 0 (quadrupole).

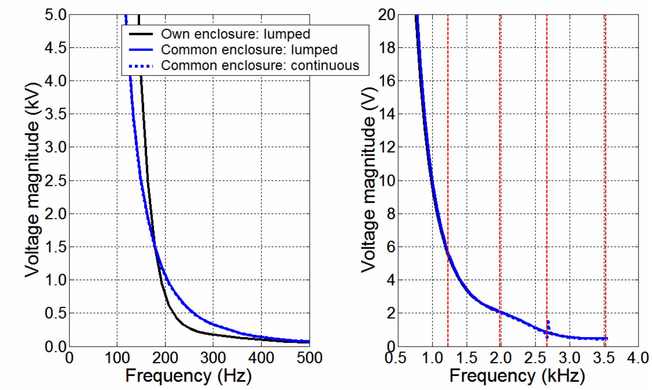

Fig.5 Voltage magnitude that must feed the most solicited drivers in order to synthesize the spherical harmonic of degree 3 and order 0.

5 Conclusion

In this work, the voltages that must feed the drivers of an icosahedral source in order to produce spherical harmonic patterns were evaluated and the effect of the enclosure design on the signals was investigated. In addition, a continuous model and a lumped one were used to evaluate the acoustic interaction between drivers that share the same enclosure and a comparison was provided.

The voltages are greatly affected by the source enclosure design, except for the monopole synthesis. At low frequencies, it is possible to obtain weaker power signals by letting the drivers share the same enclosure volume, instead of provide each one of them with its own independent sealed cavity. However, letting the drivers share the same enclosure leads to controllability problems, since some natural frequencies of the spherical cavity are in the operation range of the spherical array. It is expected that this can be dealt with by adding absorbing materials in the cavity.

No remarkable differences were observed between the voltages evaluated by the lumped and the continuous model, except for frequencies corresponding to the natural frequencies of the spherical cavity. This suggests that driver voltages can be evaluated by using a simple lumped parameter model for the enclosure and by neglecting external interaction between the drivers, so that heavy calculations involved in the continuous modeling may be unnecessary.

Acknowledgments

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