# Methods to investigate the possibilities of using a three element periodic structure to suppress the transmission of energy in an elastic tube

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**Abstract**. The present investigation concern the effect of three masses attached periodically to a pipe of small diameter. The pipe is small enough that it can be treated as a beam. The masses are eccentric to the center of the beam, to achieve a large change in the moment of inertia by the added elements. A shaker is used to excite the pipe, which is suspended elastically. Two dimensions of pipe have been investigated: a flat 3 x 30 x 100 mm aluminum beam with blocks of 1100 g, and a circular ø32 x ø28 x 4000mm steel beam with blocks of 4000 g.

The insertion loss formed by the differences in level with and without masses appears to be an appropriate way to describe the phenomena for comparison between theory and experiment. Experimentally the insertion loss is expressed by the difference between the acceleration levels. Theoretically the energy levels are compared.

The theoretical model is formulated as a system of boundary equations, which describe propagation of flexural, axial and torsion waves within each segment of a tube between periodic elements, and continuity conditions at the points, where masses are attached. The presence of masses couples propagation of waves of all these types. An exact solution of this system is obtained and it is found that appropriate locations of three identical equally spaced masses can dramatically decrease the power input into the system in some frequency 'stop bands' regardless the excitation conditions.

# 1 INTRODUCTION

In industrial applications, such as pump and compressor systems, pipe vibrations can exceed an acceptable level. This is influenced by the vibration source strength of the compressor or pump, and to a high degree also on the dynamic characteristics of the piping system. Especially for systems, where varying speed of rotation is applied, eigenfrequencies in piping system are likely to be excited in certain speed ranges. It is desirable to use passive and simple design elements to suppress this propagation of energy to parts of the piping system where high vibration levels are harmful or annoying.

Solutions to such problems require that measurements and analyses are made in situ, and that solutions can be provided preferably without dismantling the piping system. Periodically spaced masses added to the piping are an appropriate solution, and investigations to develop valid prediction models, and measurement techniques has been initiated in the Danish Makunet network.

# 2 THEORETICAL MODEL

The energy transmission in straight elastic beams bearing concentrated masses can conveniently be described within the framework of boundary integral equations method. In the general case of spatial vibrations, these equations should be formulated for the longitudinal waves, for torsion waves and for flexural waves. These waves propagate independently upon each other in a homogeneous beam, but they interact in a compound beam of spatial configuration or in a beam bearing inclusions.

$$u(\xi)=\left[EAu'(x)U(x,\xi)-EAu(x)\frac{\partial U(x,\xi)}{\partial x}\right]\Bigg|_{x=0}^{x=l}$$

$$\varphi(\xi)=\left[GI_t\varphi'(x)\Phi(x,\xi)-GI_t\varphi(x)\frac{\partial \Phi(x,\xi)}{\partial x}\right]\Bigg|_{x=0}^{x=l}$$

$$v(\xi)=\left[EI_z v'''(x)V(x,\xi)-EI_z v(x)\frac{\partial^3 V(x,\xi)}{\partial x^3}-EI_z v''(x)\frac{\partial V(x,\xi)}{\partial x}+EI_z v'(x)\frac{\partial^2 V(x,\xi)}{\partial x^2}\right]\Bigg|_{x=0}^{x=l}$$

$$\frac{dv(\xi)}{d\xi}=\left[EI_z v'''(x)\frac{\partial V(x,\xi)}{\partial \xi}-EI_z v(x)\frac{\partial^4 V(x,\xi)}{\partial x^3\partial \xi}-EI_z v''(x)\frac{\partial^2 V(x,\xi)}{\partial x\partial \xi}+EI_z v'(x)\frac{\partial^3 V(x,\xi)}{\partial x^2\partial \xi}\right]\Bigg|_{x=0}^{x=l}$$

$$w(\xi)=\left[EI_y w'''(x)W(x,\xi)-EI_y w(x)\frac{\partial^3 W(x,\xi)}{\partial x^3}-EI_y w''(x)\frac{\partial W(x,\xi)}{\partial x}+EI_y w'(x)\frac{\partial^2 W(x,\xi)}{\partial x^2}\right]\Bigg|_{x=0}^{x=l}$$

$$\frac{dw(\xi)}{d\xi}=\left[EI_y w'''(x)\frac{\partial W(x,\xi)}{\partial \xi}-EI_y w(x)\frac{\partial^4 W(x,\xi)}{\partial x^3\partial \xi}-EI_y w''(x)\frac{\partial^2 W(x,\xi)}{\partial x\partial \xi}+EI_y w'(x)\frac{\partial^3 W(x,\xi)}{\partial x^2\partial \xi}\right]\Bigg|_{x=0}^{x=l}$$

Here $U(x,\xi)$, $\Phi(x,\xi)$, $V(x,\xi)$ and $W(x,\xi)$ are Green's functions, which describe the shape of forced vibrations of an infinitely long beam excited at a given frequency by a unit concentrated axial force, torque or transverse force, respectively. These equations are written for each segment of compound structure at the edges, $\xi=0+\varepsilon$ and $\xi=l-\varepsilon$, $\varepsilon\to 0$. They are solved with continuity conditions at the interfaces between segments of a beam and appropriate boundary conditions.

The methodology of boundary equations is equally applicable for analysis of standing waves in structures of a finite length and for analysis of the energy transmission in unbounded structures. In the latter case, the total power flow contains four components,

$$N_\Sigma = N_U + N_\varphi + N_V + N_W .$$

They present energies transported by axial, torsion and flexural waves:

$$N_U=\frac{1}{2}EA\omega\,\mathrm{Re}\left[u'\cdot\overline{iu}\right],$$

$$N_\varphi=\frac{1}{2}EI_t\omega\,\mathrm{Re}\left[\varphi'\cdot\overline{i\varphi}\right]$$

$$N_V=\frac{1}{2}EI_z\omega\,\mathrm{Re}\left[v''\cdot\overline{iv'}-v'''\cdot\overline{iv}\right],$$

$$N_W=\frac{1}{2}EI_y\omega\,\mathrm{Re}\left[w''\cdot\overline{iw'}-w'''\cdot\overline{iw}\right]$$

The presence of equally spaced inertial inclusions generates the band gap effect in an infinite structure bearing an infinitely large number of these inclusions (a periodic structure) for all power flow components. The practical issue explored in this paper is an assessment of a possibility to reduce the energy transmission by means of three inertial inclusions.

Theoretical predictions of eigenfrequencies have been made and compared to experimentally obtained values. The results are seen in section 3.3

# 3 STRUCTURE INVESTIGATED

## 3.1 Structure

The layout of the structures, which are analyzed theoretically and experimentally, is presented in Figure 1 and 2.

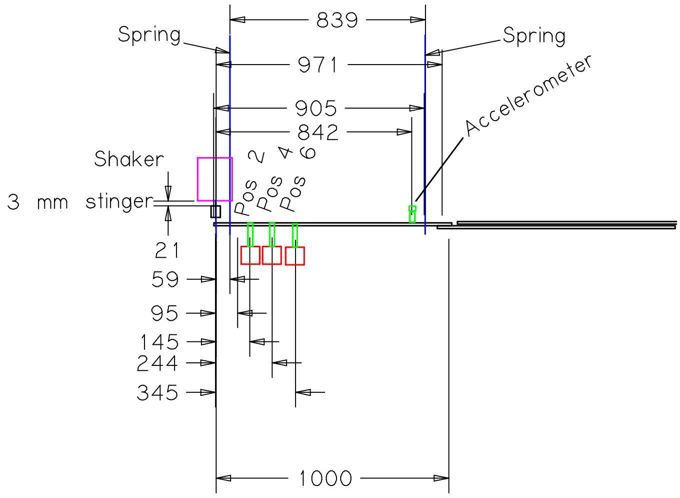


Figure 1. Analyzed structure: 1000 x 30 x 3 mm Aluminum beam

Figure 1 shows the flat Aluminum beam of length x width x thickness: 1000 x 30 x 3 mm. It is suspended in springs to two points, one in each end of the beam. The excitation of the waves is provided by an electrodynamic shaker supplied with white noise. To achieve damping of the propagating waves, a sandwich damped beam of the dimensions 1000 * 30* 7 mm is attached at the end of the test beam. Three masses are located with equal distance of 50 mm as shown in Figure 1. The masses are eccentric to the neutral axis to achieve a large change in moment of inertia.

The purpose of using the flat beam was to provide movements in one (vertical) plane to simplify the modes of vibration and thus also the analysis and calculations.

The second structure that was tested is shown in figure 2.

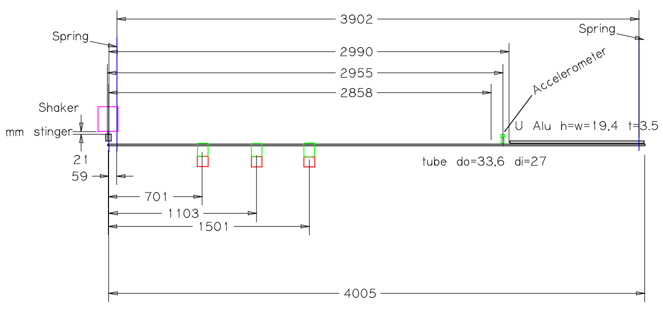


Figure 2. Pipe structure tested including periodic elements

The second structure consists of a steel pipe: 4000 x do 33.6 x di 27 mm. Damping is provided by a sandwich structure of 1000 mm length, using u shaped aluminium profiles and viscoelastic damping material. Distance between periodic elements: 400 mm.

The effect of the periodic elements is assumed to be related to half a wavelength of the beam. The first iteration of the frequencies to expect a consequence of the periodic elements is shown in figure 3.

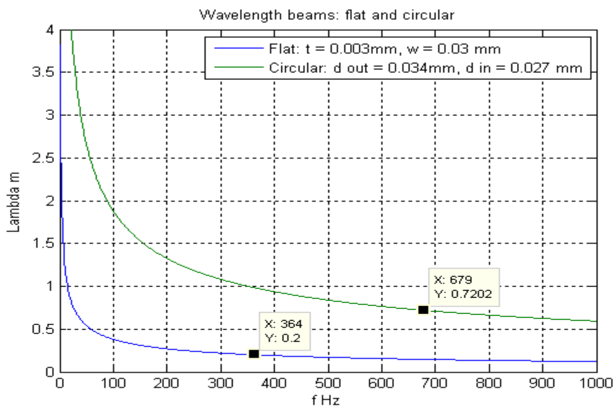


Figure 3. Wavelength of bending waves in the tested beams.

It is seen that the two beams are tested at different bending wave conditions, with frequencies of the half wavelength between periodic elements of 364 Hz and 679 Hz respectively.

## 3.2 Effect of damping

The effect of damping has been considered important for the efficiency of the periodic elements in relation to propagating waves. Application of the sandwich damping structure as seen in figure 2 is seen in figure 4. The damping effect is seen to start above 100 Hz, and become very efficient above 500 Hz. This implies that waves of frequencies higher that 500 Hz is expected not to be reflected. Note the factor of 10 reduction in amplitude.

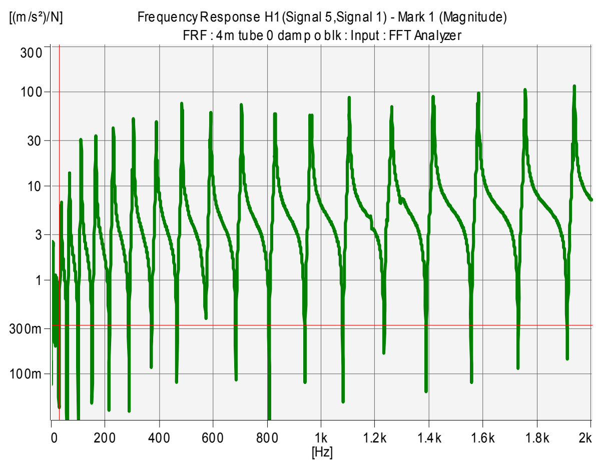

Figure 4. Frequency response of pipe with no periodic elements without damping.

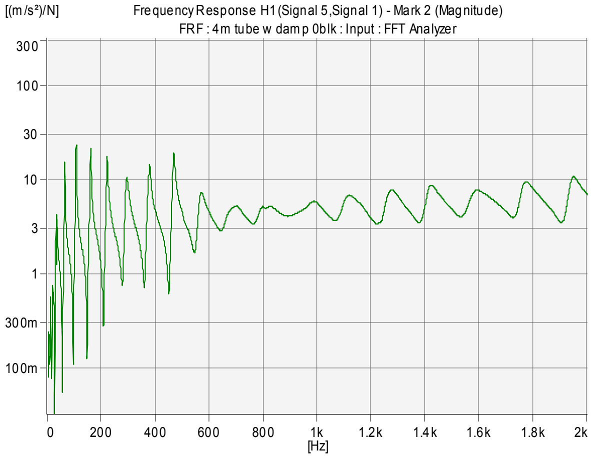


Figure 5 Frequency response of pipe with no periodic elements with damping.

## 3.3 Eigenfrequencies verification with theory

Calculations are done for the model specified in the previous section of the paper. As discussed, it is sufficient to consider only flexural motion in a vertical plane. As a validity check, eigenfrequencies of a freely suspended beam composed of four segments in the absence of attached masses were calculated by use of the methodology of boundary equations. The obtained results were identical to those given by an elementary formula for eigenfrequencies of a uniform free-free beam.

The boundary element method has been applied to the eigenfrequencies of the flat beam, with the result seen in figure 5. The predicted eigenfrequencies compare well to the predicted ones.

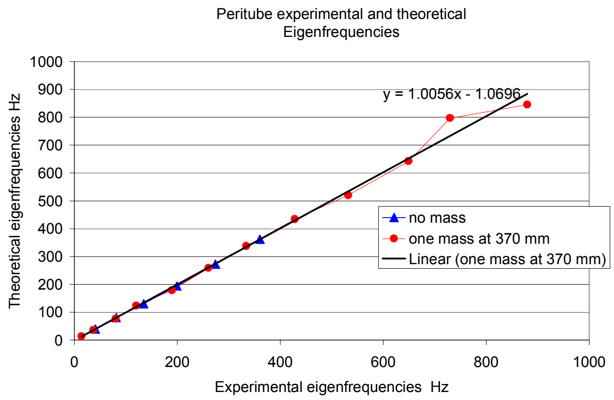


Figure 5. Experimentally and theoretically obtained eigenfrequencies of flat beam.

## 3.4 Results

A method of practical use in industrial noise control is the use of the insertion loss. It is in this case applied as the quotient between the acceleration level with and without periodic elements. The acceleration is measured in two positions in three directions, and the average is formed between all three directions in the two positions. Because of the dominant excitation in the vertical plane, see figure 2, the major acceleration levels are seen in the vertical plane. See the acceleration in the vertical direction with and without 3 blocks in figure 8.

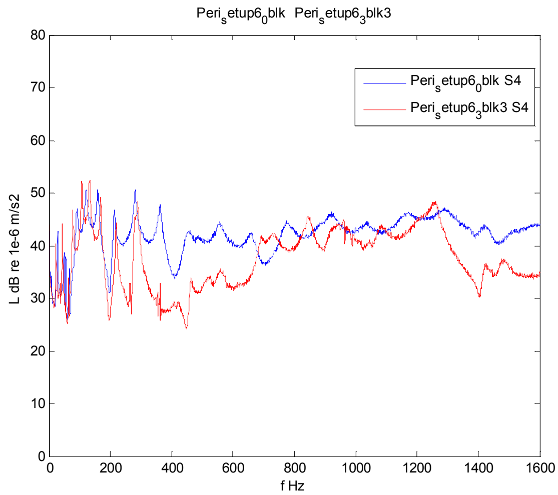


Figure 8. Acceleration in dB re 1e-6 m/s$^2$. B: no blocks, r: 3 blocks

The insertion loss for the circular pipe is seen in figure 9. Note that 0 dB in the vertical scale means no reduction. Positive response means reduction.

The insertion loss shows a significant reduction in a broad range of frequencies from 100 to 660 Hz. This is a very satisfactory result, because of the implication of this method of reduction of vibrations in pipe systems.

The insertion loss of a pipe with 1, 2 and 3 blocks is seen in figure 10. A significant effect is seen with 2 and 3 blocks. Note that the peaks in the frequency region below 500 Hz do not express eigenfrequencies, but are rather differences in the acceleration spectra when the insertion loss is determined. A shift in the eigenfrequencies when the masses are applied result in large differences in the insertion loss at frequencies different from the eigenfrequencies.

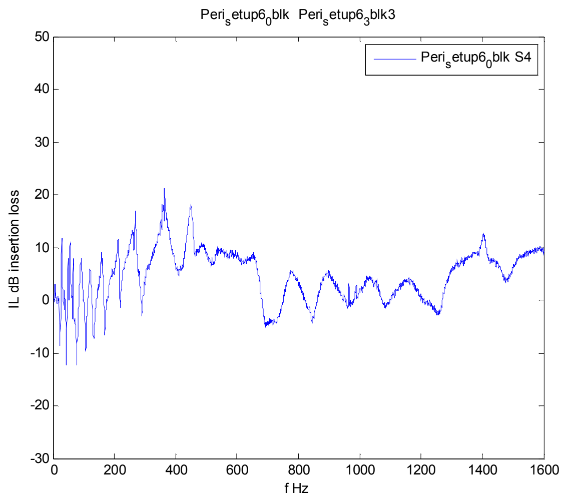


Figure 9. Insertion loss for circular pipe with three periodic elements

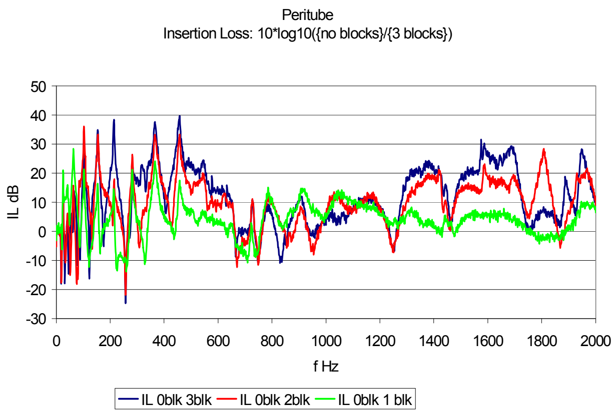


Figure 10. Experimental insertion loss, b: 3 bloks, r: 2 blocks, g: 1 block

The mass of the periodic masses is of importance for the insertion loss, see figure 11. A threshold appears to be present for the effect to be significant. This will be investigated further.

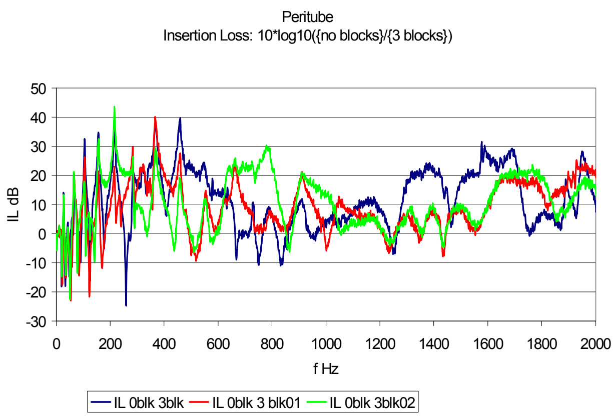


Figure 11. Experimental insertion loss with three masses vs. mass, b: heavy, g: medium, r: light

Comparison of the insertion loss calculated and determined experimentally is seen for a pipe with three masses in figure 12. I general the prediction shows the significant trends in the frequency regions 400 to 640 Hz and 900 – 1600 Hz.

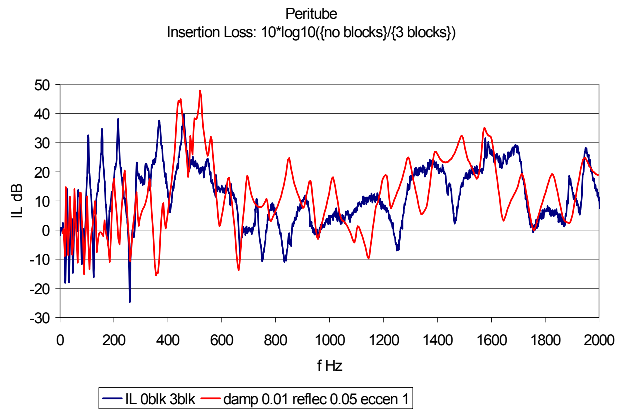


Figure 11. Comparison insertion loss. b: experiment, r: theory

## 4 CONCLUSIONS

The method of reduction of pipe vibrations by means of 3 periodic elements with a high eccentricity has been applied to two types of beams: a flat and a circular beam. The method is proven experimentally to be efficient, and the formation of stop bands is recognized.

The presently developed boundary equation method has proven efficient to predict the eigenfrequencies. Further development of an engineering tool for prediction of periodic elements in vibration reduction in pipes is encouraged by these results.

The significant effects are shown when comparing theory with experiments. Next steps are to improve the theoretical of the experimental boundary conditions and then a parametric study of the effects of size of masses, stiffness of the beams for the excentric masses, and eventual an industrial application.

## REFERENCES

S. V. Sorokin, O.A. Ershova Analysis of the energy transmission in compound cylindrical shells with and without internal heavy fluid loading by boundary integral equations and by Floquet theory , J. Sound and Vibration, **291**, 81-99 (2006).