



Instability wave control in a subsonic round jet

Georgy Faranosov

TsAGI, Acoustic Division, 17, Radio str, 105005 Moscow, Russian Federation
georgefalt@rambler.ru

The well-known problem of instability wave generation in the subsonic jet issuing from the semi-infinite cylindrical duct is examined. The jet mixing layer is simulated by a cylindrical semi-infinite vortex sheet. Instability wave is considered to be initiated by a plane time-harmonic acoustic wave propagating inside the duct in the downstream direction. The possibility of instability wave suppression by external acoustic wave is investigated. Similar problem in two-dimensional case was analyzed in [1]. It is shown that the instability wave can be damped completely on condition that the amplitude and the phase of the external forcing are chosen in accordance with those of the instability wave.

1 Introduction

It is generally accepted [2, 3, 4] that one of the main sources of acoustic radiation from a turbulent jet is due to spatial instability wave packets propagating downstream within the jet. This approach has enabled to explain and predict the principal features of sound radiated by a supersonic jet. Therefore the problem of noise control for the jet could be considered as a problem of instability wave control [5, 6]. At present the main problem in developing active jet noise control strategy comes to elaboration of suitable antiwave generators (plasma actuators, piezosensors, etc.) and to working out methods of identification of that small part of turbulence corresponding to instability wave (online measurements by antennas, microphone arrays, etc.). Furthermore it is desirable to identify instability wave and to generate an antiwave by means of actuators on the outer surface of the nozzle in the vicinity of its edge. This will prevent our interference in natural processes take place in the jet but will lead to delicate jet tuning by the external impact. On this stage we consider the control strategy of time-harmonic artificially excited instability wave as a prototype of the noise control strategy in real jet. The first goal of the work is to answer if it is possible in principle to annul an instability wave in a jet.

2 Internally excited instability wave

2.1 Formulation of the problem

Consider a semi-infinite ($x \leq 0$) cylindrical duct of radius $r = r_0$ in cylindrical polar coordinates (r, θ, x) . The duct contains a uniform axial subsonic mean flow of velocity V_0 . In the outer region the ambient fluid is at rest relative to the duct. Mean density ρ_0 and the speed of sound c are equal inside and outside the flow. The jet issuing from the duct exit is separated from the ambient medium by a vortex sheet (this assumption holds true for low frequencies). Viscosity, thermal conductivity and all nonlinearities are ignored. Let the perturbations be small, so that the motion of the gas can be considered as potential motion, and let the time dependence of perturbations be described by $\exp(-ikct)$ where the wave number k is real and positive. Let an acoustic plane wave (the simplest duct eigen mode) propagate inside the duct in the downstream direction (Fig.1). Its velocity potential is given by

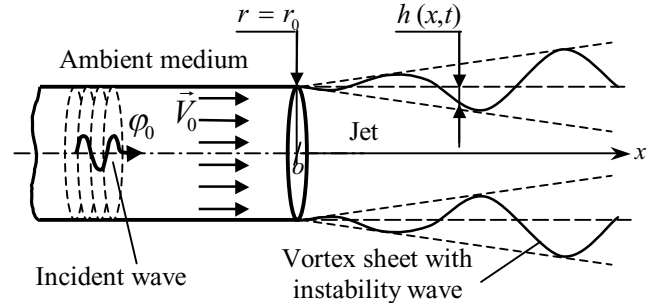


Fig.1 Sketch of the problem.

$$\varphi_0(r, x, t) = \begin{cases} 0, & r > r_0; \\ a_0 \exp\left(-ikct + \frac{k}{1+M}x\right), & r < r_0. \end{cases} \quad (1)$$

Hereafter the time factor $\exp(-ikct)$ will be suppressed.

This is a well-known problem of the interaction of sound with a round jet [7, 8, 9]. The incident wave perturbs the vortex sheet initiating an instability wave (i.e. the Kelvin-Helmholtz instability of the vortex sheet) which grows exponentially in the streamwise direction. Following Munt we will use the Wiener-Hopf procedure [10] to derive the analytical solution for the pressure and velocity potential fields.

Outside and inside the flow the velocity potential satisfies the equations

$$\Delta\varphi_I + k^2\varphi_I = 0, \quad r > r_0; \quad (2)$$

$$\Delta\varphi_{II} - \left(ik + M\frac{\partial}{\partial x}\right)^2\varphi_{II} = 0, \quad r < r_0. \quad (3)$$

where $\Delta = \frac{\partial^2}{\partial x^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}$ – two-dimensional Laplace operator in cylindrical polar coordinates and $M = V_0/c < 1$ is the jet Mach number. We note here that in case of plane incident wave the dependence on the azimuthal angle θ may be dropped.

The boundary conditions on the surface $r = r_0$ are as follows (for brevity, by p we denote the ratio of the pressure to the mean density of the medium)

$$\begin{aligned} p(x) &= p_I(x, r_0^+) - p_{II}(x, r_0^-) = \\ &= -ikc(\varphi_I(x, r_0^+) - \varphi_{II}(x, r_0^-)) - V_0\frac{\partial\varphi_{II}}{\partial x}(x, r_0^-), \end{aligned} \quad (4)$$

$$p(x) = 0, \quad x > 0;$$

$$\frac{\partial\varphi_I}{\partial r}(x, r_0^+) = -ikch(x),$$

$$\frac{\partial\varphi_{II}}{\partial r}(x, r_0^-) = -ikch(x) + V_0\frac{dh(x)}{dx}, \quad (5)$$

$$h(x) = 0, \quad x < 0.$$

Here $p_I(r, x)$ and $p_{II}(r, x)$ – are pressure perturbations in the stationary medium and in the jet respectively, and $h(x)$ is the normal displacement of the vortex sheet relative to its unperturbed position ($r = r_0$). In addition, the radiation condition should be satisfied: (i) when $r \rightarrow \infty$, the perturbations caused by the incident wave should decrease at any fixed instant of time, and (ii) the perturbations should be produced by sources positioned at the interface $r = r_0$ (causality).

2.2 Wiener-Hopf technique

We will seek the solution in the form

$$\begin{aligned} \varphi_I &= \varphi_1, \quad r > r_0, \\ \varphi_{II} &= \varphi_0 + \varphi_2, \quad r < r_0, \end{aligned} \quad (6)$$

where the incident field is separated explicitly.

Now define the Fourier transform and half-range Fourier transforms as

$$\begin{aligned} \Phi_m(\alpha, r) &= \int_{-\infty}^{+\infty} \varphi_m(x, r) \exp(i\alpha x) dx, \\ p_-(\alpha) &= \int_{-\infty}^0 p(x) \exp(i\alpha x) dx, \\ h_+(\alpha) &= \int_0^{+\infty} h(x) \exp(i\alpha x) dx. \end{aligned} \quad (7)$$

The inversion contour for these transforms must be chosen in such a way as to satisfy causality condition [11, 12]. Applying (7) to (2)-(5) we obtain

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \Phi_1}{\partial r} \right) - \gamma^2 \Phi_1 = 0, \quad r > r_0; \quad (8)$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \Phi_2}{\partial r} \right) - \beta^2 \Phi_2 = 0, \quad r < r_0; \quad (9)$$

and

$$\begin{aligned} p_-(\alpha) &= i(k + M\alpha)cQ(\alpha)I_0(\beta r_0) - \\ &- ikcP(\alpha)K_0(\gamma r_0) + i(k + M\alpha)c\Phi_0(\alpha), \\ ikch_+(\alpha) &= \gamma P(\alpha)K_0(\gamma r_0), \\ i(k + M\alpha)ch_+(\alpha) &= \beta Q(\alpha)I_0(\beta r_0) \end{aligned} \quad (10)$$

for

$$\begin{aligned} \Phi_1(\alpha, r) &= P(\alpha)K_0(\gamma r), \quad r > r_0, \\ \Phi_2(\alpha, r) &= Q(\alpha)I_0(\beta r), \quad r < r_0, \end{aligned} \quad (11)$$

where K_n and I_n are modified Bessel Functions of the order n . The radial wave numbers are

$$\begin{aligned} \gamma &= \sqrt{\alpha^2 - k^2}, \\ \beta &= \sqrt{(1 - M^2)\left(\alpha - \frac{k}{1 - M}\right)\left(\alpha + \frac{k}{1 + M}\right)}. \end{aligned} \quad (12)$$

The branch cuts of γ go from $\pm k$ to $\pm \infty$ and the branch cuts of β go from $\frac{k}{1 - M}$ to $+\infty$ and from $-\frac{k}{1 + M}$ to $-\infty$.

From Eq.(10) one can find that

$$p_-(\alpha) = c^2 H(\alpha) h_+(\alpha) + i(k + M\alpha)c\Phi_0(\alpha), \quad (13)$$

where

$$H(\alpha) = \frac{1}{\gamma\beta} \left(k^2 \beta \frac{K_0(\gamma r_0)}{K_1(\gamma r_0)} + (k + M\alpha)^2 \gamma \frac{I_0(\beta r_0)}{I_1(\beta r_0)} \right) \quad (14)$$

is the Wiener-Hopf kernel. It is very important to know the location of its poles and zeros to obtain casual solution. The properties of $H(\alpha)$ were investigated in details in [7, 13]. A group of poles and zeros is located close to the line $\text{Im}\alpha = \frac{M}{1 - M^2}$ and one zero α_0 ($\text{Re}\alpha < 0$, $\text{Im}\alpha_0 > 0$) is related to the Kelvin-Helmholtz instability of the vortex sheet (Fig.2). To obtain correct result one should first consider the problem for complex wave number $k_0 = k + ik_i$, $k_i \gg k$ and then determine the solution with real wave number k in the limit of $k_i \rightarrow +0$ by analytic continuation [11, 12]. It turns out that for $k_i \gg k$ the zero α_0 lies in the lower half of complex α -plane so that the contour C of integration in inverse Fourier transform runs above α_0 . But when we reduce k_i , the hydrodynamic zero α_0 approaches the real axis and at certain $k_i > 0$ crosses this axis. The analytical continuation of the solution will be correspond to the integral over the contour C deformed in such a way as to bypass α_0 from above (Fig.2).

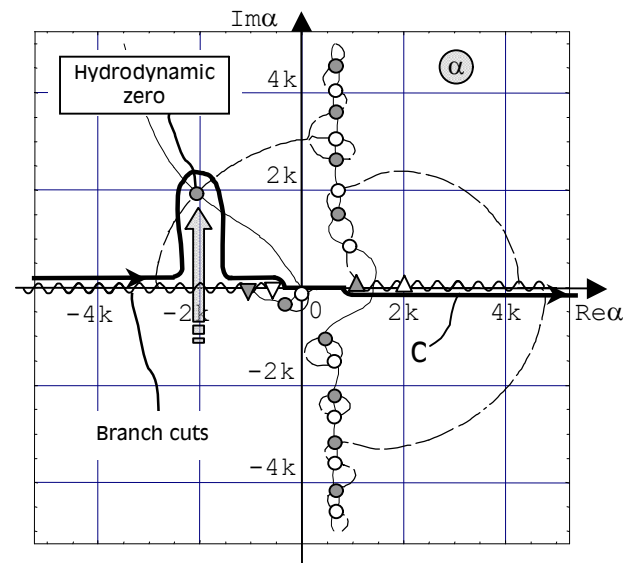


Fig.2 Poles \circ and zeros \bullet of $H(\alpha)$ and the contour of integration C . \blacktriangle , \blacktriangledown – branch points of γ , \triangle , ∇ – branch points of β . Parameters are $M = 0.5$, $kr_0 = 4$.

The most important step is the factorization of $H(\alpha)$ in the form

$$H(\alpha) = H_+(\alpha)H_-(\alpha) \quad (15)$$

where H_{\pm} are analytic, non-zero and possess algebraic behaviour at infinity in R_{\pm} . R_+ and R_- represent the half spaces above and below the contour C respectively so that it lies within the region $R_+ \cap R_-$ of overlapping of R_+ and R_- .

To perform the factorization let us introduce a new function

$$S(\alpha) = \frac{\sqrt{1 - M^2} \gamma H(\alpha)}{M^2 (\alpha - \alpha_0)(\alpha - \alpha_0^*)}, \quad (16)$$

where * denotes complex conjugation. Note that α_0 is not a zero for $S(\alpha)$. Furthermore, it is easy to show that $S(\alpha) \sim 1$ as $|\operatorname{Re} \alpha| \rightarrow \infty$. These two facts allow us to use the well-known formula to factorize $S(\alpha)$ [10]:

$$S_{\pm}(\alpha) = \exp\left(\pm \frac{1}{2\pi i} \int_C \frac{\ln S(\xi)}{\xi - \alpha} d\xi\right), \quad (17)$$

where the contour C' is taken within the region $R_+ \cap R_-$ so as to run below α for $S_+(\alpha)$ and above for $S_-(\alpha)$.

Thus, for H_{\pm} we have

$$\begin{aligned} H_-(\alpha) &= \frac{S_-(\alpha)}{\gamma_-(\alpha)}, \\ H_+(\alpha) &= \frac{M^2 S_+(\alpha)(\alpha - \alpha_0)(\alpha - \alpha_0^*)}{\sqrt{1 - M^2} \gamma_+(\alpha)}. \end{aligned} \quad (18)$$

Now we can represent Φ_0 as a sum of two functions Φ_{0+} and Φ_{0-} regular in R_+ and in R_- respectively:

$$\begin{aligned} \Phi_0(\alpha) &= 2\pi a_0 \delta\left(\alpha + \frac{k}{1+M}\right) = \\ &= \lim_{\varepsilon \rightarrow +0} \frac{ia_0}{\alpha - (-\frac{k}{1+M} - i\varepsilon)} + \lim_{\varepsilon \rightarrow +0} \frac{-ia_0}{\alpha - (-\frac{k}{1+M} + i\varepsilon)}. \end{aligned} \quad (19)$$

After some algebra one can obtain from Eqs.(13, 15, 16)

$$\begin{aligned} \frac{p_-(\alpha)}{H_-(\alpha)} - i \frac{(k+M\alpha)c\Phi_0(\alpha)}{H_-(\alpha)} + i \frac{kc\Phi_{0+}(\alpha)}{(1+M)H_-(-\frac{k}{1+M})} = \\ = i \frac{(k+M\alpha)c\Phi_{0+}(\alpha)}{H_-(\alpha)} + c^2 H_+(\alpha) h_+(\alpha) = J(\alpha) \text{ (say)}. \end{aligned} \quad (20)$$

The left-hand side of the Eq.(20) is the function regular in R_- and the right-hand side is the function regular in R_+ . Since these regions overlap the left- and right-hand sides, being equal in the zone of overlapping, determine the function $J(\alpha)$ regular in the hole α -plane. It can be shown that $J(\alpha)$ behaves algebraically at infinity, then, according to the Liouville theorem, it has the form of a polynomial of an integer degree of α . The polynomial coefficients can be determined from the behaviour of the solution at the lip of the duct. Note here that the imposition of any conditions at the nozzle edge in combination with causality condition constitutes very subtle problem especially for three-dimensional jet [9, 14]. For this kind of problems along with other types of edge conditions the full Kutta condition was often used [7, 8, 9]. It requires that all the vorticity is shed from the lip. In that case one has for the vortex sheet displacement near the edge $h(x) \sim x^{3/2}$ as $x \rightarrow +0$. This implies that the flow leaves the edge as smoothly as possible. In the present paper we adopt the full Kutta condition. Applying this condition we obtain $J(\alpha) = 0$.

Now we are able to find the unknown functions from Eqs.(10, 11, 20):

$$h_+(\alpha) = \frac{a_0 k}{(1+M)cH_-(-\frac{k}{1+M})} \frac{1}{(\alpha + \frac{k}{1+M})H_+(\alpha)}, \quad (21a)$$

$$p_-(\alpha) = i(k+M\alpha)c\Phi_0(\alpha) - i \frac{kc\Phi_{0+}(\alpha)H_-(\alpha)}{(1+M)H_-(-\frac{k}{1+M})}, \quad (21b)$$

$$\begin{aligned} \Phi_1(\alpha, r) &= \\ &= \frac{ia_0 k^2}{(1+M)H_-(-\frac{k}{1+M})} \frac{K_0(\gamma r)}{\gamma K_0(\gamma r_0)H_+(\alpha)(\alpha + \frac{k}{1+M})}, \end{aligned} \quad (21c)$$

$$\begin{aligned} \Phi_2(\alpha, r) &= \\ &= -\frac{ia_0 k}{(1+M)H_-(-\frac{k}{1+M})} \frac{(k+M\alpha)I_0(\beta r)}{\beta I_0(\beta r_0)H_+(\alpha)(\alpha + \frac{k}{1+M})}. \end{aligned} \quad (21d)$$

Thus, the solution of the problem can be written in the form of the inverse Fourier transform

$$h = \frac{1}{2\pi} \int_C h_+ \exp(-i\alpha x) d\alpha, \quad (22a)$$

$$p = \frac{1}{2\pi} \int_C p_- \exp(-i\alpha x) d\alpha, \quad (22b)$$

$$\varphi_1 = \frac{1}{2\pi} \int_C \Phi_1 \exp(-i\alpha x) d\alpha, \quad (22c)$$

$$\varphi_2 = \frac{1}{2\pi} \int_C \Phi_2 \exp(-i\alpha x) d\alpha, \quad (22d)$$

where the contour of integration C is shown in Fig.2. From Eqs.(18, 21a, 22a) one can find that the instability wave for the displacement of the interface corresponds to the residue in α_0 and is determined by the following expression

$$h_0(x, t) = \tilde{h}_0 \exp(-ikct - i\alpha_0 x), \quad (23)$$

where

$$\tilde{h}_0 = -\frac{ia_0 k}{(1+M)cH_-(-\frac{k}{1+M})} \frac{1}{\alpha_0 + \frac{k}{1+M}} \lim_{\alpha \rightarrow \alpha_0} \frac{\alpha - \alpha_0}{H_+(\alpha)} \quad (24)$$

is the complex amplitude of the instability wave. Similar expressions can be obtained for other types of duct eigen modes.

3 Control action

3.1 Formulation of the problem

Now let us set the following question. Is it possible to annul internally excited instability wave (23) by an external action on the vortex sheet?

To answer this question consider a plane sound wave propagating in the stationary medium along the x -axis in the downstream direction. We will call this wave "the control wave". This wave also induces Kelvin-Helmholtz instability of the vortex sheet. So let us try to adjust the parameters of the control wave in such a way as to destroy the internally excited instability wave (20). First of all, it is obvious that the control wave should have the same frequency as the wave to be damped. Thus, the velocity potential of the control wave is given by

$$\varphi_c(r, x, t) = \begin{cases} a_c \exp(-ikct + kx), & r > r_0; \\ 0, & r < r_0. \end{cases} \quad (25)$$

The governing equations and the boundary conditions are, of course, the same as in section 2.

3.2 Control instability wave

Omitting detailed calculations, we write the solution analogous to (21):

$$h_+(\alpha) = -\frac{a_c k}{cH_-(-k)} \frac{1}{(\alpha+k)H_+(\alpha)}, \quad (26a)$$

$$p_-(\alpha) = -ikc\Phi_0(\alpha) + i\frac{kc\Phi_{0+}(\alpha)H_-(\alpha)}{H_-(-k)}, \quad (26b)$$

$$\begin{aligned} \Phi_1(\alpha, r) &= \\ &= -\frac{ia_c k^2}{H_-(-k)} \frac{K_0(\gamma r)}{\gamma K_0(\gamma r_0)H_+(\alpha)(\alpha+k)}, \end{aligned} \quad (26c)$$

$$\begin{aligned} \Phi_2(\alpha, r) &= \\ &= \frac{ia_c k}{H_-(-k)} \frac{(k+M\alpha)I_0(\beta r)}{\beta I_0(\beta r_0)H_+(\alpha)(\alpha+k)}. \end{aligned} \quad (26d)$$

Then, for the displacement of the interface due to the instability wave generated by the control wave (25), we obtain

$$h_c(x, t) = \tilde{h}_c \exp(-ikct - i\alpha_0 x), \quad (27)$$

where

$$\tilde{h}_c = \frac{ia_0 k}{cH_-(-k)} \frac{1}{\alpha_0 + k} \lim_{\alpha \rightarrow \alpha_0} \frac{\alpha - \alpha_0}{H_+(\alpha)} \quad (28)$$

is the complex amplitude of the control instability wave.

3.3 Damping condition

For a complete mutual suppression of the instability waves, we require (due to the linearity of the problem) for the total displacement of the interface

$$h_0(x, t) + h_c(x, t) = 0. \quad (29)$$

From this it follows an expression connecting the control wave amplitude and phase to those of the initial wave

$$\frac{a_c}{a_0} = \frac{1}{1+M} \frac{H_-(-k)}{H_-(-\frac{k}{1+M})} \frac{\alpha_0 + k}{\alpha_0 + \frac{k}{1+M}}. \quad (30)$$

Hence, to completely suppress instability wave (23) by plane wave (25) incident from the stationary, it is necessary to choose its complex amplitude a_c according to Eq.(30). It can be seen that the complete suppression of the instability wave can be achieved for any values of the dimensionless parameter kr_0 .

4 Results

Fig.3 shows the dependence of the magnitude of the absolute value and the phase of the quantity a_c/a_0 on the Mach number for different values of kr_0 . It turns out that for $kr_0 > 1$ the ratio a_c/a_0 is quite close to the one for the two-dimensional problem [1]. Thus, for all kr_0 we can write $a_c/a_0 \sim 1$. This fact qualitatively conforms to the results of [1, 15] where it was shown that the plane vortex sheet is most susceptible to the perturbations propagating downstream in the stationary medium, i.e. when the source of perturbations lies far upstream from the nozzle edge. On the analogy of the two-dimensional case one can assume

that the emission of the control wave in the downstream direction is most advantageous, because, in this case, its amplitude will be minimum and on the order of the exciting field amplitude.

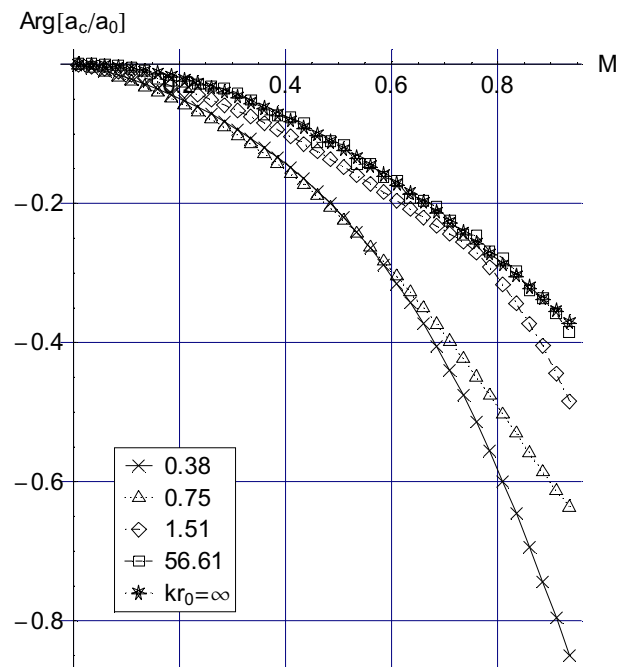
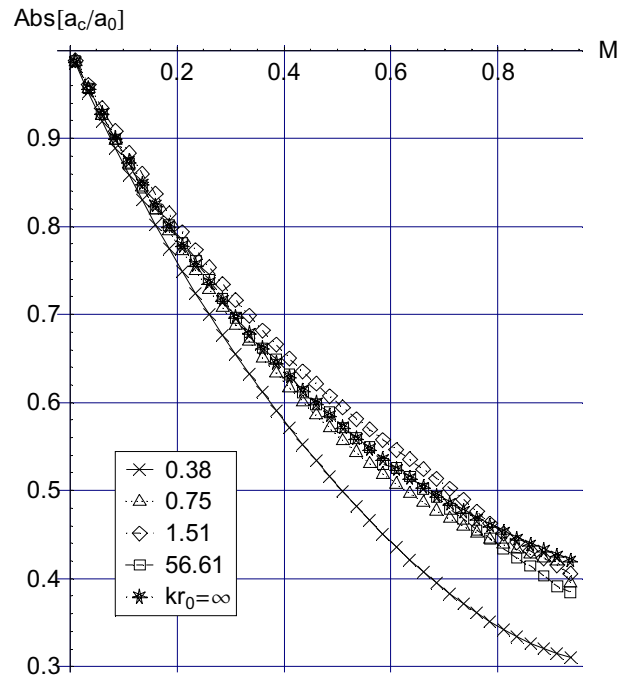


Fig.3 The dependence of the magnitude of $\text{Abs}(a_c/a_0)$ and $\text{Arg}(a_c/a_0)$ on the Mach number for different values of kr_0 .

5 Conclusions

Thus, in the framework of the chosen axisymmetric model of the jet with vortex sheet separating the flow and the stationary medium, we demonstrated the fundamental possibility of effectively controlling instability wave, artificially excited by a plane wave propagating inside the duct, by adjusting the control acoustic action. The intensity of this action is on the order of the flow perturbations that give rise to the initial instability wave. The latter fact is especially important: since the development of instability waves is caused by small perturbations of the flow, the intensity of the control field can also be small.

For the implementation of active instability wave control, it is necessary to independently determine (measure) the initial parameters of the instability wave (the set of perturbations that occur near the nozzle edge and leave the edge) and, according to them, to adjust the external action so as to completely suppress the instability wave. The near field characteristics that are necessary for the instability wave identification will be analyzed in further studies.

Acknowledgments

The author is very grateful to Professor V.F. Kopiev for the formulation of the problem and fruitful discussions. This work was supported by the Russian Foundation for Basic Researches, project no. 08-01-00697.

References

- [1] V.F. Kopiev, G.A. Faranosov, "Instability wave control in two-dimensional model of the nozzle edge", *Acoust. Phys.* 54(2), 371-379 (2008)
- [2] C.K.W. Tam, P.J. Morris "The radiation of sound by the instability waves of a compressible plane turbulent shear layer", *J. Fluid Mech.* 98, 349-381 (1980)
- [3] C.K.W. Tam, P. Chen "Turbulent mixing noise from supersonic jets", *AIAA Paper.* 93-4408.gn (1993)
- [4] V.F. Kopiev, S.A. Chernyshev, M.Yu. Zaitsev, V.M. Kuznetsov "Experimental validation of instability wave theory for round supersonic jet", *AIAA Paper.* 2006-2595, (2006)
- [5] V.F. Kopiev "On the possibility and prospects of turbulent flow noise control" *CD-ROM Proceedings. FM11-12156. XXI ICTAM.* Warsaw, Poland.xc, 15-21 August, (2004)
- [6] V.F. Kopiev, N.N. Ostrikov, S.A. Chernyshev, J.W. Elliot "Aeroacoustics of supersonic jet issued from corrugated nozzle: new approach and prospects", *Int. J. of Aeroacoustics.* 3(3), 199-228 (2004)
- [7] R.M. Munt "The interaction of sound with a subsonic jet issuing from a semi-infinite cylindrical pipe" *J. Fluid Mech.* 83(4),609-640 (1977)
- [8] A.M. Cargill "Low frequency acoustic radiation from a jet pipe – a second order theory", *J. Sound Vib.* 83(3), 339-354 (1982)
- [9] S.W. Rienstra "A small Strouhal number analysis for acoustic wave-jet flow-pipe interaction", *J. Sound Vib.* 86(4), 539-356 (1982)
- [10] B. Noble. "Methods based on the Wiener-Hopf technique" *Pergamon Press, London* (1958)
- [11] Crighthton D.G., Leppington F.G. "Radiation properties of the semi-infinite vortex sheet: the initial value problem" *J. Fluid Mech.* 64(2), 393-414 (1974)
- [12] R.J. Briggs "Electron stream interaction with plasmas" *MIT press* (1964)
- [13] J.D. Morgan "The interaction of sound with a subsonic cylindrical vortex layer" *Proc. R. Soc. Lond. A.* 344, 341-362 (1975)
- [14] D.G. Crighthton "The Kutta condition in unsteady flow" *Ann. Rev. Fluid Mech.* 17, 411-445 (1985)
- [15] D.W. Bechert "Excitation of instability waves in free shear layers. Part 1. Theory" *J. Fluid Mech.* 186, 47-62 (1988)