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Measured and calculated sounding frequencies of pipes coupled with free reeds

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The *khaen, naw, bawu, sheng* and *sho* are examples of Asian free-reed mouth organs, which incorporate approximately symmetric free reeds coupled to pipe resonators. Previous research has shown that the reeds in these instruments behave as "blown-open" reeds in which the playing frequency is above both the natural frequency of the reed and the first peak of the measured impedance curve. Detailed calculations of input impedance have been made for a variety of these instruments, taking into account the position of the reed along the pipe, tuning slots, finger holes, and non-uniform cross sections. The details of these calculations are in good agreement with the measured impedances of the same instruments. If the reed is treated as a damped, driven harmonic oscillator, the sounding frequencies of these reed-pipes can be predicted using a phase relation between the reed vibration and the phase of the complex impedance. [Research supported by National Science Foundation REU Grant PHY-0354058.]

1 Introduction

Unlike the free reeds found in Western instruments such as the reed organ, accordion, and harmonica, the reeds of the Asian free reed mouth organs are not only coupled to pipe resonators, but are approximately symmetric, so that the same reed can operate on both vacuum and pressure (inhaling and exhaling). Figure 1 shows a reed from a *sheng*. The *sheng*, *sho*, and *khaen* all employ one reed per pipe, thus requiring a separate pipe for each pitch. Also common in various parts of South and Southeast Asia are free reed pipes which allow change of pitch using finger holes. One version of this type of instrument now common in China is the *bawu*. Detailed general information on the Asian free reed mouth organs is available in the article by Miller [2].

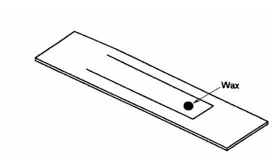

Fig. 1 A reed from a sheng, from Gellerman [1].

The free reeds used in the Asian mouth organs are typically cut from a single piece of thin metal, usually brass or a bronze alloy, and set into a bamboo pipe. In the single note per pipe instruments, a finger hole is drilled at a point that destroys the pipe resonance and prevents the reed from sounding unless the hole is closed. Wind is provided by blowing either in or out through the mouthpiece which forms the opening of the air chamber that surrounds the reeds. The instrument is usually held upright with the air chamber supported by the hands. Fingers or thumbs of both hands are available to close the holes and sound notes. It is typical in playing the instruments that several notes are sounded simultaneously, some of them serving as drones. (See References 2 and 3 for more detailed information.)

The frequency of reed vibration is determined by both the reed and the pipe. One or two tuning slots (usually two for the khaen) are cut into the pipe, determining the effective acoustical length. The vibrating frequency of the blown reed can, within certain limits, be pulled to match the pipe resonance, so that fine tuning is done by means of the position of these tuning slots. Earlier studies [3] showed that the sounding frequency of the reed-pipe combination was often close to and slightly above a measured impedance peak of the pipe, as would be expected for a "blown-open" or (+,-) reed in the classification of Fletcher.[4]

There are some features of these wind instruments that are of special interest. One is that, in instruments such as the *khaen* and related instruments including the gourd pipe *naw*, the reed is mounted in the side of a pipe open at both ends, rather than at a (closed) end of the pipe as in typical wind instruments. Another is that, in cases in which the natural frequencies of the pipe and the reed are not close, the sounding frequency of the reed-pipe combination is not close to either. This occurs in particular with some pipes of the sheng, in which the reed is mounted in a closed, tapered end of a mostly cylindrical pipe, and in the *naw*, in which the reed is mounted in the side of an open pipe of non-uniform cross section.

The main objective of the research reported here was to calculate the input impedance at the position of the reed for pipes from various Asian instruments of non-uniform cross section, sometimes including toneholes or tuning slots, by using a method of transmission matrices [5, 6]. These calculated curves can be compared with experimental results. Using the calculated impedance, modeling the reed as a damped harmonic oscillator, and using the phase relation between the reed and the pipe as done by Fletcher in [8], the sounding frequencies of the reed-pipe combinations can be calculated and compared with experimental results.

2 Calculation of Input Impedance

2.1 The *naw* as an example

The methods used will be illustrated using the example of the *naw*, a Southeast Asian gourd pipe pictured in Figure 2. The construction of the *naw* is fairly simple.

Fig. 2 The *naw*.

Five bamboo pipes open at both ends and of different lengths (some containing tuning slots) are arranged in a circular cluster. In each pipe, a hole is placed so that it destroys the pipe resonance; the pipe will not sound unless the hole is covered.

A free reed, constructed of bamboo (Figure 3), is mounted near the lower end of the pipe. The cluster of pipes is sealed in a dried gourd, which acts as a windchest, with a waxy mixture so that the lower ends protrude through the base of the gourd. Although the pipes appear to be cylindrical, each has a slight taper near the reed end. This taper has a significant impact on the acoustical properties of the pipe.

Fig. 3 Reed from a *naw*.

Notes are played by blowing into the neck of the gourd and covering the holes of different pipes so that they will sound. Although the tuning of the open pipes varies from region to region, and even from instrument to instrument, it generally follows a pentatonic scheme. When the bottom ends of the pipes are covered with the thumb, the pitch is bent slightly downward, about one or two semitones lower than the open pipe note.

2.2 Calculation of acoustic impedance using transmission matrices

The input impedance at the reed position as a function of frequency for the pipes of these free-reed mouth organs was calculated using transmission matrices. The method used was that of Keefe [6] as described by Scavone [5]. In some important cases the calculation involved pipes of non-uniform cross section. In these cases the non-cylindrical pipe bores were measured using x-rays and approximated by a set of cylindrical sections. This was true in particular of the tapered pips of the *naw* (Figure 4).

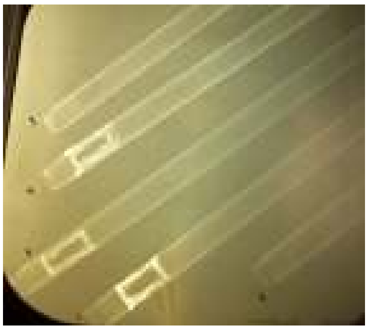

Fig. 4 x-rays of pipes from the *naw*.

An additional complication in the calculations occurred for pipes such as those from the *naw* and *khaen*, in which the reed is mounted in the side wall of a pipe which is open at both ends (Figure 5). This case required calculating input impedances for the sections above and below the reed and adding them in parallel to calculate the input impedance at the reed position.

$$\frac{1}{Z_{total}} = \frac{1}{Z_1} + \frac{1}{Z_2} \qquad (1)$$

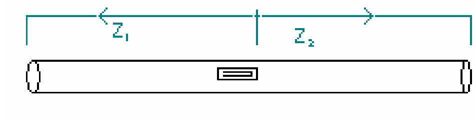

Fig. 5 A pipe with the reed mounted in the side wall as in the naw or khaen.

2.3 Calculating the transmission matrix for a single pipe section – simplified case

In the simplest case, wall and viscothermal losses are ignored. The transmission matrix m for a single cylindrical section is given by (where $k=2\pi f/c$):

$$m = \begin{bmatrix} \cos(kL_i) & jZ_0 \sin(kL_i) \\ j\frac{1}{Z_0}\sin(kL_i) & \cos(kL_i) \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad (2)$$

From this transmission matrix the input impedance is given by

$$Z_{in} = \frac{b + aZ_L}{d + cZ_L} \qquad (3)$$

To approximate a pipe of non-cylindrical cross section, matrices m_i representing each cylindrical section are cascaded, starting at the excitation point, to get the pipe matrix M:

$$M = \prod_{i=1}^{n} m_i = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \qquad (4)$$

where each section matrix is

$$m_i = \begin{bmatrix} \cos(kL_i) & jZ_0 \sin(kL_i) \\ j\frac{1}{Z_0}\sin(kL_i) & \cos(kL_i) \end{bmatrix} \qquad (5)$$

The resulting input impedance is then given by

$$Z_{in} = \frac{B + AZ_L}{D + CZ_L} \qquad (6)$$

In the low-frequency approximation that $Z_L = 0$ for an open pipe end, the input impedance is then

$$Z_{in} = \frac{B}{D} \qquad (7)$$

2.4 The transmission matrix for a single pipe section – some refinements

To achieve more realistic results, and to account for complications such as tuning slots, some refinements were necessary. For pipes with tuning slots or tone holes, additional matrix elements (not shown here) were included in the product for the transmission matrix M. Losses were accounted for by replacing the real wave number k with a complex wave propagation number Γ, defined as follows (where α is the phase attenuation, v_p the phase velocity, and $\omega=2\pi f$):

$$\Gamma = \alpha + j\frac{\omega}{v_p} \qquad (8)$$

A corrected value [7] was also used for the load impedance (where ρ is the density of air and r the pipe radius):

$$Z_L = 0.25\frac{\rho\omega}{\Gamma\pi r^2}\Gamma^2 r^2 + j0.6\frac{\rho\omega}{\Gamma\pi r^2}\Gamma r \qquad (9)$$

For the case in which the pipe extends in two directions from the input point (reed location), the single section impedances can be combined in parallel using equation (1).

$$Z_{in} = \frac{B + AZ_L}{D + CZ_L} \qquad (10)$$

3 The Phase Relation

A requirement for sustained oscillation is phase matching between the oscillations of the reed and the pipe input impedance Z_{in}. To calculate this phase relation we follow the model of Fletcher [8] in which the reed is modeled as a damped, driven simple harmonic oscillator with natural frequency f_r and damping coefficient χ_r. In this case the phase of the reed oscillation is given by

$$\tan\varphi = \frac{\chi_r f f_r}{f^2 - f_r^2} \qquad (11)$$

Since the input impedance is a complex number $Z_{in} = |Z_{in}|e^{j\theta}$ calculated by the method outlined above, once Z_{in} has been calculated the phase is obtained immediately from

$$\tan\theta = \frac{\Im(Z_{in})}{\Re(Z_{in})} \qquad (12)$$

The required condition for sustained oscillation is given by Fletcher [8] as

$$\tan\theta = -\tan\varphi \qquad (13)$$

Obtaining tan φ as a function of frequency involves measurements or approximations of the reed frequency and damping coefficient. Solving the phase relation Eq.(13) graphically gives the condition for sustained oscillation.

Since Eq.(13) generally holds for multiple values of f, additional information is needed from earlier experimental studies to choose the point of intersection corresponding to the sounding frequency. Of particular use is the condition that "blown-open" reeds must sound above both the natural reed frequency and the pipe resonant frequency.

In addition there is the physical limitation that the free reed can normally only match a sounding frequency less than an octave from its natural frequency of oscillation.

4 Measurements of Pipe Input Impedance

Measuring the magnitude of the input impedance of these pipes at the reed position was done by a method which followed the basic principle of Benade and Ibisi [9], using a swept sine wave to excite the air column inside the pipe at the reed position and then measuring the response of the pipe with a probe microphone. A mechanical vibrator was used to excite a latex membrane stretched over the reed opening (Figure 6).

Fig. 6 Experimental setup for impedance curve measurements.

By machining an aluminum foot for the mechanical vibrator to closely match the size of the reed opening, and using the latex membrane as an air seal, good results were obtained for an effective range of measurement to 5000 Hz for most pipes. The measured impedance curves obtained are not calibrated, and the variation in frequency response of the mechanical vibrator is not accounted for, but these curves are nevertheless useful for obtaining the location of the impedance peaks.

5 Results

5.1 The *naw* pipe

Figure 7 shows calculated and measured impedance curves for a *naw* pipe. The upper pair of curves is for the pipe open at both ends. The lower pair is for the pipe with the lower end closed.

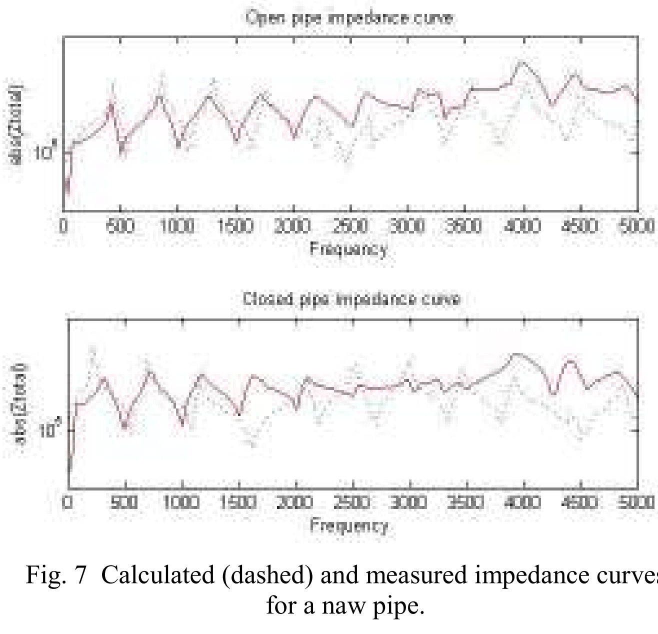

Fig. 7 Calculated (dashed) and measured impedance curves for a naw pipe.

The graphs of -tan θ and tan φ for the open naw pipe in Figure 7 are shown in Figure 8. The circled point of intersection represents the sounding frequency at 440 Hz, which agrees well with the experimental value.

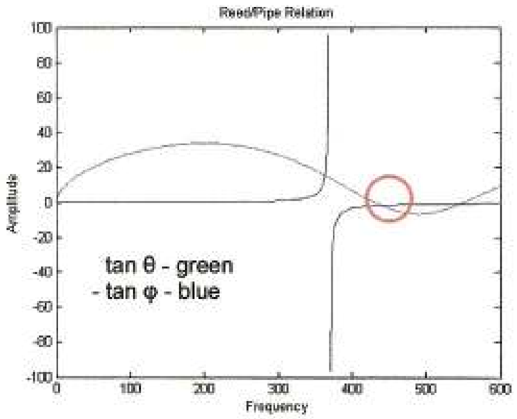

Fig. 8 The sounding frequency of a naw pipe determined by the phase relation.

5.2 A khaen pipe

Figure 9 shows the input impedance of a khaen pipe calculated by the method of the preceding sections. Although the pipe has a uniform cross section, the end sections formed by the presence of the two tuning slots as shown in the diagram below (Figure 10 on the following page) complicate the spectrum. The location of the reed at approximately $L/4$ results in the near absence of harmonics 4, 8, 12, etc.

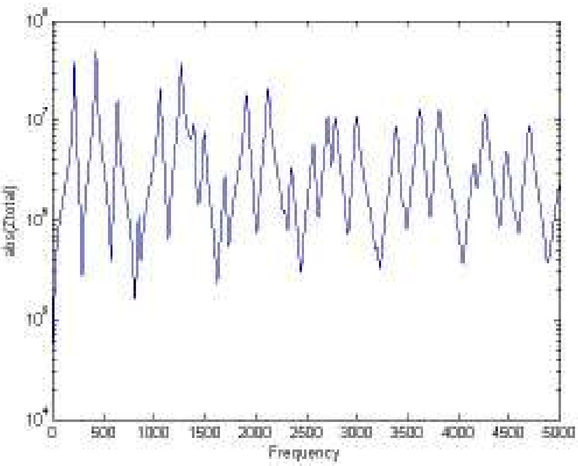

Fig. 9 Magnitude of the calculated input impedance for a khaen pipe.

6 Conclusion

The impedance curves calculated by the methods in this paper agree well with experimental results, even in smaller features and at higher frequencies. A sampling of the results obtained for pipe sounding frequencies is given in Table 1 on the following page (all frequencies in Hz).

The calculated and experimental pipe frequencies represent the first peak of the calculated or measured impedance curve. The agreement between calculated and experimental values is good, even for cases in which the sounding frequencies are not close to the pipe frequencies.

Acknowledgments

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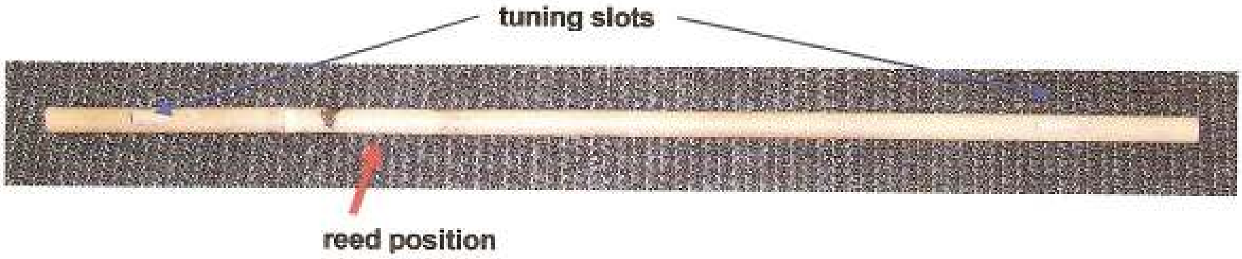

Fig. 10 The khaen pipe for which the calculated impedance is shown in Figure 9.

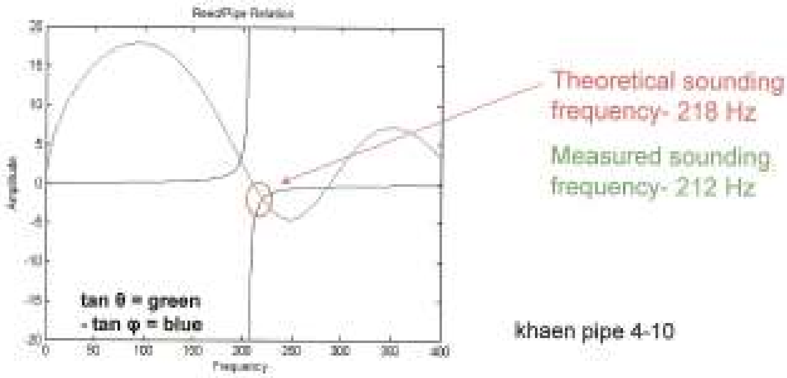

Fig. 11 Determining the sounding frequency of the khaen pipe of Figure 10.

Pipe	Reed Frequency	Calculated Pipe Frequency	Experimental Pipe Frequency	Calculated Sounding Frequency	Experimental Sounding Frequency
Sheng Pipe 6	511	379	350	528	520
Sheng Pipe 7	490	341	350	508	495
Khaen Pipe 4-9	434	421	N/A	450	452
Khaen Pipe 4-10	206	210	N/A	218	212
Naw 1-5 Open	370	432	430	440	440
Naw 1-5 Closed	370	230	220	374	395
Bawu 1 6 holes closed	250	327	330	328	340
Bawu 4 holes closed	250	401	400	402	405

Table 1: Summary of calculated and experimental results for a variety of pipes