Quasi-static evaluation of mechanical properties of poroelastic materials: static and dynamic strain dependence and in vacuum tests

Paolo Bonfiglio, Francesco Pompoli and Paresh Shravage

Dipartimento di Ingegneria - University of Ferrara, Via Saragat 1, 44100 Ferrara, Italy
paolo.bonfiglio@unife.it

A complete description of the vibro-acoustical behavior of a poroelastic material requires the knowledge of both geometrical quantities, related to the structure of the fluid-filled pores to account the sound propagation within them, and mechanical parameters (i.e. Young's Modulus, Poisson ratio and loss factor) in order to model the wave propagation through the elastic structure constituting its skeleton. Because the non-linear nature of poroelastic media, these mechanical properties are observed to depend on static preload and dynamic strain applied to them.

In the present work well established quasi-static methods, based on the measurement of mechanical impedance and the use of adequate polynomial relations, have been used to determine the dependence of the mechanical properties on the applied deformations. Furthermore, tests have been also carried out in a vacuum chamber in order to evaluate the real contribution of the filling fluid on the total vibro-acoustical response of the material.

1 Introduction

The acoustic behavior of the poroelastic materials is completely described by Biot poroelasticity theory [1]. In order to model sound propagation through the elastic frame of poroelastic material, Biot model requires five geometric parameters (airflow resistivity, open porosity, tortuosity, viscous and thermal characteristic lengths); moreover it also requires three elastic properties of the skeleton, Young's modulus *(E)*, Poisson ratio (ν) and loss factor (η). Various static, quasi-static and dynamic methods for determining these elastic properties were proposed in the literature. In this paper two quasi-static methods are investigated [2,3]. The effect of dynamic and static deformations is investigated for both methods; furthermore the effect of air inside the porous material on the elastic properties is also evaluated by repeating mechanical impedance measurements in vacuum conditions.

2 Description of the measurement methods

In this section, two quasi-static methods for the determination of mechanical properties of porous materials are described in detail. The mechanical properties of porous materials are complex and frequency dependent in nature because of viscosity of the frame. Therefore dynamic tests should be carried out to evaluate these properties. However, even when the dynamic values of elastic parameters are higher than its static value, it has been shown from simulations and measurements that the constant value of elastic properties measured at low frequency provides reliable results [2].

2.1 Method A: Method based on the measurement of mechanical stiffness and lateral deformation

This method was proposed by Marietz **et al.** [2]; the scheme of the set-up is shown in Figure 1.

The experimental set up consists of a cylindrical porous sample sandwiched between two rigid plates. The lower plate is excited by electrodynamics shaker and upper plate is rigidly fixed. During the test, the lower plate is excited and, because of this, the sample gets deformed along its diameter. This effect is also known as "bulge effect". This lateral deformation D_2 and the vertical deformation D_1 are measured by a laser vibrometer. Also the force transmitted, F, through the sample is measured by force transducer. Using these quantities it is possible to calculate transfer function and mechanical impedance, which are complex and frequency dependent, as follows:

$$T^* = \frac{D_2}{D_1} \text{ and } K^*(\omega) = \frac{F}{D_1} \quad \left[\frac{N}{m}\right] \tag{1}$$

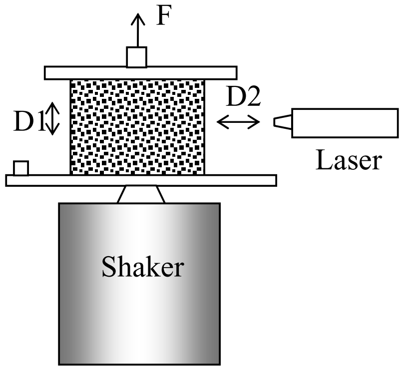

Fig. 1 Measurement setup for quasi-static method

From these quantities, their real parts (named T and K respectively) and finite element simulation of static case of porous sample can be used to calculate mechanical properties. Simulations are executed for an arbitrary value of Young's modulus and loss factor with varying Poisson ratio. Therefore one can obtain graph of simulated function T as a function of the Poisson ratio.

Taking this simulated curve as an abacus, it is possible to:

I] calculate the Poisson ratio of the material by using minimizing experimental and simulated transfer function *T*.

II] now fixing the measured value of Poisson ratio in FEM model subsequent variation in Young's modulus are carried out. This gives the simulation curve for compression stiffness *K*.

III] by minimizing experimental and compression stiffness values, it is possible to calculate the Young's modulus of the porous sample.

IV] lastly the loss factor is calculated from measured $K(\omega)$ as follows:

$$\eta = \frac{\mathrm{Im}\left[K(\omega)\right]}{\mathrm{Re}\left[K(\omega)\right]} . \tag{2}$$

2.2 Method B: Method based on the measurement of mechanical stiffness of two cylindrical samples of different diameters

Langlois *et al.* [3] have proposed a method for determining mechanical properties based on the measure of the mechanical stiffness of two disc shaped samples of different diameters and the use of polynomial relations for simultaneously calculating Young's modulus, Poisson ration and loss factor. The measurement setup is completely analogous to setup shown in Figure 1.
The compression test is performed on two samples with different shape factors. The shape factor is defined as the half the radius to thickness [*R/2L*]. In this configuration the sample will try to bulge as it is compressed in between rigid plates. For a long column of porous sample (typically $s < 0.025$), the static compression stiffness will not be affected by Poisson's effect or the boundary conditions. In such a case the column can be characterized by Young's modulus given by

$$E = \frac{L}{A} K \tag{3}$$

where A is the cross-sectional area of the column and L is the thickness of the sample. In case of high shape factors, the Poisson's effect and boundary conditions produce lateral deformation and they can not be neglected. In this case the young's modulus is rewritten as:

$$E = \frac{K.L}{A.P_s(\nu)} \tag{4}$$

where the term $P_s(\nu)$ is introduced as a correction factor to obtain true value of Young's modulus from apparent value of Young's modulus.

For the determination of the mechanical properties, FEM simulations are used. The poroelastic sample are modeled as solid material using elasticity theory. These FEM simulations are executed for a fixed value of Young's modulus with varying values of shape factor and Poisson ratio. The ranges of Poisson ratio are 0 to 0.48 and shape factor from 0.1 to 2.1. These simulated values are related by polynomial curves (order 10) in *s*. Once these polynomial curves are ready, then it is possible to calculate the mechanical properties using following procedure.

I] first measurement of compression stiffness (K_1 and K_2) is carried out on two samples having different shape factors (S_1 and S_2).

II] then polynomial curves (order 8) are fitted for these samples with two shape factors given by

$$E_1 = \frac{K_1 \cdot L_1}{A_1 \cdot P_{s_1}(\nu)} \quad \text{and} \quad E_2 = \frac{K_2 \cdot L_2}{A_2 \cdot P_{s_2}(\nu)} \tag{5}$$

III] since these two samples are cut from same piece of material, mechanical properties are same for both of them and so it is possible to calculate Young's modulus and Poisson ratio with equations 5 using E_1-E_2=0.

IV] the loss factor is calculated from the mechanical impedance by using equation 2.

3 Measurement Setup

The measurement test rig is shown in Figure 2. It consists of two disc shaped metallic plates. The lower plate is fixed on an electrodynamic shaker which excites the plate with sine-sweep in the frequency range 10-80 Hz. The measurements were carried out below any resonance of the system. An accelerometer is mounted on lower plate. The upper plate is mounted on a rigid structure through a static force transducer. The porous sample is placed in between these two plates with double-sided adhesive tape on lower plate. The sample is slightly compressed, by using a crank system, and strain applied is very low so that material behavior should maintain linear. The upper plate is coated with sandpaper so that the sample can not move sidewise during the tests. The measurements can also be repeated in vacuum to evaluate the effect of air inside the sample. The measurement set-up consists of following instruments:

- Shaker B&K Type 4809
- Power Amplifier B&K Type 2716C
- Force Transducer PCB 288D01
- Accelerometer B&K Type 4501
- Signal Conditioner B&K Nexus e MESA MUX10A
- Lase Vibrometer Polytech OFV3001
- PC equipped with Sound Card ESI Wami Rack 192X
- Edwards RV12 Vacuum pump.

LabVIEW® and Matlab® routines have been developed for signal acquisition and post-processing.

To calibrate the system and minimize the uncertainties from the transfer functions of the tested sample calibration procedure proposed by Marietz *et al.* [2] is used. In this case a spring is tested on the system in the frequency range of interest and stiffness of the spring is assumed to be constant in the frequency range and equal to its static value K_0. Then it is possible to define a calibration function H_c (complex and function of the frequency) and the calibrated compression stiffness K_c in the following way:

$$H_c(\omega) = \frac{1}{K_0} \frac{F(\omega)}{D_1(\omega)}\bigg|_{spring} \quad \text{and} \quad K_c = \mathrm{Re}\left\{\frac{K^*(\omega)}{H_c(\omega)}\right\} \tag{6}$$

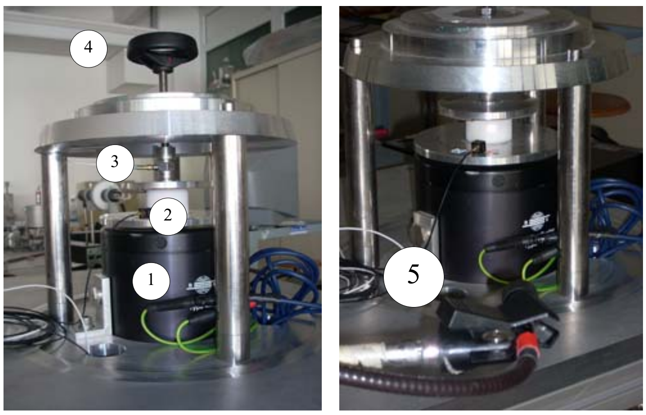

Fig. 2 Test Rig: 1-shaker, 2-accelerometer, 3-force transducer, 4-system for compression, 5-laser

4 Results and Discussions

In this paragraph, experimental results on three poroelastic materials are presented and discussed. The effect of static compression as well as effect of the variation in the level of dynamic excitation is investigated. The description of the tested materials is summarized in the Table 1. Two samples from each material were cut with diameters 45 mm and 30 mm, named S_1 and S_2 for each sample respectively.

Materials	Thickness [mm]	Density [Kg/m^3]	Flow Resistivity [Ns/m^4]	Porosity [-]
Melamine **[M30]**	28	10	10800	0.99
PU-Foam **[PU20]**	20	30	30372	0.99
Rockwool **[RW20]**	20	110	77290	0.99

Table 1 - Description of the tested materials

4.1 Comparison between the two measurement Techniques

Table 2 shows values of compression stiffness K_1 and K_2 and T for samples S_1 and S_2, calculated as the average value between 20 and 40 Hz. In the same table the values of the Young's modulus (E), Poisson ratio (ν) and the damping loss factor (η) are given calculated from the methods A and B. It has to be underlined that variations lower that 5% for K_1 and K_2 and not higher than 2% for T have been found during tests.

				Method A		**Method B**		
Material	K_1 [N/m]	K_2 [N/m]	T [-]	E [Pa]	ν [-]	E [Pa]	ν [-]	η [-]
M30	6816	3345	0.196	114079	0.27	106343	0.48	0.07
PU20	15532	6353	0.330	169374	0.33	154309	0.36	0.17
RW20	20411	9258	0.008	256787	0.01	238400	0.09	0.15

Table 2 - Comparison between the two methods

From the table it is possible to observe that the values Young's modulus from both methods are consistent. There is some variation in values of Poisson ratio obtained from two methods mainly for material M30. Differences can be explained as follows. Method B assumes that the measurement of compression stiffness for two samples of same material is carried out in same mounting condition. Moreover it is assumed that poroelastic materials are perfectly elastic, homogenous and isotropic in nature. Consequently viscoelasticity and non homogeneity of the same material samples, measurement uncertainties (mainly due to the mounting of the specimens) and also non parallelism between the two rigid plates (found in the equipment) could lead to variations in the compression stiffness, which are justifiable only by means of overestimated values of the Poisson ratio.

As an example for sample S_2 of all materials the expected compression stiffness was simulated with FEM model using Young's modulus and Poisson ratio from Method A; the results are reported in Table 3 with the measured values (with 5% relative error in K_2).

Materials	Measured (K_2) [N/m]	Expected from FEM [N/m]
M30	3345 ± 167	3008
PU20	6353 ± 318	6548
RW20	9258 ± 463	9073

Table 3 - Comparison between the measured compression stiffness and expected for the sample S_2

From the analysis of results in the previous table, it is found that even if the values of compression stiffness are comparable with the results from FEM model, the polynomial relations could lead to large errors in the determination of the mechanical properties.

4.2 Effect of the static and dynamic amplitudes

In the present paragraph the values of mechanical parameters obtained from varying the level of excitation of the shaker (*dynamic deformation*) and the initial deformation (*static deformation*) are reported. At first the sample was mounted with some static pre-compression so that it will not slide during the measurement. After that, the compression was increased with linear step 0.1 mm.

Regarding dynamic deformations, they have been used *rms* values of the displacement of lower plate equal to 0.06 mm, 0.08 mm, 0.12 mm, 0.17 mm e 0.24 mm.

Here it is interesting to underline that the applied deformations on all materials were lower than 5%; such condition should ensure linear behavior of the material [4].

Because of limitations in method B, it is not possible to calculate mechanical parameters for all the tested materials for both static and dynamic loads. Thus results obtained by means methods A will be presented.

In figures 3-5, the graphs for the mechanical parameter for melamine foam are shown.

The average values of all the elastic parameters, calculated by varying both static and dynamic deformations, are listed in tables from 4 to 6. Also the difference between the maximum and minimum values is also given for all materials.

From graphs, it can be observed that:

- Young's modulus increases with increase in the static load and diminishes when the dynamic deformation increases. Similar results have been found for materials PU20 and RW20.
- Poisson ratio is practically constant for all the values of static and dynamic deformations. Similar results have been found for materials PU20 and RW20.
- The damping does not follow a regular trend for all the tested materials.

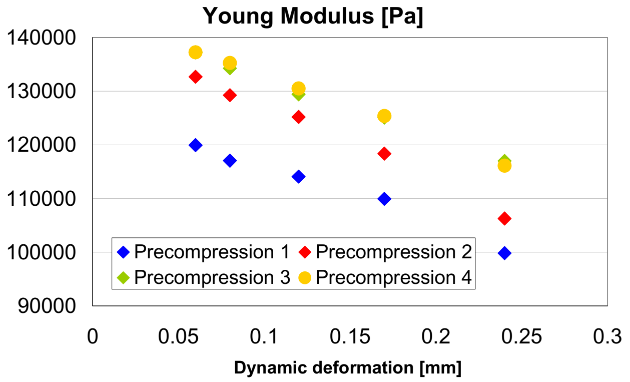

Fig. 5 Graph for Young's modulus as a function of the dynamic deformation

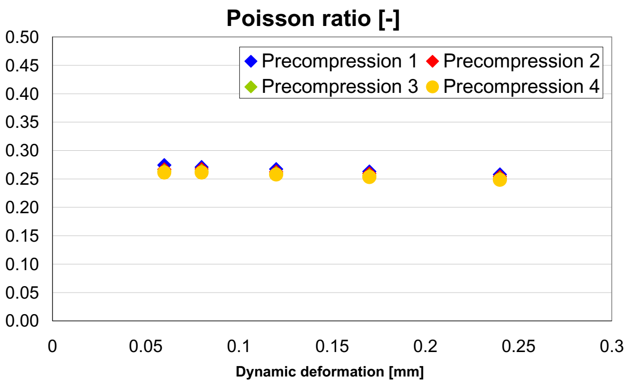

Fig. 6 Graph for Poisson ratio as a function of the dynamic deformation

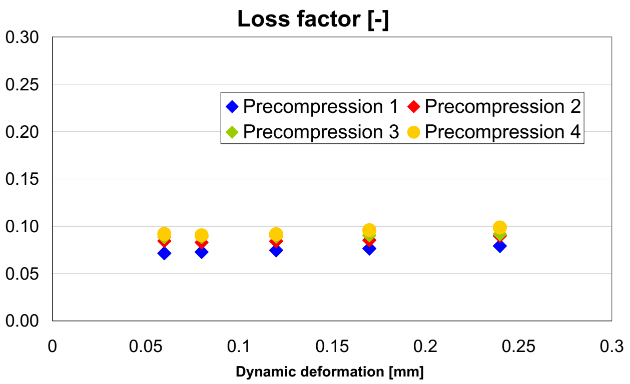

Fig. 7 Graph for loss factor as a function of the dynamic deformation

Parameters	Parameter Values	Difference (Max- Min) Values
E [Pa]	122999	23179
ν	0.26	0.01
η	0.09	0.01

Table 4 - Average Material Properties for M30

Parameters	Parameter Values	Difference (Max- Min) Values
E [Pa]	178004	61488
ν	0.34	0.17
η	0.07	0.05

Table 5 - Average Material Properties for PU20

Parameters	Parameter Values	Difference (Max- Min) Values
E [Pa]	294293	96139
ν	0.01	0.002
η	0.15	0.01

Table 6 - Average Material Properties for RW20

The analysis of data shown in previous tables allows to underline the strong dependence of Young modulus from the deformations (variations around 30% for materials can be found for materials PU20 and RW20). Regarding the Poisson ratio and the loss factor, material PU20 exhibits variable behavior. On the contrary the effect of deformations on ν and η can be neglected for materials M30 and RW20.

5 Effect of Vacuum on compression stiffness of the materials

In this section the effect of vacuum on melamine foam is discussed. Tests were carried out at different pressures from 3 mbar up to the atmospheric pressure.
In vacuum chamber, pressure is increased from 3 mbar to the atmospheric pressure.

Because of the geometry of the vacuum system, it is important to underline that tests of lateral deformation by means of laser vibrometry can not be carried out at the present. For this reason only compression stiffness and loss factor have been measured and compared.

The results for the compression stiffness of sample S_1 are shown in the Figure 8. In Figure 9 the same analysis is reported for loss factor.
The uncertainties in the measurements of compression stiffness and loss factor were found lower than 200 [N/m] and 0.01 respectively.

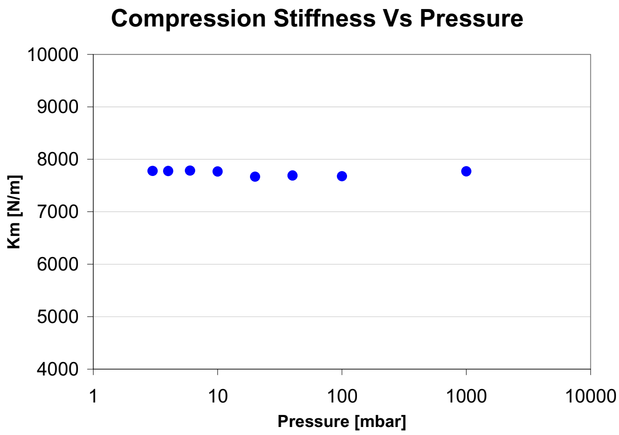

Fig. 8 Effect of Pressure on Compression Stiffness

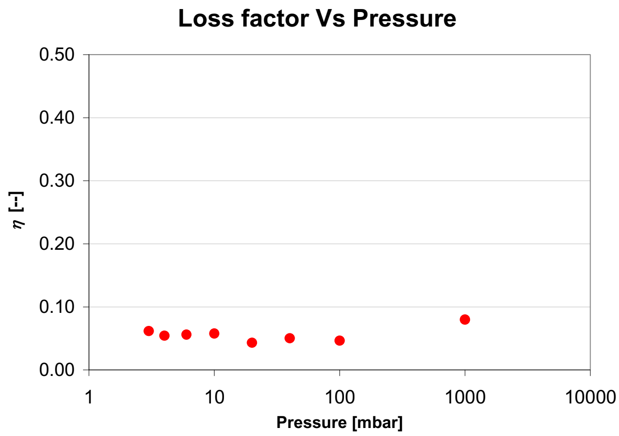

Fig. 9 Effect of Pressure on Loss factor

From these graphs it can be noticed that there is no significant effect of vacuum on compression stiffness and loss factor.

6 Concluding remarks

In the present paper two different quasi-static methods for the determination of the mechanical properties of poroelastic materials have been investigated. The analysis has pointed out some limitations of the measurement test-rig, mainly related to the test procedure of the two samples method. Among the possible causes, the non homogeneity of the specimens of the same materials has been taken into account.

Furthermore, tests by means of the method based on mechanical impedance and lateral deformation measurements have been carried out by varying the excitation level from the shaker and the static initial preload. The analysis has permitted to notice that the effect of such deformations on the mechanical properties is not negligible and it could lead to variations of around 30% for Young modulus.

Future investigations of the proposed research will be devoted to the enhancement of the measurement device. In particular measurements will be repeated by using a static force transducer in order to control the correct positioning of samples and ensure the same static pre-compression when two different samples of the same material are tested. Moreover the system will be modified in order to investigate the vacuum effect on the intrinsic mechanical properties.

Finally, in order to overcome limitations due to the approximation of the test sample as a linear elastic solid, FEM viscoelastic models will be developed for taking into account the effect of deformations.

References

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