# The hydrophone free-field calibration in the non-anechoic water tank using continuous radiation mode

Alexander Isaev and Anton Matveev

VNIIFTRI, Mendeleevo, 141570 Moscow, Russian Federation
matveev@vniiftri.ru

The method of direct projector signal selection in the non-anechoic measuring tank at continuous radiation mode is considered. The method is based on complex averaging of the frequency response of reduced transfer impedance projector and hydrophone. It is allow reducing influence of reflection and obtaining frequency response closed to the free-field frequency response. Experimental results of this method and its application for hydrophone reciprocity calibration in the non-anechoic measuring tank at continuous radiation mode are presented.

# 1 Introduction

In the some applications the hydrophone calibration on 1/3 octave frequencies is insufficient and it is required to measure a practically continuously frequency response with the steady step of frequency tuning. In these cases, tone signal and tone burst radiation technique usage leads to the unacceptable measuring efforts. Use of signal with the distributed in frequency band power allows to reduce time of measurement significantly, but requires in continuous radiation mode. At continuous radiation mode in non-anechoic measuring tank it is possible to provide free-field conditions with reducing the reflection influences by the use of special processing of receive signal.

Procedures allowing significantly to reduce the reflection influences in a hydro acoustic channel (such as complex conjugation or time reversal of wave, phase conjugation) are widely popularized in present. At measurements in natural and laboratory conditions the time-delay spectrometry (TDS) is widely applied. TDS represents one of ways of realization of a principle of selection of a direct signal of the radiator based on properties of complex transfer function of measuring tank.

The method of direct projector signal selection based on complex averaging of the frequency response of reduced transfer impedance RTI projector and hydrophone and its application for free-field calibration of hydrophone will be considered below.

# 2 Theory

For a point projector and the receiver carried in infinite homogeneous space on distance $r$, the current through a projector $i(t)$ represents non-zero process at frequency interval $[f_0 - \Delta f/2, f_0 + \Delta f/2]$ with spectral density $I(f) = F[R_i(\tau)]$ (where $R_i(\tau)$ – autocorrelation function of process, F – means Fourier transform).

Denote by $\dot{U}_{i,u}(f) = F[R_{i,u}(\tau)]$ the cospectral density of a current through a projector and voltage on an output of the receiver $u(t)$ ($R_{i,u}(\tau)$ – cross-correlation function of processes $i(t)$ and $u(t)$). Transfer function of system a projector-receiver $\dot{Z}_{PH}(f)$, carried on distance $r$ in a free field of a spherical wave, we represent as product of a reduced transfer impedance of a projector and the receiver (RTI projector-receiver) $\dot{Z}_{PH,sph}(f)$ on complex spatial function $\dot{\Omega}(f) = \frac{r'}{r}\exp[-jk(r-r')]$ [1]:

$$\dot{Z}_{PH}(f) = \dot{Z}_{PH,sph}(f)\dot{\Omega}(f) \quad (1)$$

Using a ratio for spectra of processes on an input and an output of linear system [2] and expression for transfer function $\dot{Z}_{PH}(f)$ Eq.(1), cospectral density at frequency spacing $[f_0 - \Delta f/2, f_0 + \Delta f/2]$ we represent as

$$\dot{U}_{i,u}(f) = \dot{Z}_{PH,sph}(f)\dot{\Omega}(f)I(f), \quad (2)$$

from which follows

$$\dot{Z}_{PH,sph}(f) = \frac{\dot{U}_{i,u}(f)}{I(f)}\dot{\Omega}(f)^{-1}. \quad (3)$$

At continuous radiation in water tank, sound pressure $\dot{p}_\Sigma(f)$ in the receiving point is created by a direct wave of a projector and the waves reflected by a bottom, walls and a surface of water. In the assumption of final number of significant reflections:

$$\dot{p}_\Sigma(f) = \dot{p}_0(f) + \sum_{i=1}^{n}\dot{p}_{ref,i}(f), \quad (4)$$

where $\dot{p}_0(f)$ – sound pressure of a direct wave, $\dot{p}_{ref,i}(f)$ – sound pressure of $i$-th reflection. Formula describing a sound pressure in the receiving point in the presents of reflection is indenting the formula of sound pressure in the presents of scattereres [1]. It allows to use for the describing transfer function of tank in continuous radiation the expression similar to expression for transfer function of scattering irregularity [1].

Let us express RTI projector-receiver in a water tank through RTI in undisturbed field of spherical wave and complex functions describing phase delays of reflected waves with respect to direct wave of a projector [3]:

$$\dot{Z}'_{PH,sph}(f) = \dot{Z}_{PH,sph}(f)\left[1 + \sum_{i=1}^{n}\dot{W}_i(f)\dot{\Omega}_{ref,i}(f)\right]. \quad (5)$$

The multiplier in square brackets in Eq.(5) represents the transfer function of a water tank $\dot{\Omega}_{WT}(f)$; $\dot{\Omega}_{ref,i}(f) = \frac{r}{r_i}\exp[-jk\Delta r_i]$ – function with oscillating at frequency real and imaginary parts; $\Delta r_i = r_i - r$ – a propagation difference of the direct wave and $i$-th reflection; $\dot{W}_i(f) = \frac{\dot{p}_{ref,i}(f)}{\dot{p}_0(f)}\dot{\Omega}^{-1}_{ref,i}(f)$ – factor of $i$-th reflection.

Problem of providing of the conditions equivalent to the free field conditions we shall interpret as a task of the restoration $\dot{Z}_{PH,sph}(f)$ in the frequency dependence $\dot{Z}'_{PH,sph}(f)$ received at continuous radiation in a water tank.

Let's make following typical assumptions about behavior of functions $\dot{Z}_{PH,sph}(f)$ and $\dot{W}_i(f)$ in the considered frequency interval: parameter $\dot{W}_i(f)$ weakly changes with frequency (constant); period of oscillation of function $\dot{Z}_{PH,sph}(f)$ is much more the period of oscillation of function $\dot{Z}'_{PH,sph}(f)$ caused by functions $\dot{\Omega}_{ref,i}(f)$. Not breaking a correctness of the further reasoning parameters $\dot{W}_i(f)$ we shall accept equal to one, and a constant multiplier $r / r_i$ for reduction of records we shall omit.

Estimation of RTI in undisturbed field at frequency $f_0$ receive by a complex averaging $\dot{Z}'_{PH,sph}(f)$ at frequency range $[f_0 - \Delta f/2, f_0 + \Delta f/2]$:

$$\hat{\dot{Z}}_{PH,sph}(f_0) = \frac{1}{\Delta f} \int_{f_0-\frac{\Delta f}{2}}^{f_0+\frac{\Delta f}{2}} \dot{Z}'_{PH,sph}(f)df = \dot{Z}_{PH,sph}(f_0,\Delta f) + \sum_{i=1}^{n} \dot{\varepsilon}_i(f_0,\Delta f) \quad (6)$$

where:

$\dot{Z}_{PH,sph}(f_0,\Delta f) = \frac{1}{\Delta f} \int_{f_0-\frac{\Delta f}{2}}^{f_0+\frac{\Delta f}{2}} \dot{Z}_{PH,sph}(f)\,df$ – average value of RTI projector-receiver at frequency interval in free field; $\dot{\varepsilon}_i(f_0,\Delta f) \approx \dot{Z}_{PH,sph}(f_0,\Delta f) \frac{1}{\pi \Delta f \Delta t_i} \sin(\pi \Delta f \Delta t_i) e^{-j2\pi f_0 \Delta t_i}$ – summand, caused by influence of $i$-th reflection; $\Delta t_i = \Delta r_i / c$ – time delay of $i$-th reflection, $c$ – sound speed in water. The right part of Eq.(6) includes an oscillated real function which decreases with a widening of frequency interval $\Delta f$ and becomes equal zero for $\Delta f$ multiple to $1 / \Delta t_i$. From this follows, that at the certain choice of a frequency interval $\Delta f$ the estimation $\hat{\dot{Z}}_{PH,sph}(f_0)$ will practically coincide with average value of RTI projector and receiver at frequency interval in free field.

Usually the first reflection makes a largest contribution to distortion of frequency dependence of RTI projector and receiver. Influence of the first reflection eliminates practically completely by complex uniformly precise averaging of RTI projector-receiver in the strongly defined frequency interval $\Delta f_1 = 1 / \Delta t_1$. Residual influence of second reflection is eliminated by repeated averaging at frequency interval $\Delta f_2 = 1 / \Delta t_2 < \Delta f_1$. As a result total reducing of influence of third and following reflection makes not less than 25 times that appears sufficient.

If frequency response $\dot{Z}'_{PH,sph}(f)$ is measured at interval much exceeding $\Delta f_1$ the estimation of frequency response RTI projector-receiver in free field reduced to the uniformly precise moving averaging of $\dot{Z}'_{PH,sph}(f)$ at frequency interval $\Delta f_1$ and repeated uniformly precise moving average at frequency interval $\Delta f_2$. Such processing is equivalent to the single weighted averaging with use of weighting function $h(f')$ set at interval $[-(\Delta f_2 + \Delta f_1) / 2, (\Delta f_2 + \Delta f_1) / 2]$, looking like a trapezoid with top and bottom bases accordingly $\Delta f_1 + \Delta f_2$ и $\Delta f_1 - \Delta f_2$, received by convolution of two rectangular weighted functions with width $\Delta f_1$ and $\Delta f_2$ [3]. Thus, the minimal frequency interval of created test process should coincide with the minimal interval of averaging $\Delta f_1 + \Delta f_2$.

Frequency interval $\Delta f_{eff} = \Delta f_1 + \Delta f_2$, at averaging in which the conditions close to conditions of a free field are provided, we shall name an effective bandwidth of measurements in water tank, by analogy to an effective bandwidth of casual process. Effective bandwidth $\Delta f_{eff}$ we shall express by sound speed in water and propagation differences of the direct wave and reflections:

$$\Delta f_{eff} = c \frac{\Delta r_2 + \Delta r_1}{\Delta r_2 \Delta r_1}. \quad (7)$$

Relation

$$\Delta r_{eff} = \frac{\Delta r_2 \Delta r_1}{\Delta r_2 + \Delta r_1} \quad (8)$$

- effective size of water tank. At continuous radiation the ratio of effective bandwidth and effective size

$$\Delta f_{eff} \times \frac{\Delta r_{eff}}{c} = 1 \quad (9)$$

can be interpreted as a relation for time-and-frequency transformation $\Delta f \times \Delta t_1 = 1$. The more the difference of a propagation of the direct wave and reflected signals in water tank, the more its effective size and narrowly effective bandwidth, at measurements in which tank becomes "sound transparent" and represents "boundless" space.

The considered method of direct projector signal selection realizes the property of transfer function of measuring tank $\dot{\Omega}_{WT}(f)$ at effective bandwidth, which can be expressed as:

$$\left| \frac{1}{\Delta f_{eff}} \int_{-\frac{\Delta f_{eff}}{2}}^{\frac{\Delta f_{eff}}{2}} h(f') \dot{\Omega}_{WT}(f - f')df' \right| \approx 1 \quad (10)$$

The complex averaging of frequency response of RTI projector-receiver allows at measurements in water tank to receive the response of RTI close to free field measuring. Result of such processing is frequency dependence RTI in free-field, average in an effective bandwidth.

$$\dot{Z}_{PH,sph}(f,\Delta f) = \frac{1}{\Delta f} \int_{-\frac{\Delta f_{eff}}{2}}^{\frac{\Delta f_{eff}}{2}} h(f') \dot{Z}_{PH,sph}(f - f')df' \quad (11)$$

Because weighing function $h(f')$ is known, the distortions of frequency response caused by averaging can be substantially reduced by means of proven methods of spectrum reconstruction under the form of the filter.

# 3 Experiment

## 3.1 Direct projector signal selection in the non-anechoic water tank at continuous radiation mode

Quality of selection at measurements was estimated experimentally by check inversely proportional low of dependence of sound pressure with distance at continuous radiation in non-anechoic water tank (reverberation time 200 ms) width 6.5 m, length 10 m, water level 5 m. Violation of inversely proportional low was estimated on the dependence of transfer impedance values in free field, received by complex averaging from distance between projector and receiver. For this purpose were graphed $1/Z_{PH}$ and a line of the best approximation of this dependence. Because in real the response $1/Z_{PH}$ can be distorted by the scattering at mounts and bodies of hydrophones, it was compared with response received at tone-burst radiation. For scattering reduction as a radiator and the receiver were applied bodiless hydrophones representing strengthened on the cable piezoelectric sphere, which were placed in water tank at depth of 2.5 m by means rigid mounts in form of needle. Distance between projector and receiver changed from 0.83 m to 4.3 m with step 0.5 m. Difference of propagation of direct wave and first reflection remained a constant and equaled 2 m. Difference of propagation of direct wave and second reflection changed from 4 to 2.1 m. Frequency interval of averaging calculated for all positions of transducers and changed from 1071 Hz at minimal distance to 1412 Hz at maximal. Frequency responses of RTI at continuous radiation were measured with use chirp signal, frequency of which was changed no less than 1412 Hz. For each measuring distance complex averaging of frequency response of RTI was carried out at corresponding frequency interval.

On Fig.1, responses of values of $1/Z_{PH}$ from distance between projector and receiver at continuous and tone-burst radiation at 10 kHz and 20 kHz and lines of the best approximation of responses are shown.

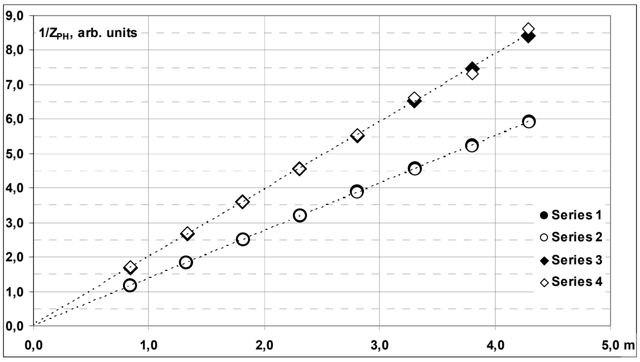


Fig.1 Responses of $1/Z_{PH}$ with distance between projector and receiver, received at frequencies: 10 kHz at tone-burst (series 1) and continuous (series 2) radiation, 20 kHz at tone-burst (series 3) and continuous (series 4) radiation.

For 10 kHz mismatches of responses at continuous and turn-burst radiation are not exceeds 0.5 % at distances up to 3 m and reaches 2 % at distances more than 3 m. Approximately likewise response at continuous radiation deviates from line of the best approximation of response at tone-burst radiation. At frequency of 20 kHz deviations of responses at continuous radiation from tone-burst radiation and from line of the best approximation of response are not exceeds 0.5 %.

On Fig.2 are presented similar shown on Fig.1 to response for frequencies 63 кГц and 100 кГц.

Mismatching for these frequencies at continuous and tone-burst radiations and their deviations from lines of the best approximation of response at tone-burst radiation are not exceeds 0.8-1.7 %. It indicates that the applied method of moving averaging of complex frequency response yields the results coincides with accuracy demanded for standard measuring with results, received by method of tone-burst radiation. Proximity of the received responses to theoretical lines allows to apply the specified method in procedure of a method of reciprocity.

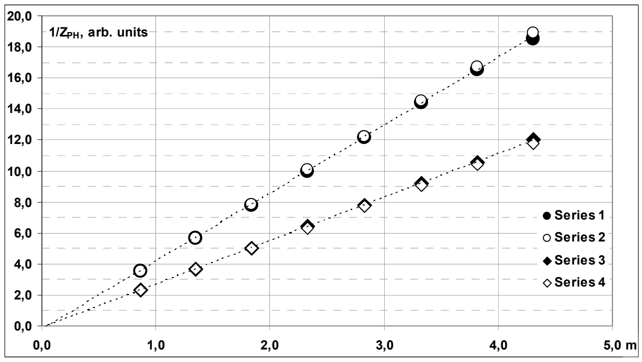


Fig.2 Responses of $1/Z_{PH}$ with distance between projector and receiver, received at frequencies: 63 kHz at tone-burst (series 1) and continuous (series 2) radiations, 100 kHz at tone-burst (series 3) and continuous (series 4) radiations.

For the decision of some measuring tasks, the receiver should be measured “on diffuse field”. Diffuse field is define as field simultaneously being isotropic (directions of distribution of a sound wave in each point are equiprobable) and homogeneous (the density of energy in the set area of space is statistically uniform). The consequences following from this definition are absence of inversely proportional law of dependence of sound pressure with distance between projector and receiver and absence in diffuse field property of “directivity” of receiver. Diffuse field create in reverberation chamber at radiation continuous band process (deterministic or random). On Fig.3 directional responses of hydrophone measured in non-anechoic (reverberation) water tank at continuous radiation with use considered method of direct projector signal selection (series 2 and 4) and at tone-burst radiation of harmonic signal (series 1 and 3) are presented.

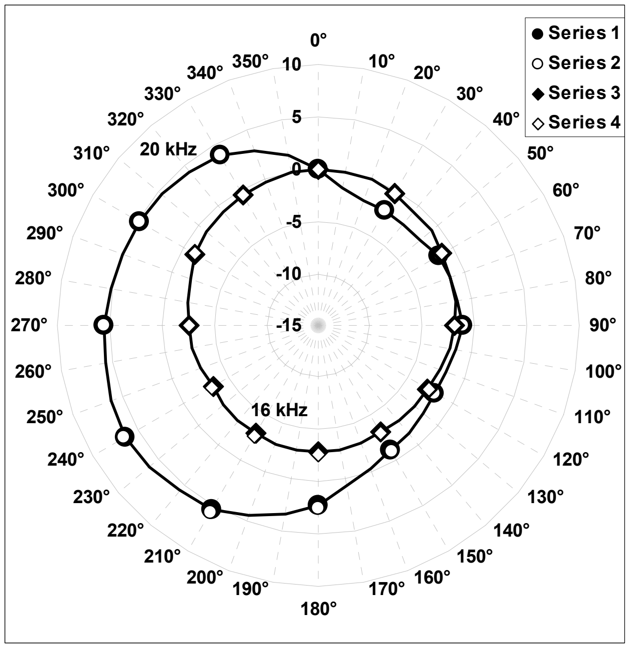


Fig.3 Directional responses of hydrophone measured for 16 kHz и 20 kHz at continuous (series 2 and 4) and tone-burst (series 1 and 3) radiation accordingly.

The received directional responses practically do not differ (mismatching of series 1 with a series 2 and of series 3 with a series 4 do not exceed 0.2 dB). Results show that at continuous radiation in reverberation chamber (non-anechoic water tank) consideration of a field as diffuse field is incorrect. The reverberation field that appears as completely determined is in such a way created. Application correspond processing of received signal allows to allocate amplitude and a phase as direct wave as reflection waves. This fact explains the reason of that in practice at measuring of the receiver "on diffuse field" it is necessary to change a relative positioning of the receiver and a projector in reverberation chamber and to average the results received at various arrangements. Thus, contrary to a diffused opinion, a possibility of receiving in reverberating field of the responses close to responses in diffuse field, does not exclude a possibility of receiving of the responses close to "a free field".

## 3.2 Hydrophone calibration at continuous radiation in non-anechoic water tank

Considered method of selection was applied for reciprocity calibration of hydrophone at continuous radiation in non-anechoic water tank with the minimal size 6 m and reverberation time 200 ms. Transducers were placed on depth of 3 m. Distance between projectors and receivers was 0.5 m. Delays of the first and second reflection concerning a direct signal were 3.8 ms and 4.1 ms, accordingly. RTI projector-receiver averaged at frequency interval $\Delta f_{eff} = 505$ Hz. Sensitivity of a hydrophone $M_H(f)$ was calculated as:

$$M_H(f) = \sqrt{J_{sph}(f)\frac{\left|\dot{Z}_{PH,sph}(f,\Delta f_{eff})\right|\left|\dot{Z}_{TH,sph}(f,\Delta f_{eff})\right|}{\left|\dot{Z}_{PT,sph}(f,\Delta f_{eff})\right|}}, \qquad (12)$$

where:
$J_{sph}(f)$ – reciprocity factor at frequency $f$;
$\dot{Z}_{PH,sph}(f, \Delta f_{eff})$, $\dot{Z}_{TH,sph}(f, \Delta f_{eff})$, $\dot{Z}_{PT,sph}(f, \Delta f_{eff})$ – accordingly, RTI projector-hydrophone, reversible transducer-hydrophone, projector-reversible transducer in undisturbed field of spherical wave at frequency $f$ average at effective bandwidth $\Delta f_{eff}$. Correction of estimations on the form of the filter was not spent.

On Fig.4 by series 1 and 2 the results of calibration of hydrophones type 1 and type 2 at continuous and tone-burst radiation on 1/48 octave frequency are shown. The amount of the values received at continuous radiation, considerably exceeds 48 in an octava interval of frequencies.

The presented responses show very good agreement of results of calibration of hydrophone type 1 (differences between series 1 and 2 do not rank over 0.03 dB). Results for a hydrophone type 2 also show good agreement, except for areas of antiresonances. Mismatching of results on these areas achive 1 dB, thus the width of areas makes a small part of an octave. The continuous radiation causes an error depended by substitution in the formula of measurements of values RTI, received non-uniformly precise averaging in an effective bandwidth. Difference of sensitivity the result received on harmonious signals by a standard method of reciprocity, was showed on frequencies close to the expressed resonances of hydrophones where sensitivity sharply changes with change of frequency. It makes "payment" for decrease in laboriousness of measurements.

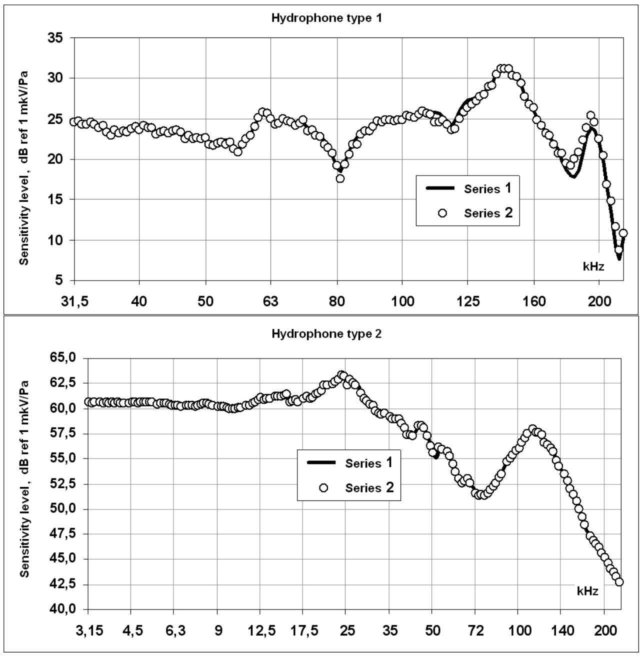


Fig.4 Frequency responses of hydrophones type 1 and 2 measured at continuous radiation (series 1) and tone-burst radiation with frequency step of 1/48 octave (series 2).

The calibration method at continuous radiation with use complex averaging frequency response RTI projector-receiver on effective bandwidth is realized and successfully applied in working standard for hydrophone calibration.

# References

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[2] J. S. Bendat, A. G. Piersol, "Random data: analysis and measurement procedures", Moscow, Mir, 1974, (in Russian)

[3] A.E. Isaev, "Transfer function of water tank and effective "frequency window" of free-field measurements", *Proc. of conf. "Metrological problems of hydrophysical measurements"*, VNIIFTRI, p. 1, 2006 (in Russian)