IC engine velocity fields, spherical harmonics and evanescent waves

Igor Machetta and Eugenius Nijman

Centro Ricerche FIAT, Strada Torino, 50, 10043 Orbassano (TO), Italy
igor.machetta@fptpowertrain.crf.it

Abstract. Spherical nearfield acoustical holography (SNAH) has been used to reconstruct the volume acceleration distribution close to the surface of an IC engine from the measured sound pressure distribution on a sphere enclosing it. Such reconstruction frequently requires the calculated target spheres to intercept the strongly non-spherical powertrain structure violating, at first sight, the conventional rules of SNAH. In this paper the SNAH results are compared with the results of two alternatives, i.e. an inverse and a power based, source reconstruction method. Notwithstanding the alleged violation of the rules it is shown that surprisingly good results may be obtained provided the spherical spectrum components associated with evanescent waves are omitted from the back propagation calculation.

1 Introduction

During the EC Framework 6 SILENCE project spherical acoustical holography has been used to reconstruct the volume acceleration distribution close to the surface of an IC engine, in order to acoustically characterise it. Since the engine shape is not spherical, the calculation of the acoustic field very close to the source surface causes the target spheres to intercept the powertrain, violating, at first sight, the conventional rules of SNAH. Actually, the formulation adopted to back propagate the measured field would also be applicable in the intercepting case provided the Rayleigh hypothesis is met ([1],[2]), which is for instance the case when the reconstruction surface close to the source is a Lyapunov surface ([3]). Unfortunately, however, it is impossible to a priory know whether the source under investigation satisfies such condition. The other important limiting factor for the practical surface velocity reconstruction by means of SNAH is the experimental accuracy. Measurements errors are amplified during back propagation and may introduce considerable inaccuracies. This effect turns out to become particularly important when high order harmonics are included in the back propagation. In order to limit the error built-up a truncation criterion has been introduced in a previous publication ([4]). In the present paper we adopted a new criterion which consists of limiting the wave spectra components to those associated with propagating waves. From an experimental point of view, it's very difficult to obtain a careful evaluation of the nearfield of the engine and even a tiny imperfection may give a large error in the holographic computation of the field on smaller target spheres. In the present paper the volume acceleration distribution of a powertrain obtained using this new criterion is compared to the results derived by two other substitution monopole techniques ([8], [9]).

2 Theory

For an exhaustive treatment of the fundamentals of spherical near-field acoustical holography the reader is referred to [5]. A brief overview is given here. For the present application the general solution for exterior problems is used, meaning that in principle all sources must be contained within both the measurement and the target spheres and no reflecting or scattering objects must be present in the region between the two spheres. Any arbitrary pressure distribution $p(\theta,\varphi)$ on a sphere of radius R can be expanded in terms of spherical harmonics $Y_n^m(\theta,\varphi)$:

$$p(\theta,\varphi) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} P_{mn} Y_n^m(\theta,\varphi) \quad (1)$$

where the spherical harmonic of order n and degree m is an angle function defined as

$$Y_n^m(\theta,\varphi) = \sqrt{\frac{(2n+1)}{4\pi}\frac{(n-m)!}{(n+m)!}} P_n^m(\cos\theta) e^{im\varphi} \quad (2)$$

and P_n^m are associated Legendre functions, θ is the polar angle ($0 \le \theta \le \pi$) and φ is the azimuthal angle ($0 \le \varphi \le 2\pi$). The complex coefficients P_{nm}, also referred to as the spherical wave spectrum, in this application, were calculated with inversion of Eq. (1) (see [6]), without regularization of the spherical harmonics matrix (already good conditioned). If the pressure distribution is due to an arbitrary source distribution which is fully contained within the sphere of radius R and which is radiating into free space, than the spherical wave spectrum on any other concentric target sphere of radius r may be obtained as:

$$P_{nm}(r) = \frac{h_n^{(2)}(kr)}{h_n^{(2)}(kR)} P_{nm}(R) \quad (3)$$

Where k represents the wavenumber and $h_n^{(2)}$ is the spherical Hankel function of the second kind and of order n. This is always true as long as the target sphere of radius r completely contains the source distribution. The spherical wave spectrum W_{nm} of the velocity on the target sphere of radius r is obtained applying Euler's equation:

$$W_{nm}(r) = \frac{1}{i\rho_0 c} \frac{h_n'^{(2)}(kr)}{h_n^{(2)}(kR)} P_{nm}(R) \quad (4)$$

where ρ_0 is the air density, c is the sound speed in the air and i is the square root of -1.

In practice the sound field is sampled at a finite number of discrete positions. This spatial sampling limits the number of spherical harmonics which can be included in the analysis (angular Nyquist criterion). This number not only depends on the amount of microphones on the sphere, but also on their spatial collocation. For the current constant angle distribution (see test rig description in [4]) the following expression applies for the maximum order (see [7]):

$$n_{\max} = \frac{\sqrt{N}+1}{2} \quad (5)$$

In the present case the number of microphones is N = 146 and the antenna treated in this report is consequently suited until n = 6, that is, 49 spherical harmonics.

3 Propagating filter

In the present application to calculate the acoustic field on the target spheres only the spherical components associated with the propagating waves are included in Eq. (3), more in particular the spherical series is truncated at $n = kr$, in order to discard the elements that describe the evanescent behaviour ([5]).

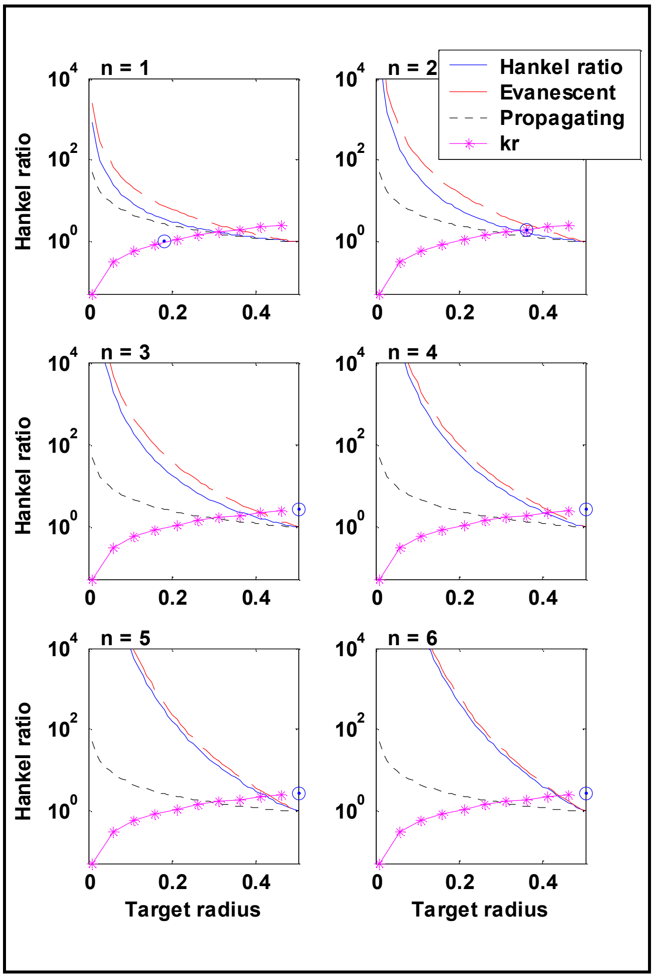

Fig. 1 Hankel functions ratio at 300 Hz.

The six graphics in Figure 2 show an example of the Hankel functions ratio of the Eq. (3), as a function of the radius of the target sphere at 300 Hz, for n from 1 to 6. The evanescent (red dashed line) and propagating (black dotted line) trends are compared to Hankel ratio for target radius from r = 0.5 to r = 0.01m. Moreover the fuchsia line (with asterisks) shows the typical kr behaviour. The blue circle indicates the limit $n = kr$. At low orders the propagating trend is predominant, while as n grows the behaviour becomes similar to that evanescent. The limit $n = kr$ indeed seems to give a good approximation of the change between propagating and evanescent trend. The spherical components calculated on the target spheres with a radius positioned to the left of blu circle will be associated to the subsonic waves, while those to the right will represent the propagating waves. Moreover the Hankel function ratio of higher orders grows rapidly to very high values for decreasing r, considerably amplifying the unavoidable measurement inaccuracies.

4 Description of the acceleration estimate

To calculate the acceleration distribution close the engine surface it is adopted a technique very similar to the power based smt ([8]). In practice the powertrain surface is sampled in a finite number of areas (equivalent monopole sources). The acoustic intensity close to each areas is calculated through holographic computation starting by measured pressure on a sphere of radius 0.5 meter. To estimate the acoustic power it is necessary to associate a surface to the intensity. This is obtained comparing the radiation resistance of a known monopole placed on each sub-area of the engine, measured with an intensity probe and estimated with holography. In detail by intensity probe measurements we get

$$R = \frac{I \cdot S}{|\dot{Q}|} \quad (6)$$

where R is the radiation resistance, I the measured known monopole intensity, $|\dot{Q}|$ the modulus of the volume acceleration and S is the associated scanned surface. By holography it is obtained that

$$R = \frac{W}{|\dot{Q}|} \quad (7)$$

where W is the acoustic power generated by known source estimated on the measurement sphere. Dividing Eq. (6) by Eq. (7) the surface to associate with the measured intensity on the operating engine is obtained. Now the acoustic power generated by each area is known. The volume acceleration distribution is found making the ratio between the acoustic power and the radiation resistance (equation (7)) of each engine area.

5 Intensity calculation

The engine surface has been divided in 58 monopole sources. They are located on the spheres (concentric to the measurements sphere) with a radii between 0.133 and 0.496 meter. In this paper the results on two target spheres are presented: one slightly intercepts the engine (Area A, radius 0.442m, Figure 2), the other strongly cuts it (Area B, radius 0.147m, Figure 2).

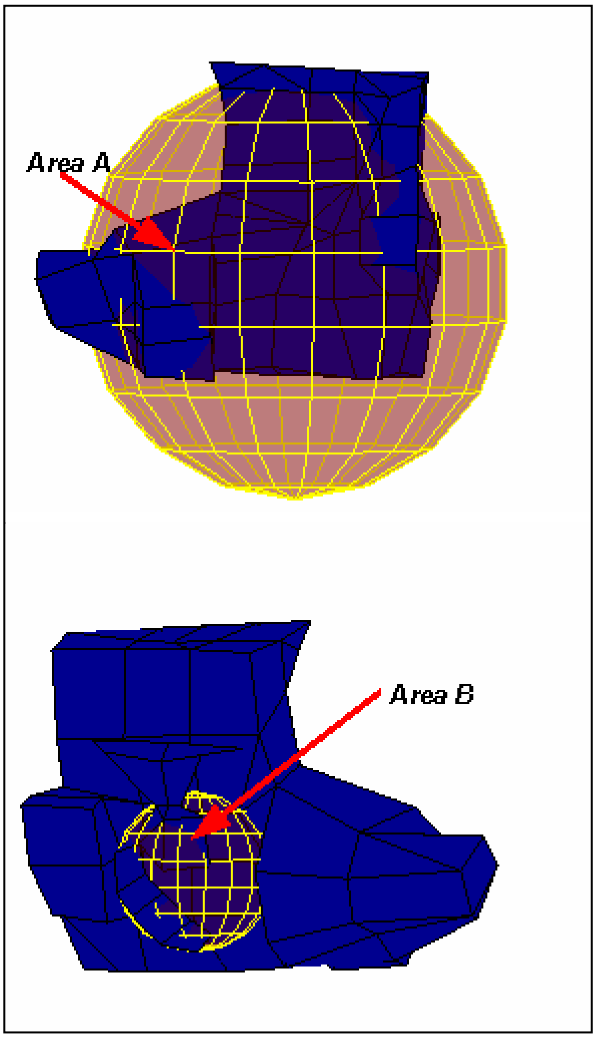

Fig. 2 Calculated target spheres.

Under operating engine conditions the pressure field is measured with a spherical antenna which doesn't intercept the source. From this measurements, pressure and radial velocity on the target spheres are calculated adopting equation (3) and (4) for the back propagation of the field and the real part of their product is the intensity.

The results are shown in Figure 3. The intensity measured with dual microphone probe (black) is compared with the holographic calculation including all 49 harmonics (red dashed), limiting them to the number obtained by comparison between BEM and measurements (see[4]) (blue dotted) and keeping only propagating waves (green dot-dashed). There are no differences between intensity regarding the Area A (little interference of the target sphere with source) obtained using or not BEM filter, whereas including only propagating waves a little improvement at the lower frequencies is gained. Bigger differences are present for the case where the target sphere strongly intercept the powertrain (Area B). With all spherical components included in the back propagation of the field, a very large imprecision occurs. Adopting the BEM filter only results in an improvement at lower frequencies, while limiting the analysis to the propagating waves produces good correspondence in the full frequency range. Most probably also the typical approximations of the BEM code, produce an imperfection (even if small) in the estimate of the acoustic field. This error is enormously amplified when the field is back propagated towards very small target spheres, limiting the usefulness of the BEM filter .

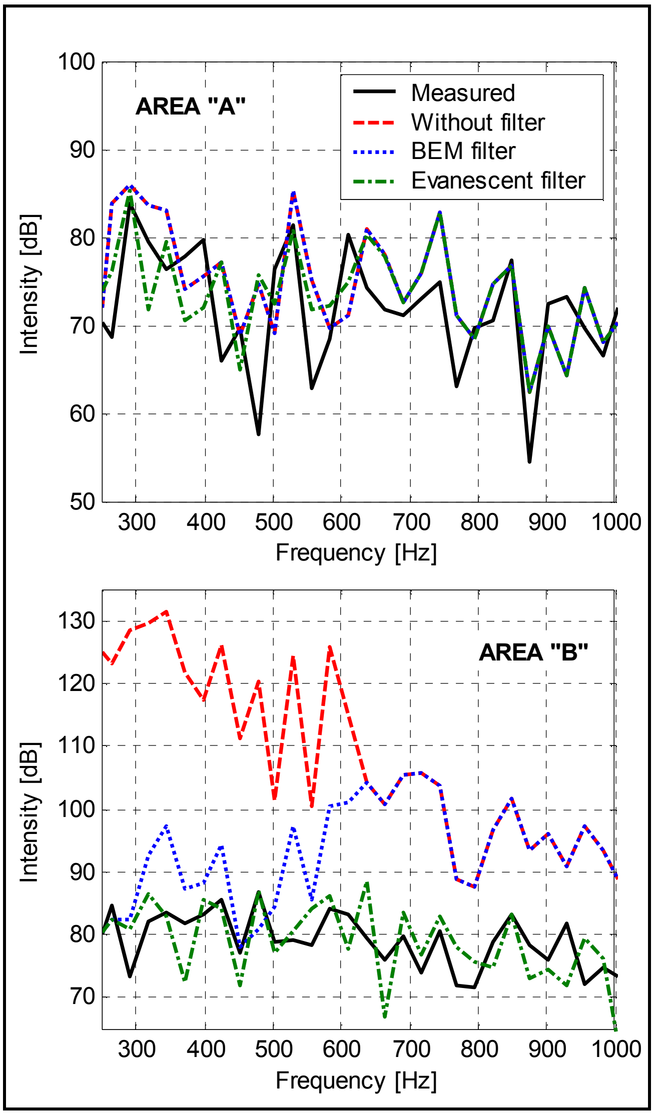

Fig. 3 Acoustic intensity of the two investigated areas.

6 Acceleration results

To reconstruct the volume acceleration distribution of the powertrain, the intensity obtained by only propagating waves is used here. The results adopting intensity calculated with all harmonics and BEM filter were, obviously, worse and they are omitted.

The acceleration derived by holography is compared to that obtained by other two substitution monopole techniques: power-based method ([8]) and inverse method with logarithmic cross validation criteria to regularize the problem ([9]).

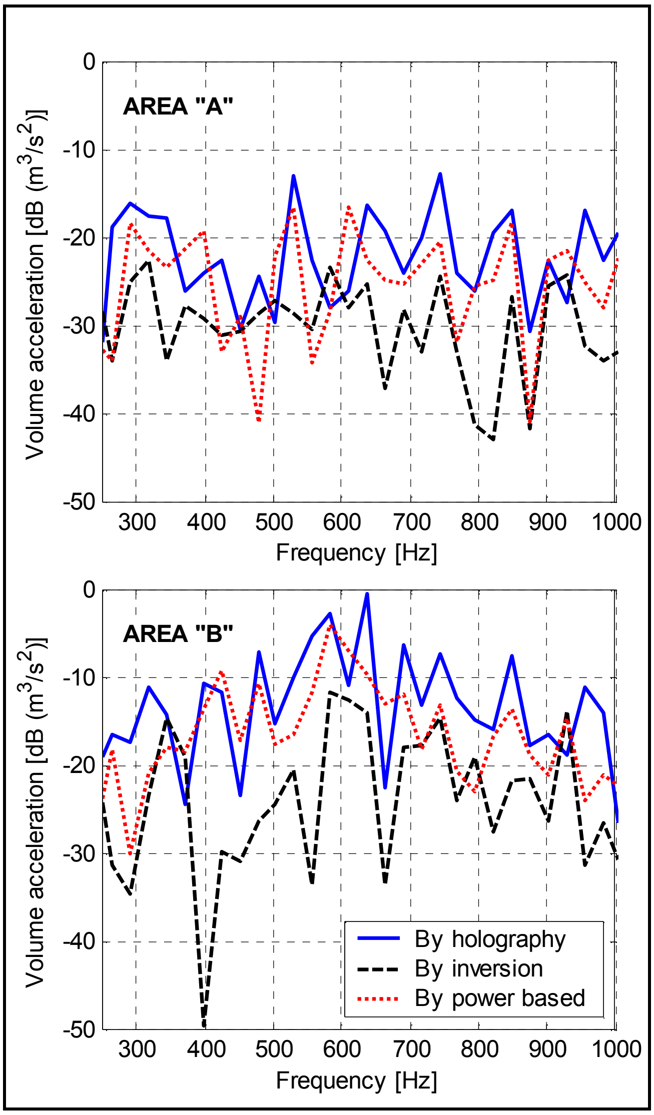

Fig. 4 Volume acceleration of the two investigated areas.

Figure 4 shows the volume acceleration of the two investigated areas. A good correspondence with the results obtained by the other two methods is visible. Only for the Area B, at middle frequencies, the inverse method produces a volume acceleration lower than those obtained by holography and power-based.

7 Summary

In order to acoustically characterize an IC engine, spherical holography was used to estimate its surface volume acceleration distribution. To back propagate the acoustic field close the source surface it is necessary that the target spheres intercept the engine, considering its strongly non spherical shape. In previous work we showed that the measurements layout imposed a limited dynamic range on the spherical wave spectra. Therefore a truncation of the spherical series was necessary and we proposed a criteria to select the spherical components. With the final aim to further improve the results a new method for the selection of spherical elements has been presented here. Only the elements associated with to propagating waves must be included in the spherical transformation. The volume acceleration distribution obtained applying this truncation criterion shows satisfactory agreement with the results of the reference substitution monopole techniques, notwithstanding the target spheres interfere, sometimes very strongly, with the investigated source. Most probably, due to the limited dynamic range of the system, the non perfect measurements of the nearfield doesn't permit the use of the subsonic waves to back propagate the acoustic field on smaller cutting spheres.

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