Influence of acoustic waveguides lengths on self-sustained oscillations: Theoretical prediction and experimental validation

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Human vocal folds and lips of brass instruments players produce self-sustained oscillations due to the interaction between airflow, acoustic waveguides and deformable tissues. This interaction is commonly modeled as a distributed one or two mass-spring system coupled with a simple airflow and acoustic description. This study focuses on the influence of the acoustic waveguide length on the resulting self-sustained oscillation characteristics, i.e. the minimum pressure required to sustain oscillations, the oscillation frequency. Both fixed and varying waveguide lengths are considered. Theoretical predictions with the simplified interaction model are compared to experimental data obtained with a deformable in-vitro replica suitable to produce self-sustained oscillations in presence of an upstream (12, 24 or 32cm) and downstream (varying from 0 up to 235cm) acoustic waveguide. The current study shows the strong influence on the minimum pressure regardless the waveguide length. In addition the waveguide length is shown to impose the resonance frequency for waveguide length superior to 40cm. A rapid change in waveguide length introduces bifurcations between different oscillation regimes.

1 Introduction

Inside the phonatory apparatus, human vocal folds are in interactions with the airflow coming from the lungs and with the resonators (trachea, vocal and nasal tracts). The situation of the trombonist lip's is comparable. One of the main differences is the length of the acoustical resonator. In both cases, this triple interaction between deformable structure, airflow, and acoustics results in the production of self sustained oscillations. Classically, the same types of models are used to describe this phenomenon: a distributed one or two mass-spring system coupled with a simple airflow and acoustic description.

The first objective of this study is to determine experimentally the influence of the acoustical resonators length on deformable structures oscillations (vocal folds, vibrating lips). The second objective is to validate the theoretical predictions of this influence obtained with a two mass model.

2 Theoretical modeling

The theoretical model of the vibrating structure consists in a quasi-steady flow model taking into account viscous effects, a mechanical two mass model, and an acoustical propagation model based on linear acoustic. This theoretical model, schematized in figure 1, is analyzed with two different methods (stability analysis, dynamical analysis).

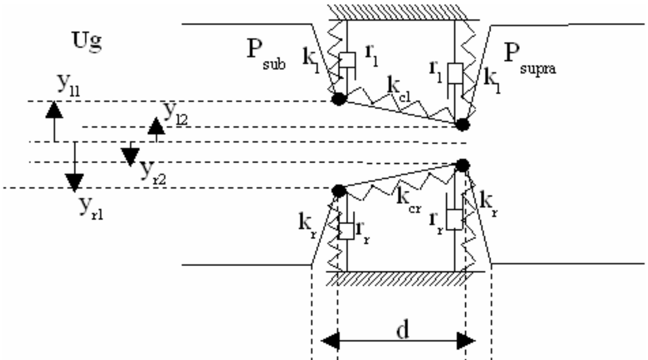

Fig.1 Two mass mode of vocal folds or trombonist lip's. U_g is the volume flow rate. P_{sub} is the upstream pressure. P_{supra} is the downstream pressure. Lg the constriction width, d constriction length. y_{l1}, y_{l2}, y_{r1}, y_{r2} are the mass positions compared to the central axis. k_l and k_r are the spring stiffnesses. k_{cl} and k_{cr} are the coupling spring stiffnesses. r_l and r_r are the damping constants.

2.1 Mechanical description

Each vocal fold (or lips) is represented by two coupled oscillators. It consists in two punctual masses linked to the body and coupled by springs (k_l, k_r, k_c) and dampers (r_l, r_r). Interactions with the airflow are taken into account using the pressure forces F_1 and F_2. The movement of the two vocal folds is supposed to be symmetric. Consequently, we considered the apertures $H_1=y_{l1}+y_{r1}$ and $H_2=y_{l2}+y_{r2}$ to describe the movement, the spring constant $k_l=k_r=k$, $k_c=0.5k$, and the damping constant $r_l=r_r=r$. It results in Eq.(1):

$$
\begin{aligned}
\frac{\partial^2 H_1(t)}{\partial t^2} &= -\frac{2k}{m}(H_1(t)-H_{10}) - \frac{2k_c}{m}(H_1(t)-H_{10}-H_2(t)+H_{20}) \\
&\quad -\frac{2r}{m}\frac{\partial H_1(t)}{\partial t} + \frac{4}{m}F_1(P_{sub},P_{supra},H_1,H_2) \\
\frac{\partial^2 H_2(t)}{\partial t} &= -\frac{2k}{m}(H_1(t)-H_{20}) - \frac{2k_c}{m}(H_2(t)-H_{20}-H_1(t)+H_{10}) \\
&\quad -\frac{2r}{m}\frac{\partial H_2(t)}{\partial t} + \frac{4}{m}F_2(P_{sub},P_{supra},H_1,H_2)
\end{aligned}
\tag{1}
$$

2.2 Airflow description

The airflow coming from the lungs is modeled as incompressible, and quasi-steady. Viscous effects are taken into account by a Poiseuille corrective term. Airflow is separating from the geometry at a point x_s at which energy is dissipated by turbulence effects. The pressure distribution along the constriction can be described by the following equation:

$$
\begin{aligned}
P(x,t) &= P_{sub} - \frac{1}{2}\rho U_g^2\left(\frac{1}{A^2(x,t)}\right) - 12\mu L_g^2 U_g \int_{x_0}^{x}\frac{dx}{A^3(x,t)} \qquad si\ x < x_s \\
P(x,t) &= P_{supra} \qquad si\ x > x_s
\end{aligned}
\tag{2}
$$

where P_{sub} and P_{supra} are the upstream and downstream pressures, ρ the air density, U_g the volume flow rate, μ the dynamic viscosity coefficient, L_g the glottal width, A(x,t) the section area for a given x-coordinate and at time t.

The volume flow rate is supposed to be constant within the constriction and determined by taking into account the pressure drop between the entrance of the constriction and the airflow separation point:

$$U_g = \frac{12\mu L_g^2 \int_{x_0}^{x_1} \frac{dx}{A^3(x,t)} + \sqrt{\left(12\mu L_g^2 \int_{x_0}^{x_1} \frac{dx}{A^3(x,t)}\right)^2 + 2\left(P_{sub} - P_{supra}\right)\rho\left(\frac{1}{A_s^2}\right)}}{\rho\left(\frac{1}{A_s^2}\right)} \quad (3)$$

where A_s is the section area at the separation point.

2.3 Acoustical modeling

We assumed that the two mass model is coupled with an upstream resonator and a downstream resonator. The section area of the resonators are fixed to S=2.5cm. The length L of the upstream resonator is fixed to12cm. The length L of the downstream is varying from 0 to 235cm. The oscillating constriction (vocal folds or trombonist lip's) is considered like an acoustical point source, due to its dimensions and the frequencies band of interest (50-2500Hz). Impedance at the entrance of the resonator is calculated in the following equation:

$$Z = p/u = Z_0 \frac{Z_L \cos(kL) + iZ_0 \sin(kL)}{iZ_L \sin(kL) + Z_0 \cos(kL)} \quad (4)$$

where p is the acoustic pressure, u is the acoustic volume flow rate, $Z_0 = \rho c / S$, c the speed of sound, $Z_L = \pi a^2 \rho c \left(1 - J_1(2ka)/(ka) + S_1(2ka)\right)$, J_1 is the first kind Bessel [4] function of order 1, and S_1 the Struve function of order 1 a the resonator radius, k=2Πf/(c.(1-1.65.10^{-3}/(a.sqrt(f))-3i.10^{-5}sqrt(f)/a, f the frequency.

The three first resonances of this impedance are extracted. It results in three equations of the following form:

$$\frac{\partial^2 \psi(t)}{\partial t^2} + \frac{\omega}{Q_A}\frac{\partial \psi(t)}{\partial t} + \omega_A^2 \psi(t) = \frac{Z_A \omega_A}{Q_A} u \quad (5)$$

$p = \partial\psi / \partial t$, ω_{ai} the resonance pulsation, Q_{ai} the quality factor of this resonance, Z_{ai} the amplitude, i the resonance number.

2.4 Stability and dynamical analysis

We use two methods for studying this theoretical model. First, a stability analysis can be performed [1, 6]. Equations are linearised around and equilibrium state. For a given set of control parameters (mechanical, aerodynamical and acoustical parameters), the eigenvalues of the system give access to two pertinent values in speech: upstream pressure threshold of oscillation and fundamental oscillation frequencies. Then, it is also interesting to observe the temporal evolution of the characteristic values of the system, and to compare it directly with measurements performed on the experimental set-up. The described set of equations needs to be discretized ([2]).

3 Experimental set-up

3.1 Description

As shown in Fig. 2, this experimental set-up is composed of a pressure reservoir ("the lungs") fed by a compressor.

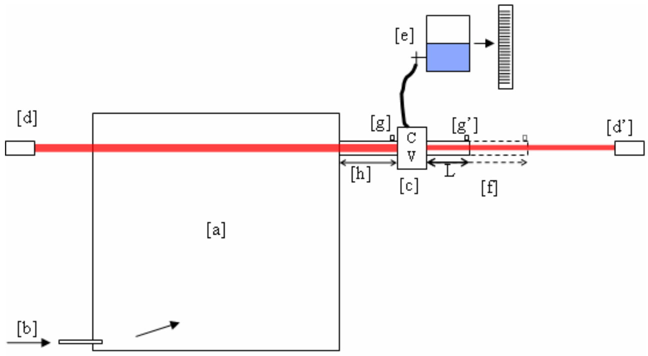

Fig.2 Experimental set-up. [a] Pressure reservoir (lungs). [b] Air supply. [c] Vocal fold/lips replica. [d][d'] Laser diode and photo sensor. [e] Water pressure reservoir. [f] Downstream resonator of variable length. [g][g'] Pressure sensors. [h] Upstream resonator of fixed length.

A uniform section pipe (the upstream resonator) of 12cm length is fixed on it. A deformable replica is fixed on this resonator. This replica (Fig. 3) is made of metal pieces covered with latex and filled with water under pressure. Water pressure is controlled by the reservoir [e].

Fig.3 Picture of the oscillating replica. Metallic pieces are covered by latex of 0.3mm thickness and filled with water under controlled pressure.

A uniform section pipe is then fixed on the replica (the downstream resonator). Its length is varying from 0cm (absence of resonator) to 235cm. Airflow is forced through the replica and, under certain conditions, self-sustained oscillations appear.

Thanks to pressure sensors, we measure the pressure upstream the replica, the pressure downstream the replica and the pressure at the end of the resonator. The variations of the constriction aperture are measured with an optical system. The measured data allow to extract the quantities which are predicted with the theoretical model, the oscillation threshold, fundamental frequency, dynamical evolution.

3.2 Link between theoretical model parameters and experimental replica

The relationship between the vocal fold replica and the theoretical model is not trivial for all the control parameters. Some of them, like the subglottal pressure, the initial geometry or the acoustical characteristics are directly connected to reality (upstream pressure of the deformable replica, aperture measured by the optical system, length and section of the resonators). Some parameters have to be estimated (masses, spring stiffnesses, damping constants). The vibrating mass is estimated as m_{cv}=m_{water}/2, the quantity of water contained in the latex vocal fold replica divided by a factor 2. Spring stiffnesses and damping constant are connected to the mechanical resonance of the replica as follows, $\omega_0 = \sqrt{2k/m}$, $Q_0 = m\omega_0/2r$ where ω_0 is the resonance pulsation and Q_0 the quality factor of this mechanical resonance.

The mechanical responses are measured as described by [6]. In this study, the water pressure in the replica is fixed to 2500Pa. We extracted three resonances from the mechanical response. 118Hz, 144Hz, 166Hz are the respective resonance frequencies, and 12, 10, 8 are the respective quality factors of the resonances. In addition to that, the initial measured aperture H_{10}=H_{20}=2mm.

4 Results and discussion

4.1 Oscillation pressure thresholds and fundamental frequencies

The length of the upstream resonator is fixed to 12cm. The length of the downstream resonator is varied from 0 to 235cm. For each downstream resonator, we measure the upstream pressure (P_{sub}) thresholds and the fundamental frequency of oscillation F_0. Fig. 4 shows the experimental data. Concerning the oscillation threshold, we can observe that in most of the case, the Onset pressure is higher than the Offset pressure. Such hysteresis phenomenon has been theoretically described by [3] in the context of voiced sound production.
We also notice that we can divide the length variation domain of the downstream into two band of interest. The first zone (the “vocal folds domain”) is situated before 40cm. In this domain, the acoustical Eigen frequencies of the resonator are far from the mechanical resonance frequencies of the replica. In this zone, the pressure threshold decreases as the resonator length increases. The oscillation fundamental frequency, which is closed to the mechanical resonance frequencies of the replica, does not seem to be affected by the length variation of the resonator. The second zone (“vibrating lips domain”) is situated after 40 cm. In this case, the acoustical Eigen frequencies are closed to the mechanical resonance frequencies. We observed that the oscillation frequency is strongly influenced by the resonators length.

A length variation from 40cm to 100cm implies an oscillation frequency variation from 153.5Hz to 117.5Hz (i.e. a decrease of 20%) and a pressure onset threshold increase from 350Pa to 1150Pa. These variations are correlated with the difference between the acoustical Eigen frequencies and the mechanical resonance frequencies.

If we increase the resonator length from 117Cm to 150cm, we observe a frequency step. At this point, the onset pressure threshold decreases and reaches a minimum value for a resonator length of 175cm. Finally, between L=213cm and 213cm, we find a second frequency step.

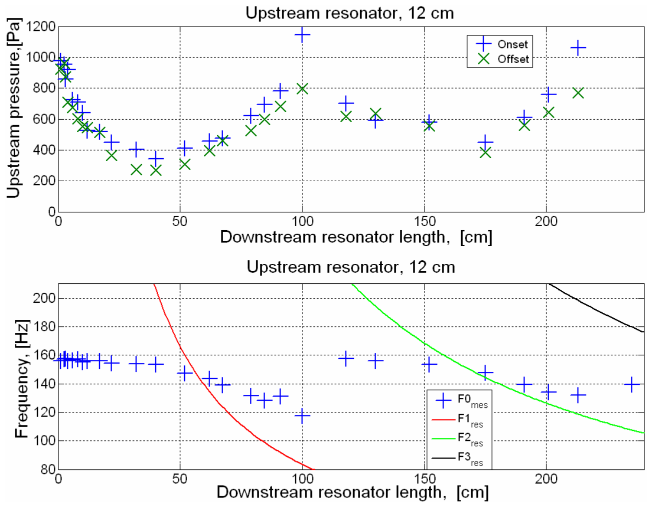

Fig.4 Experimental measurement of the downstream resonator influence. [a] Upstream pressure thresholds. Upstream pressure is increased until oscillations appear (+ Onset pressure). Then pressure is decreased until oscillations cease (x Offset pressure). [b] Fundamental frequency of oscillations. The continuous lines represent the theoretical resonance of the downstream resonator.

We have compared the theoretical prediction of the two mass model with these experimental data. The mechanical parameters (k, kc, r) have been estimated using the mechanical response of the replica. The initial measured aperture was H_0=2mm. The stability analysis is then realized for each of the three mechanical resonances and for a downstream resonator length varying from 0 to 235cm. The results of this comparison are depicted in Fig. 5. Theoretical predictions are qualitatively in agreement with the experimental data. Quantitatively, the oscillation frequencies are theoretically more influenced by the resonator that what we observed. Thus, for L>40cm, the predicted oscillation frequency is imposed by the resonator Eigen frequency.

The best agreement between theoretical predictions and experimental data is obtained with the third mechanical resonance of the replica, except for very small resonator length (L<4cm). In this last case, the second mechanical resonance gives better agreement. But the error rate is high since it can be up to 50% for the pressure threshold and up to 30% for the oscillation frequency.

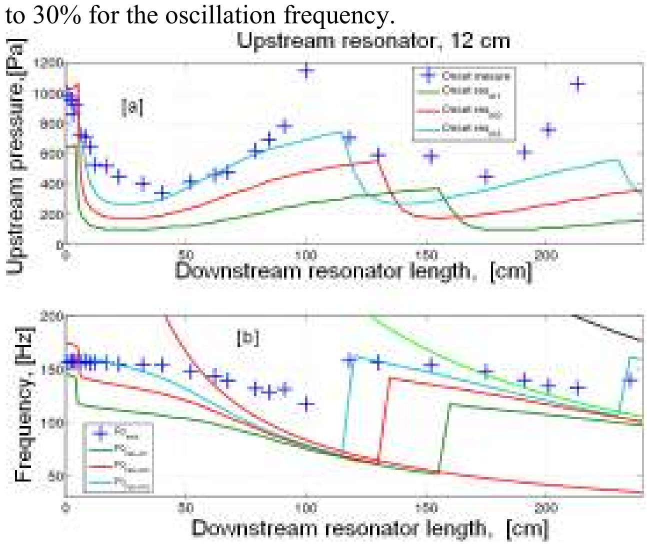

Fig.5 Comparison between theoretical prediction and experimental data, for an upstream resonator of 12cm length, and a downstream resonator of varying length (0 to

235cm). [a] Upstream pressure thresholds. [b] Fundamental frequency of oscillations.

4.2 Effect of a dynamical variation of the resonator length

We simulate the effect of a continuous length variation of the resonator on oscillation and on acoustical pressure radiated at the end of the resonator. Fig. 6 shows an example of such a simulation. Resonator length is continuously increased from 4 to 160cm during 7 seconds.

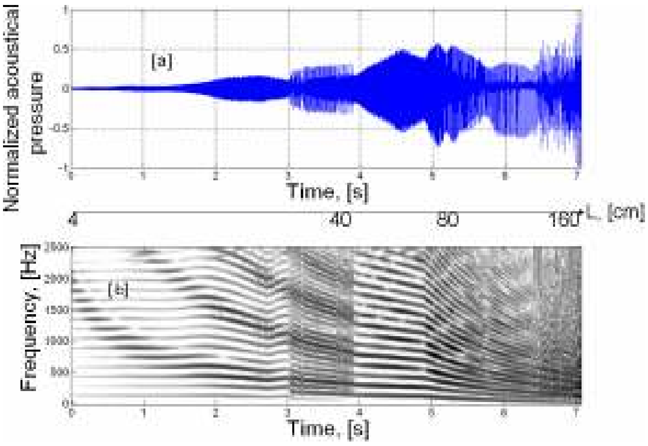

Fig.6 Dynamical simulation. The length of the downstream resonator is time-varying from 4 to 160cm. [a] Normalized acoustical pressure. [b] Spectrogram of the normalized acoustical pressure.

What we observed is consistent with the previous conclusions. For a resonator length lower than 40cm, the fundamental frequency of oscillation is not strongly influenced. Yet, we notice the amplitude of the acoustic pressure is continuously increasing. Then, for resonator length higher than 40cm, we first observe a bifurcation phenomenon, which can be correlated with the step frequency observed in Fig.5. After this bifurcation, the fundamental frequency is strongly influenced, as we have seen previously, by the resonator length.

5 Conclusion

We have presented an experimental and theoretical study of the influence of the acoustical coupling on oscillation pressure thresholds, fundamental frequencies and dynamical behavior of vocal folds or brass player lip's. We have shown that the resonator length variation domain can be divided into two band of interest: first zone for L<40cm, second zone for L>40cm. In the first zone, the resonator length is close to human vocal tract length. The resonator doesn't influence the oscillation frequency. If its length increases, oscillations are easier to produce (decrease of the onset pressure threshold). In the second zone, the resonator has a strong influence on the oscillation frequency as well as on the pressure threshold. These conclusion are in accordance with the one of [5]. The effect of acoustical coupling is significant when the mechanical resonance frequencies are close to the Eigen frequencies of acoustical the resonator.

The use of the two mass model to predict the experimental data shows its ability to reproduce qualitatively what observed during the measurements. Quantitatively, the theoretical predictions are not good. Moreover, different mechanical parameters (extracted from different mechanical resonances) have to be used. Thus, the two mass model and its limited number of mechanical modes is insufficient to predict the effect of a large variation of the resonator length.
Finally, numerical simulation of a continuous resonator length variation shows theoretical the presence of bifurcation phenomenon which can be linked to frequency step observed experimentally.

References

[1] J. Gilbert, S. Ponthus, J.F. Petiot, "Artificial buzzing lips and brass instruments: Experimental results", *J. Acoust. Soc. Am.* 104, 1627-1632 (1998)

[2] N.J.C. Lous, G.C.J. Hofmans, R.N.J. Veldhuis, A. Hirschberg, "A symmetrical two-mass vocal-fold model coupled to vocal tract and trachea, with application to prosthesis design", *Acta Acustica* 84, 1135-1150 (1998)

[3] J.C. Lucero, "A theoretical study of the hysteresis phenomenon at vocal fold oscillation onset-offset", *J. Acoust. Soc. Am.* 105, 423-431 (1999)

[4] P.M. Morse, K. Uno Ingard, "Theoretical acoustics", Mc Graw-Hil Book Company 1 vol. New York, 383-388 (1968)

[5] M. Rothenberg, "Acoustic interaction between the glottal source and the vocal tract", *Vocal Fold Physiology*, University of Tokyo Press, 305-328 (1980)

[6] N. Ruty, X. Pelorson, A. Van Hirtum, I. Lopez Arteaga, A. Hirschberg, "An in-vitro setup to test the relevance and the accuracy of low-order vocal folds models", *J. Acoust. Soc. Am.* 121, 479-490 (2007)