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Guided waves in plates with linear variation of thickness

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Guided waves represent promising non-destructive evaluation (NDE) techniques. Their advantage of long distance propagation is however hampered by complex wave scattering at each discontinuity along the investigated structure. These scattered waves can be used to locate and size possible defects. Theoretical investigation is a prerequisite step in the development of a NDE technique. Planar metallic structures with localized thickness variation imposed by design represent possible new industrial structures that could be tested using guided waves. The waves scattered at the region of variable thickness must be theoretically predicted and experimentally measured before any defect could be detected. Very few authors have investigated waveguides with continuous variation of thickness. Our previous numerical and experimental studies have proven the complexity of the scattering phenomenon in such cases. Among other remarks is the curved shape of the wave front.

The present work investigates properties of the waves in the linear thickness variation region of the waveguide. A plain strain model is theoretically investigated and numerically tested using specialized finite elements based algorithms. Specific properties such as stress and displacements fields are determined by numerical simulation at several frequencies and slope angles.

1 Introduction

The guided waves propagate in elastic structures for relatively long distances and are scattered by defects along this path. This main advantage has led to many scientific works during the last decades. The extended use of numerical computations using the fast increasing computer capabilities, has offered many solutions to particular problems concerning guided waves. Dispersion curves for the most general case of guided waves of any cross-section constant along the waveguide, made of any anisotropic, viscoelastic materials are now available [1], [2]. The scattering of any incident guided mode can be theoretically and numerically investigated and modal amplitudes, modal energy and surface displacements can be obtained with good accuracy [3]. Another problem related to waveguides is the wave propagation along variable cross-section regions. Technical applications are plates with variable thickness used in aeronautical and automotive industries, the Gaussian shape of the welding seam between two plates, etc.

A fluid layer of variable thickness is investigated in [4], using a coupled mode theory which allowed a new intrinsic mode of the wedge to be deduced. Another study of cylindrical waveguides of variable radius and/or wall thickness and material properties is presented in [5]. Experiments doubled by numerical simulations on wave propagation in an aluminum wedge are presented in [6] and interesting phenomena such as the presence of critical thickness corresponding to modal cut-offs are deduced. Guides waves have also been investigated recently [7] for plates with one plane side and the other side having a Gaussian variation of thickness. The global geometry in this case is not symmetric about the middle plane of the plate. As expected, experiments and numerical predictions indicate a modal conversion between the symmetrical and anti-symmetrical modes at ratios depending on the local thickness. Higher slope on the Gaussian side involves rapid conversion to the Lamb modes at the local frequency*thickness value. This so-called adiabatic behavior of modes is not easily determined, because the conversion is continuous and classical experimental techniques require a certain distance for modal identification.

The present study aims a more in-depth study of the adiabatic character of guided waves. In order to simplify the approach, the geometry has a plane of symmetry which includes the wave vector. This configuration avoids modal conversion between symmetric and anti-symmetric modes. Only the local thickness of the waveguide, depending on the slope will influence the modal conversion.

2 Theoretical aspects

A plane strain model is used in the following and the material is assumed to be isotropic and homogeneous of mass density ρ and elasticity coefficients C_{11}, C_{66}, ($C_{12}=C_{11}-2C_{66}$).

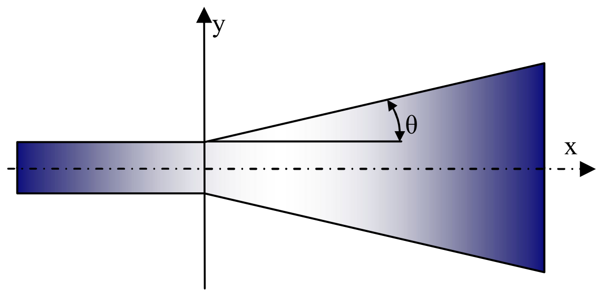

Fig. 1 Geometrical configuration of the tapered plate

The displacements field has two components in this model. Assuming harmonic waves of angular frequency ω, the displacements functions become:

$$\begin{bmatrix} u(x,y,t) \\ v(x,y,t) \end{bmatrix} = \begin{bmatrix} u(x,y) \\ v(x,y) \end{bmatrix} \exp(-i\omega t) = \bar{U} \exp(-i\omega t) \tag{1}$$

The linear elasticity relations between strains and stresses are written as:

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 \\ C_{21} & C_{22} & 0 \\ 0 & 0 & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_6 \end{bmatrix} \tag{2}$$

in which

$$\varepsilon_1 = \varepsilon_{11} = \frac{\partial u}{\partial x}; \quad \varepsilon_2 = \varepsilon_{22} = \frac{\partial v}{\partial y}; \quad \varepsilon_6 = \varepsilon_{12} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}.$$

The differential equations of motion

$$\frac{\partial}{\partial x}\left[C_{11}\frac{\partial u}{\partial x}\right]+\frac{\partial}{\partial y}\left[C_{66}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right]=\rho\ddot{u}$$
$$\frac{\partial}{\partial x}\left[C_{66}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right]+\frac{\partial}{\partial y}\left[C_{22}\frac{\partial v}{\partial y}\right]=\rho\ddot{v} \quad (3)$$

can be cast into matrix form as:

$$a\overline{U}-\nabla\cdot\left(c\nabla\overline{U}\right)=\overline{0} \quad (4)$$

with

$$[a]=\begin{bmatrix}-\rho\omega^2 & 0\\ 0 & -\rho\omega^2\end{bmatrix};$$
$$[c]=\begin{bmatrix}\begin{bmatrix}C_{11} & 0\\ 0 & C_{66}\end{bmatrix} & \begin{bmatrix}0 & C_{12}\\ C_{66} & 0\end{bmatrix}\\ \begin{bmatrix}0 & C_{66}\\ C_{21} & 0\end{bmatrix} & \begin{bmatrix}C_{66} & 0\\ 0 & C_{22}\end{bmatrix}\end{bmatrix} \quad (5)$$

The solution of the harmonic wave propagation can be obtained using the finite elements method and wave absorbing regions as presented in [8]. Pure Lamb waves such as S_0 are generated in the x<0 region by imposing on the two parallel faces, appropriate functions of x for several wavelengths. Taking advantage of the constant thickness, for a given frequency, the corresponding wavelengths are obtained from the dispersion curves. The elastic coefficients are real values in the numerical model, except for short regions at the two extremities along the *x* coordinate of the model. In these regions, imaginary parts of the elastic coefficients are added, increasing their values with the third power of distance from the elastic domain, such that no reflections take place from the model vertical boundaries (x=const). The displacements along the y=0 line, as obtained from the finite element simulation are extracted and used to determine the characteristics of the adiabatic modes. If S_0 is the incident mode, then *u(x,0)* displacements provide information such as the successive positions at which *u(x,0)=0*, which are denoted by x_1, x_2, …, x_n. The distance x_{i+1}- x_i is thus considered as the local half-wavelength $\lambda(x_i)/2$, which can be converted to local wavenumbers $k(x_i)$. Due to the geometric symmetry of the structure, no modal conversion can occur. This method remains to be compared with the sliding window Fourier transforms.

Several slope angles have been chosen for the numerical tests. In the following, only S_0 mode at a single frequency is assumed incident into the wedge region. The local wavenumbers are compared in non-dimensional form (*k.h*, in which *h* is the local distance between the free surfaces at a given *x*) versus the *f.h* value at the position *x*.

3 Numerical results

A 2 mm thick aluminium plate (c_L= 6380 m/s, c_T= 3100 m/s, ρ=2700 kg/m^3) is submitted to a harmonic excitation at 800 kHz. The excitation is produced as normal pressure with *exp(kx)* dependency, in which *k* is the wavenumber of the S_0 mode in the 2 mm plate, modulated by a Gaussian envelope along 10mm, and centred at 50mm from the wedge angle. An example of the harmonic solution is presented in Fig. 2 as *u(x,y)* displacements on the cross-section of the tapered plate. As the wave enters the variable thickness region, its pattern changes and the lines constant displacements become curved at some x positions. Gradually the displacements become confined to the free surfaces of the wedge, as expected from the S_0 mode at high frequency·thickness (fh) products.

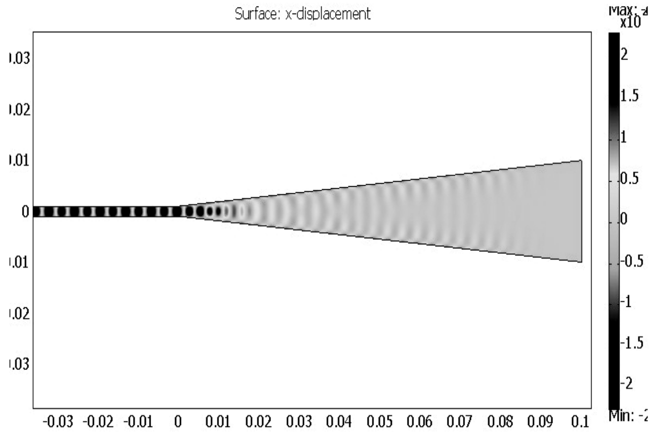

Fig. 2 Longitudinal displacements for the cross-section

The *u(x)* displacement along the y=0 line, extracted from this finite elements solution is presented in Fig. 3. Its harmonic pattern in -60mm to 0mm region can be used to confirm by Fourier transform, the presence of a pure S_0 mode. For x<-60 mm and x>70mm the wave is damped so that no reflected waves come from those model boundaries. Several slope angles and lengths of free propagation have been tested.

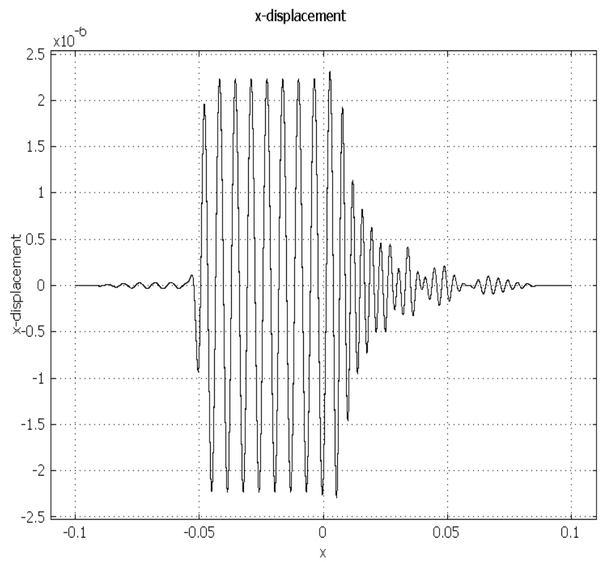

Fig. 3 Longitudinal displacements along the center line of the cross-section for a 3.72° wedge

A very good agreement between the local wavenumbers obtained as presented in the previous paragraph and those of the S_0 Lamb mode at the same thickness has been obtained for a slope angles less than θ=3°. The comparison for θ=1.53° is presented in Fig. 4 for fh less than 6.5 MHz·mm. The adaptation of the S_0 mode to the local thickness of the waveguide in this case, is the reason for which, some authors call such a mode as adiabatic.

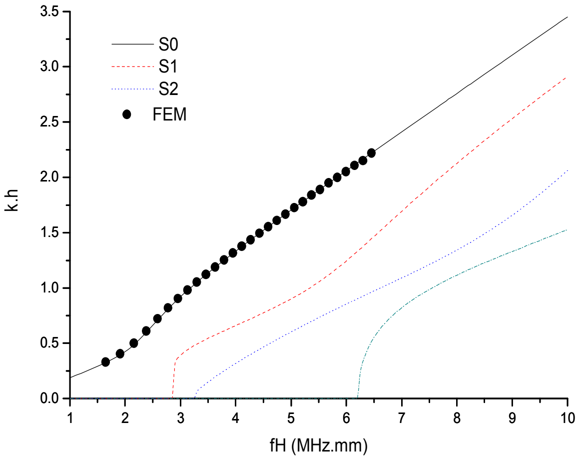

Fig. 4 Wavenumbers (FEM) vs. frequency for a 1.53º wedge compared to dispersion curves (S_0, S_1, S_2)

As the wedge angle has been increased, some discrepancies have been noticed. For one tested slope angle θ=3.72º (Fig. 5), above f.h=5 MHz·mm, three narrow peaks at f.h=5.6, 6.6 and respectively at 7.9 MHz·mm have been obtained and between these peaks, the S_0 curve is no longer followed. Even more important variations are obtained at higher angles. For example, a wedge slope angle θ=5.14º has led to more ample variation indicated in (Fig. 6). These angles correspond to different selected maximum thicknesses and are not critical values for a specific phenomenon. In fact, for angles above 3º, such phenomena occur.

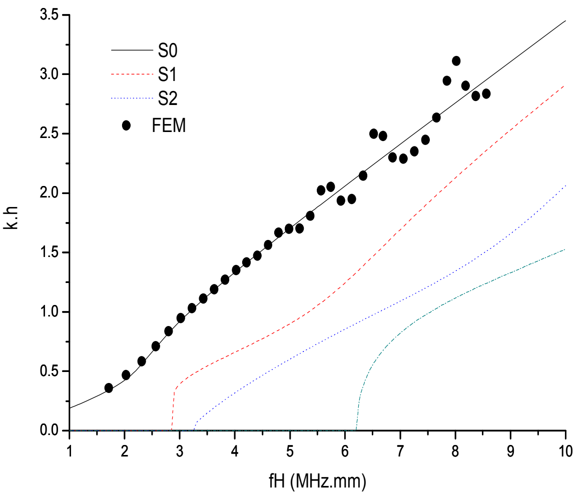

Fig. 5 Wavenumbers (FEM) vs. frequency for a 3.72º wedge compared to dispersion curves (S_0, S_1, S_2)

Local peaks at approximately f.h=5.7 and 7.2 MHz·mm have been obtained. Since the mesh size used in all these cases is almost identical and it has been proven to predict well the wave propagation up to f.h=6.5 MHz·mm, finite element mesh error has been discarded. It remains as possible explanation, the conversion of the incident mode S_0 to other modes S_1, S_2 and S_3, specific to this type of waveguide. However, these peaks do not occur at the cut-off frequencies of these modes, as can be seen from a comparison of Fig. 4 with Fig. 5 or Fig. 6. The modal identification procedure based on longitudinal displacements is inappropriate in distinguishing individual modes in the case of multiple modes existing simultaneously.

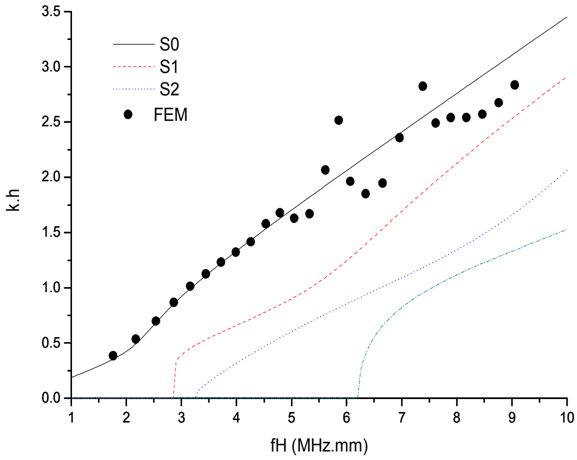

Fig. 6 Wavenumbers (FEM) vs. frequency for a 5.14º wedge compared to dispersion curves (S_0, S_1, S_2)

Another remark concerns the Fourier transform (FFT) of the longitudinal displacements as functions of the x coordinate. The wavenumbers obtained from the signal in Fig. 3 using a rectangular window for 0<x<0.6 is presented in Fig. 7. There are three wavenumbers, indicating the possible presence of modal scattering, which agrees with the previous conclusions. However, the values are inaccurate, representing averaged values of the wavenumbers, far from the obtained variation of wavenumbers with the position on the Ox axis.

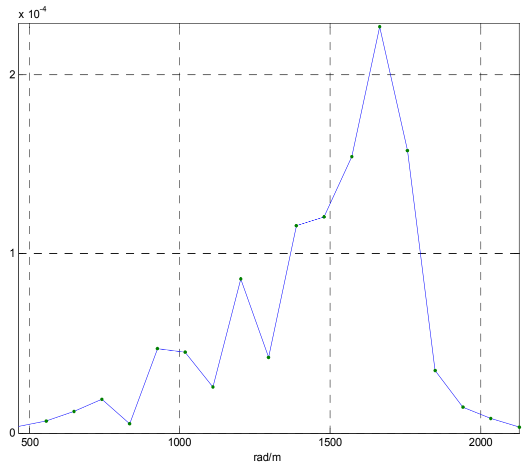

Fig. 7 Fourier transform of the longitudinal displacements at 0<x<0.6, y=0, for a 5.14º wedge

This fact indicates that classical signal processing techniques are less appropriate in this case of linear thickness variation. One alternative is to use the so-called "sliding window" which includes in the FFT only a selected short sub-domain of the signal. The whole signal is divided in such short windows and the obtained results are grouped as parametric solutions. Even these shorter signal windows, which are producing more accurate results by FFT, are not as accurate as the above procedure in the single mode regions, but represent a reasonable alternative for small slopes.

4 Conclusion

The usual approach used to describe the propagation of guided waves in tapered plates is that of adiabatic modes. The wavenumbers are in fact continuously variable with the position along the wedge symmetry plane. A simple method has been used to determine the wavenumbers, based on the longitudinal displacements in the symmetry plane. The adiabatic modes hypothesis has been numerically tested for frequency·thickness product up to 6.5 MHz·mm and proved to be valid for small slope angles, less than approx. 3º without modal conversion. At these angles, the wavenumbers at every tested position along the symmetry plane coincide with the wavenumbers of the S_0 Lamb mode in a plate having the same thickness as the local thickness of the wedge.
For higher values of the slope angles, local discrepancies occur in the form of wavenumbers peaks, which are indicating modal conversion. The continuously variable signal remains a problem for future development of new signal processing techniques and more detailed investigation of modal conversion.

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