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Structure / cavity coupling using Statistical Energy Analysis : Coupling Loss Factors and energy maps into subsystems

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Prediction of interior noise is one of the most concerning issues of vehicle industry. Statistical Energy Analysis theoretically allows to determine energy spread over a structure divided into subsystems when one subsystem is submitted to a rain-on-the-roof excitation. Subsystems can be either a part of the structure or a cavity.
Recently, a method (SmEdA) based on modal bases of uncoupled subsystems have been derived. This method allows to compute CLF using standard FEM software. This method has been successfully applied on structure/structure coupling and is extended to estimation of CLF between a structure and a cavity in the present article.
In addition, in the case of interior noise, SEA can only provide a global energy into the cavity on frequency bands. No information on energy repartition into the subsystem is given. In the present article, an extension of SmEdA method is proposed to quickly estimate energy repartition into subsystems (structure or cavity).

1 Introduction

Statistical Energy Analysis is a simple method to predict energy spread over a structure. Indeed, it is based on power balance between parts of the structure called subsystems. A SEA model only depends on coupling loss factors (CLF) between subsystems, damping loss factors (DLF) of subsystems and power injected into subsystems. In addition, CLF are theoretically independent of DLF which permits to modify damping of subsystems without changing the whole model. Thus SEA method could be a tempting method for early design stage.

However, even if SEA is particularly well adapted for some applications, it becomes difficult to apply SEA to industrial structures like cars, train or planes. Indeed, there are several problems which must be addressed to establish a real SEA model : definition of SEA subsystems, CLF estimation and verification of basic SEA relations. Mace [1] distinguish "proper" and "quasi" SEA models.

First of all definition of subsystems of a real industrial structure is not obvious. However, because of the lack of methods, subsystems are usually defined arbitrarily. In that case, there is no insurance that these subsystems are SEA subsystems. To overcome this issue, Totaro et al. [2] proposed a substructuring method based on analysis and classification of energy transfer functions.

Then, assuming that subsystems are well defined, it is necessary to estimate coupling loss factors (CLF) between them. Very few methods exist to compute CLF between subsystems. Lyon [3] proposed asymptotic expressions for some kind of couplings (point, line coupling). The Power Injected Method (PIM), firstly introduced to estimate CLF experimentally, is now used numerically with Finite Elements (FE) software [4].

Maxit et al. [5] have proposed an original approach to compute CLF using FE data. This method based on a dual modal formulation permits, not only to compute CLF, but also to extend SEA to subsystems with low modal overlap. Indeed, as the SmEdA (Statistical modal Energy distribution Analysis) method is based on modal energies, one of the constraining assumptions of SEA (modal energy equipartition) is no more needed.

The present paper deals with the SmEdA method. It demonstrates that SmEdA method can be used to compute CLF between a structure and a cavity. In addition, it proposes an extension of SmEdA approach : the estimation of energy distribution into subsystems.

2 Structure / cavity coupling loss factors

The SmEdA approach has been successfully applied to estimate CLF between two structures. In this paper, the SmEdA approach is used to compute coupling loss factor between a structure and a cavity.

2.1 SmEdA approach

The principle of SmEdA approach is shown in Figure 1. If two subsystems are coupled, Coupling Loss Factors can be computed using modal bases of uncoupled subsystems. In that case, one of the subsystems has to be blocked on the interface (stress mode shapes) and the other one has to be free (displacement mode shapes).

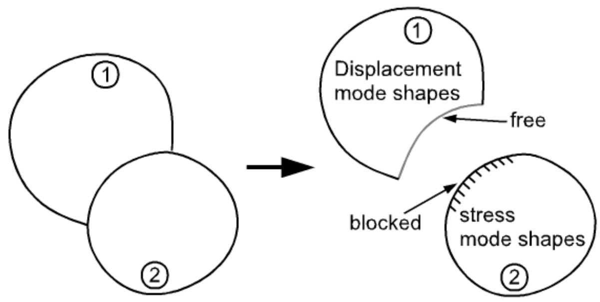

Figure 1 : principle of SmEdA approach

Then, modal coupling loss factors (coupling loss factors between one mode p of subsystem 1 and one mode q of subsystem 2) are obtained using equation 1.

$$\beta_{pq}^{12} = \frac{\left(\mathbf{W}_{pq}^{12}\right)^2}{M_p^1 M_q^2 (\omega_q^2)^2}\left[\frac{\eta_p^1\omega_p^1(\omega_q^2)^2 + \eta_q^2\omega_q^2(\omega_p^1)^2}{\left((\omega_p^1)^2 - (\omega_q^2)^2\right)^2 + \left(\eta_p^1\omega_p^1 + \eta_q^2\omega_q^2\right)\left(\eta_p^1\omega_p^1(\omega_q^2)^2 + \eta_q^2\omega_q^2(\omega_p^1)^2\right)}\right] \quad (1)$$

Where, ω_p^1 and ω_q^2 are the angular frequencies of, respectively, mode p of subsystem 1 and mode q of subsystem 2. M_p^1, M_q^2, η_p^1 and η_q^2 are corresponding modal mass and modal damping. W_{pq}^{12} is the modal work between modes p and q.

This formulation permits to express SEA in terms of modal energies rather than global energies. Finally, standard SEA CLF can be obtained using modal coupling loss factors.

In the case of a structure coupled to a cavity, it can be proven that equation 1 holds [6]. However, the definition of blocked and free subsystems is easier. It is obvious that the structure has to be free on the coupling surface (displacement mode shapes) whereas the cavity has to be blocked (rigid wall condition, pressure mode shapes).

Thus, SmEdA approach permits to compute CLF loss factors whatever structure or cavity complexity using standard FE software.

2.2 Analytical validation

To validate the SmEdA approach in the case of structure/cavity coupling, a simple analytical test case has been used. As presented in figure 2, a rectangular plate is coupled to a box via a coupling surface.

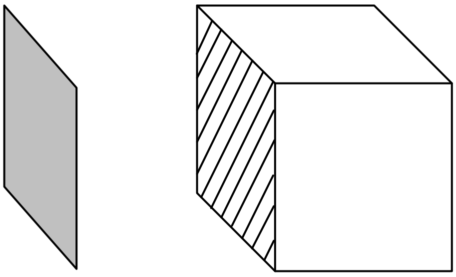

Figure 2 : analytic test case : a rectangular plate coupled to a cavity via a coupling surface (hatched surface)

In that case, displacement mode shapes of the plate and pressure mode shapes of the cavity are known. It is then possible to compute CLF between the plate and the cavity. Results are presented in figures 3 and 4. SmEdA results are compared to Lyon's relation [3] and to analytic resolution of coupled problem (simulating Rain-on-the-roof excitations by several uncorrelated point forces).

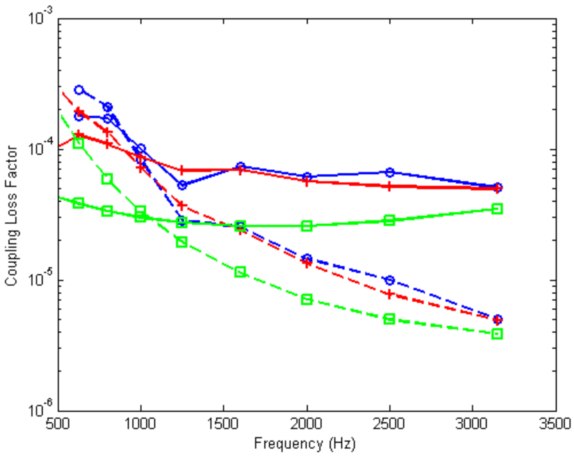

Figure 3 : Coupling Loss Factors between a plate (length=1m , width=0.8m ,thickness=1mm, young modulus=210GPa, Poisson's coefficient=0.3, density=7820kg/m^3) and a cavity (length=1m, width=0.8m, depth=0.9m, air). Solid line : structure to cavity coupling; dashed line : cavity to structure coupling. Blue circles : analytic resolution; red cross : SmEdA approach; green squares : Lyon's relation.

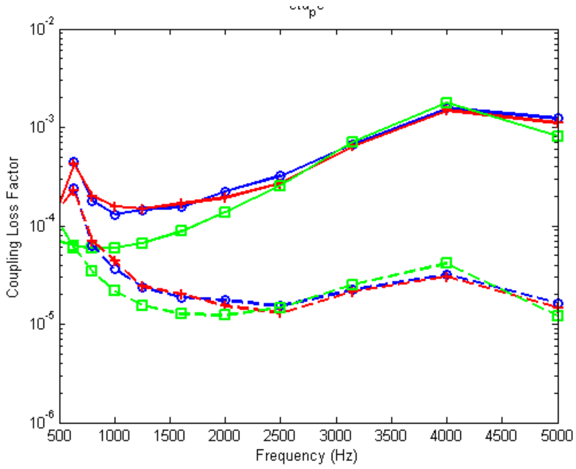

Figure 4 : Coupling Loss Factors between a plate (length=1m, width=0.8m, thickness=3mm, young modulus=210GPa, Poisson's coefficient=0.3, density=7820kg/m^3) and a cavity (length=1m, width=0.8m, depth=0.9m, air). Solid line : structure to cavity coupling; dashed line : cavity to structure coupling. Blue circles : analytic resolution; red cross : SmEdA approach; green squares : Lyon's relation.

As demonstrated in figure 3 and 4, SmEdA approach compared really well with analytic resolution of the coupled problem. It gives better estimation of CLF than Lyon's relation particularly below critical frequency of the plate.

2.3 Numerical test case

The main advantage of SmEdA approach is the possibility of using FE software to compute modal bases of uncoupled subsystems.

The present numerical test case has been used to demonstrate that such calculations are possible and give good results. Figure 5 presents meshes (structure and cavity) of the problem. These meshes can be incompatible.

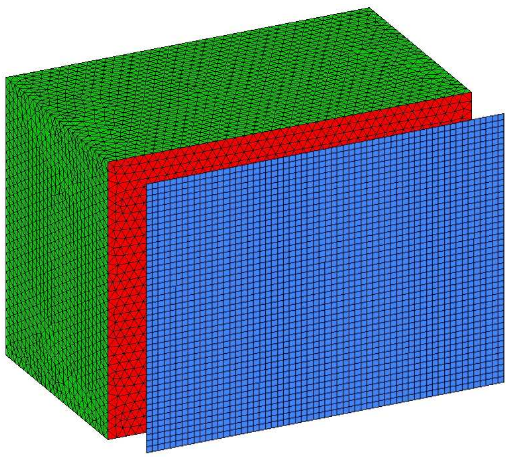

Figure 5 : meshes of the numerical test case.

Results are presented in figure 6. Numerical computations using FE data compares well with analytical calculations.

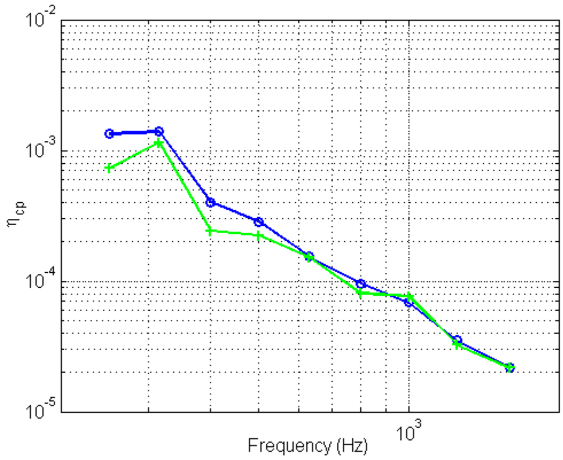

Figure 6 : Coupling Loss factors. Blue circles : analytical calculations; Green cross : numerical computations using FE software.

3 Energy distribution into subsystems

3.1 Modal energies of subsystems

SmEdA approach deals with modal energies of subsystems. Modal coupling loss factors and modal damping loss factors link modal injected powers to modal energies (equation 2).

$$\Pi_p^1 = \eta_p^1 \omega_p^1 E_p^1 + \sum_{q=1}^{q_{\max}} \beta_{pq}^{12} \left(E_p^1 - E_q^2 \right) \quad (2)$$

Where E_p^1 and E_q^2 are modal energies between mode p of subsystem 1 and mode q of subsystem 2. Π_p^1 is the power injected into mode p of subsystem 1. The systems of linear equations is similar to the SEA one.

Knowing modal coupling loss factors β_{pq}^{12} and modal injected powers, it is possible to deduced modal energies of subsystems.

In the present paper, a method is proposed to estimate energy distribution into SEA subsystems. This information can not be given by classical SEA. However, it could give useful information for the early design stage of cars for example.

3.2 Energy distribution into subsystems

The following method is voluntarily based on simplifying assumptions. The aim of the method is not to give an exact energy distribution into subsystems but to give a qualitative information. In that case, post-process of SmEdA approach is almost instantaneous.

The local energy into one subsystem over a frequency band $\Delta\omega$ is given by equation 3.

$$e(M, \Delta\omega) = \sum_r \sum_s e_{rs}(M, \Delta\omega) \quad (3)$$

Where $e_{rs}(M, \Delta\omega)$ is the modal interaction energy between two modes r and s of the subsystem. The frequency average over a frequency band $\Delta\omega$ is defined by $\langle \bullet \rangle_{\Delta\omega} = \frac{1}{\Delta\omega} \int_{\Delta\omega} \bullet \, d\omega$.

The assumption of the method is to assumed that off-diagonal terms of the sum in equation 3 can be neglected. In that case, equation 3 can be written as :

$$e(M, \Delta\omega) = \sum_r e_{rr}(M, \Delta\omega) \quad (4)$$

Diagonal terms can be then written as :

$$e_{rr}(M, \Delta\omega) = \xi \left\langle \left| a_r(\omega) \right|^2 \right\rangle_{\Delta\omega} \phi_r^2(M) \quad (5)$$

Where ξ is a mass operator depending on subsystem (structure or cavity), $a_r(\omega)$ is the modal amplitude and $\phi_r(M)$ is the mode shape.

Integrating equation 5 on the domain D of the subsystem and using the property of orthogonality of modes of the subsystems, one can write equation 6.

$$\int_D e_{rr}(M, \Delta\omega) dM = E_r = \xi \left\langle \left| a_r(\omega) \right|^2 \right\rangle_{\Delta\omega} N_r \quad (6)$$

Where N_r is the norm of mode r. E_r is the modal energy defined by SmEdA approach.

Then, using equation 6, it is possible to write :

$$\left\langle \left| a_r(\omega) \right|^2 \right\rangle_{\Delta\omega} = \frac{E_r}{\xi N_r} \quad (7)$$

Finally, local energy at point M over the frequency band $\Delta\omega$ is given by equation 8.

$$e(M, \Delta\omega) = \sum_r \frac{E_r}{N_r} \phi_r^2(M) \quad (8)$$

This expression is very simple and can be applied whatever the subsystem (cavity or structure) provided that mode shapes and modal energies are known. These quantities are used or given by SmEdA approach. This method is thus a direct extension of SmEdA approach.

3.3 Analytical test cases

In the present paper, an analytical test case is presented. A plate, excited by a point force (random noise) radiates into a cavity. Modal coupling loss factors have been computed using SmEdA approach. Modal energies of subsystems (plate and cavity) are computed using the system of linear equations (2).

Equation (8) has then be used to compute energy distribution into the cavity. The exact energy maps are presented in figure 7. Energy maps obtained with SmEdA approach are presented in figure 8.

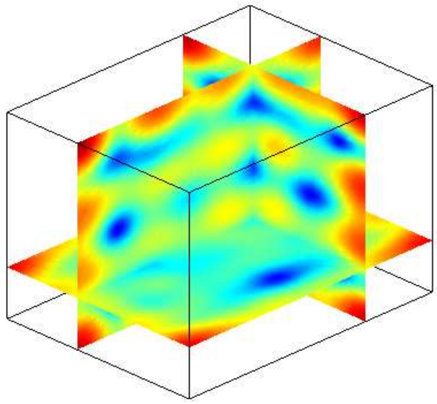

Figure 7 : energy maps into a cavity coupled to a plate excited by a point force. Frequency band 600-800Hz. Exact calculation.

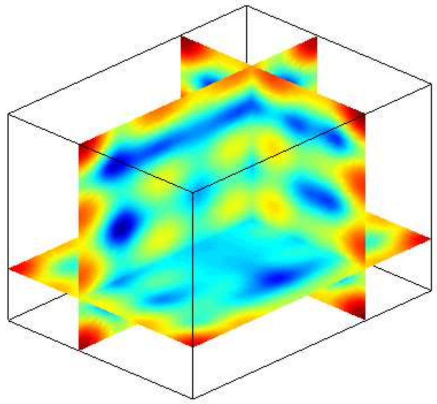

Figure 8 : energy maps into a cavity coupled to a plate excited by a point force. Frequency band 600-800Hz. SmEdA approach.

As it can be seen in figures 7 and 8, SmEdA approach gives a good estimation of energy distribution in the cavity. It is then possible to localize zones with high energy density.

4 Conclusion

SmEdA approach is used to compute Coupling Loss Factors between a plate and a cavity. It is demonstrated that this method compares well with exact resolution of the coupled problem. The main advantage of SmEdA approach is the possibility to use Finite Elements data (mode shapes of uncoupled subsystems).

In addition, SmEdA approach deals with modal energy of subsystems rather than global energies.

These modal energies are used, in the paper, to estimate energy distribution into subsystems (structure or cavity). This method, based on a simplifying assumption, gives good results and can give useful information in early design stage of a car for example.

Acknowledgments

Application of SmEdA approach on industrial applications would not have been possible without advanced post-processing functions developed by Free Field Technologies in the framework of ACTRAN/HF software. These powerful functions allows to treat really complex structures.

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